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Describe Quantum Mechanics in Dual 4 d Complex Space-Time and the Ontological Basis of Wave Function

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Abstract

Micro-object is both particle and wave, so the traditional Particle Model (mass point model) is actually not applicable for it. Here to describe its motion, we expand the definition of time and space and pick up the spatial degrees of freedom hidden by particle model. We say that micro-object is like a rolling field-matter-ball, which has four degrees of freedom including one surface curvature degree and three mapping degrees in the three-dimensional phenomenal space. All the degrees are described by four curvature coordinate components, namely "k_1, k_2, k_3, k_4", which form the imaginary part of a complex phase space, respectively. While as to the real part, we use "x_1, x_2, x_3, x_4" to describe the micro object's position in our real space. Consequently, we build a Dual 4-dimensional complex phase space whose imaginary part is 4-dimension k space and real part is 4-dimension x space to describe the micro-object's motion. Furthermore, we say that wave function can describe the information of a field-matter-ball's rotation & motion and also matter-wave can spread the information of micro-object's spatial structure & density distribution. Matter-wave and probability-wave can transform to each other though matter-wave is a physical wave. The non-point property is the foundational source of the probability in Quantum Mechanics.

Keywords

Field-Matter, Matter-Wave, Probability, Curvature, Curvature Coordinate

1. Introduction

De Broglie came up with his assumption that each object has its wave-like characteristics in 1923-1924 by making an analogy of the wave-particle duality of light, and it was later confirmed by experiments. To explain the wave-particle duality of a micro-object, Schrodinger supposed the motion of a micro-object is like the oscillation of "Elastic Fluid Limited in a Shell". Using the solution of pressure equation of "Elastic Fluid", we can get the wave function of a kinetic electron. Then putting it into the pressure equation of "Elastic Fluid", we will get the Schrodinger wave equation [1]. Schrodinger thought the wave-like nature is more basic than the particle-like nature for a micro-object, and particle is a wave packet composed of waves. The wave function squared is the weight of partition function of the electric charge *e*. Schrodinger's assumption was gave up by physicists because of its difficulty in multi-dimensions space and the diffusion of wave packet.

De Broglie assumption: a micro-object corresponds to a wave (vibration) [2]. This model is a mass point model, too. The motion of a mass point corresponds to a matter wave (phase wave) [3]. It can be described that particle is "riding on a wave" and lead by wave to go forward. Later, De Broglie persisted in the "Dual Solution Theory", namely, the nonlinear solution was in the center and formed the particle, while the plane wave solution was in the outer and guided the particle. However, it's difficult to understand "the motion of a mass point corresponds to a matter wave (phase wave)" and is too complex to get the nonlinear solution in math. So the De Broglie assumption was not accepted widely.

Then the Copenhagen School gave the uncertainly principle that position and momentum cannot be determined at the same time [4]. A single particle has its volatility, wave-function is a probability-function and the wave function squared equals to the distribution probability. The Copenhagen School is the mainstream thought now, though it has suffered the arguments between the Determinism and Indeterminism, Locality and Non-locality, Realism and Anti-realism for many years until this century [4].

To explain the determinism fact beyond the probability function, Bohm thought out the "Hidden Variable Theory". He reformulated the Schrodinger wave equation and loaded it with new significance according with the classical Hamilton Jacobi theory while avoiding the "Von Neumann Argument" [4]. Bohm thought the micro-particle should be treated as a real particle with continuous movement. It can be affected by the classical potential **U** as well as the quantum potential **Q** which is related to the wave function and determined by the solution of Schrodinger Equation [5]. Quantum Potential is the main difference between classical theory and quantum theory. Though Bohm had received many agreements, he failed to challenge the predominant Copenhagen School in Quantum Mechanics because of the unclear of the quantum potential in physics.

Many-worlds interpretation: different measurements correspond to different world. We step into a different parallel world when we measure it once, twice and so on. The world we are in now is the result we have stepped into before. World has infinitely many sliders. However, we can only in one at a time. The real world can never collapse and the real collapse is the many worlds in our mind collapse into the real world we are in. In fact, the Many-worlds interpretation cannot eliminate the collapse, it just shift the place of collapse. So it is more like a category in philosophy.

Sakata Shyoichi (Japanese), yukawa hideki (Japanese) and Tohm (French) both think the main difficulty in Quantum Mechanics comes from the unreasonable mass point model [5] [6]. String theory is a beautiful non-point theory which has satisfying results in the combination of general relativity and quantum mechanics. But it is really an unverifiable conjecture that string is the basic cell for the whole world. What we need is verifiable. Considering of the success in combination of unifying gravity and quantum mechanics, the Ring theory is another wonderful theory who can match the String theory. However, it is even more difficult to be understood because of its nets and conjunctions are really hard.

The fowling rotating flied-matter-ball model may be able to solve the difficulties in Quantum Mechanics. For example, we can know the ontological foundations of the matter-wave by deducing the wave function [7]. Field-matter is the basic form of matter. The structure of field-matter-ball consists of the rotation radius, frequency, and matter density distribution. We argue the field-matter-ball model is a proper model for micro-object and can satisfy the wave-particle dualism at the same time. But it must be pointed that the matter-field-ball is not the same as the classical ball we see in our world, and it is a "quantum ball" which has its only intrinsic quantities.

Each model is based on its original assumption. Point, String, Ring and Field-Matter-Ball model are the same on this point. They are all based on its foundational assumption and they are equal to some degree because there is no priority level among the models. When we judge a model is good or not, the key point is whether the model is handy to help us to well understand something or not. Concise is the basic requirement in Physics. The main defect in Physics now is not the short of methods in math, actually is the over excess of math.

Smolin (One of the founder of the Ring theory) had said in "The trouble with Physics" like this: there is an insurmountable difficulty in Point modeling, String theory and Ring theory. He expected a combination of the Quantum mechanics and general relativity by some new thought on the intrinsic quality of the space-time [8]. Complex number is used in space-time is exactly a new breakthrough. On one hand, it is convenience we use a field-matter-ball model to describe the quantum phenomenon. On the other hand, we are talking a Dual 4-dimensional complex space not an 11-dimensions or 26-dimensions space or even higher dimensions space which are unimaginative.

The Dual 4-dimensional complex space description can cover all quantum phenomena now, and it can have a clear connection with Gravitational Field. The flowing work is to integrate the Quantum Mechanics, Space-time and Gravitation. We expect we can offer a new direction to make breakthrough in Physics.

2. Some Physical Concepts

2.1. Micro-Object

To avoid the trouble caused by mass point model, we call the micro particle as micro-object.

2.2. Some Concepts

1) Static micro-object Compton momentum: $P_0 = m_0 c$, where m_0 is the static mass;
2) Kinetic micro-object Compton momentum: $P_1 = mc$, where m is the kinetic mass;
3) The relativistic momentum: $P_i = m_i c$ (i = 2, 3, 4), where c is the light velocity;
4) Static micro-object Compton wavelength: $\lambda_0 = h/m_0 c$, where h is the Planck constant;
5) Kinetic micro-object Compton wavelength: $\lambda_1 = h/mc$;
6) Micro-object De Broglie wavelength: $\lambda_i = h/P_i = h/mv_i$;
7) Eigen Radius of static micro-object: $R_0 = \hbar/m_0 c$; Eigen Curvature of static micro-object: $K_0 = 1/R_0 = m_0 c/\hbar$;
8) Eigen Radius of kinetic micro-object: $R_1 = \hbar/mc$; Eigen Curvature of kinetic micro-object: $K_1 = 1/R_1 = mc/\hbar$;
9) Radius $R_i = \hbar/mv_i$ and Curvature $K_i = mv_i/\hbar$. These two are the mapping of micro-object R_0 in three dimension phenomenon space. They are the main study objects in Quantum Mechanics.

Notes: the subscript "0' represents the static micro-object's property. The subscript "1, 2, 3, 4" represents the kinetic micro-object's properties. The subscript "*i*" represents some physical quantity in the three-dimension phenomenon space.

2.3. The Relationship between the Relativistic Energy Momentum and Curvature

If m and m_0 is the micro-object's kinetic mass and static mass, using the relativistic energy equation $E^2 = (mv)^2 c^2 + m_0^2 c^4$ and $(mc)^2 = (mv)^2 + (m_0 c)^2$.

Then we can get $P_1^2 = P_i^2 + P_0^2$ and $K_1^2 = K_i^2 + K_0^2$, where $\overrightarrow{K_1} - \overrightarrow{K_i} = \overrightarrow{K_0}$.

Here we have such vectors: $\boldsymbol{p}_1$, $\boldsymbol{p}_i$, $\boldsymbol{P}_0$, $\boldsymbol{k}_1$, $\boldsymbol{k}_i$, $\boldsymbol{K}_0$. $\boldsymbol{k}_1 - \boldsymbol{k}_i = \boldsymbol{K}_0$, $\boldsymbol{k}_1$ and $\boldsymbol{k}_i$ can construct a 4-dimension Curvature coordinate associated with kinetic micro-object's motion.

3. Geometry of a Micro-Object

3.1. Micro-Object Is Like a "Rolling Field-Matter-Ball"

1) For static state,

$$\text{Curvature Radius:} \quad R_0 = \hbar/m_0 c \tag{1}$$

which represents the field extension in a micro-object.

$$\text{Curvature:} \quad K_0 = 1/R_0 = m_0 c/\hbar \tag{2}$$

which represents the camber of micro-object's surface.

2) For kinetic state,

$$\text{Curvature Radius:} \quad R_1 = \hbar/mc \tag{3}$$

$$0 < R_1 = \hbar/mc \le R_0 \tag{4}$$

$$\text{Curvature:} \quad K_1 = 1/R_1 = mc/\hbar \tag{5}$$

where *m* is the kinetic mass. And the faster the object is moving, the larger the kinetic mass is, the smaller the Radius is and the larger the Curvature is [9]. It is easy to know:

$$K_0 \le K = mc/\hbar < \infty \tag{6}$$

3) For mapping in three-dimension phenomenon space,

$$\text{Radius:} \quad R_i = \hbar/mv_i \tag{7}$$

where $0 < R_i = \hbar/mv_i \le \infty$.

$$\text{Curvature:} \quad K_i = mv_i/\hbar \tag{8}$$

where $P_i = mv_i$ is relativistic momentum and $0 \le K_i < \infty$. It can be deserved in 3-dimension phenomenon space. So micro-object's image (Curvature Radius) can be very large or very small in three-dimension phenomenon space (this is the observable space) [9]. But it is not equal to its spatial structure.

4) For rotational frequency,

$$\nu_0 = E_0/h, \quad \nu_1 = E_1/h \left(\nu_i = E_i/h\right) \tag{9}$$

where $E_0 = m_0c^2$, $E = mc^2$ and $E_i = m_0 v_i^2/2$, which are just the same as we talked in Quantum Mechanics and the Theory of Relativity.

5) For density of the field-matter,

$$\eta = m/V = \eta(K) \tag{10}$$

where *V* is the volume of the field-matter-ball. *V* is function of Curvature *K* because of $V = V(R)$, $R = R(K)$. $\boldsymbol{K = k_0, k_1, k_i, (k_1 - k_i = K_0)}$. The smaller *V* is, the higher *K* is and the larger η is. $\eta(K)$ has a positive relationship with K.

3.2. The Hidden Degrees of Freedom by Particle Model

The curvature components K_1 and K_i $(i = 2,3,4)$ can form the 4-dimension Curvature coordinate $K(k_1, -k_2, -k_3, -k_4)$, where $R_1 = \hbar/mc$ and $K_i = mv_i/\hbar$. These are the hidden degrees of freedom by Particle Model.

The Radius and Curvature of a micro-object represent the state of its motion. R_0 and K_0 are not observable in our 3-dimension phenomenon space. It is the ontology of itself. While R_i and K_i is visible. R_1 and K_1 is like a bridge between noumenal world and phenomenon world (the same as light cone). If R_0 and K_0 do not exist, R_i and K_i, R_1 and K_1 won't exist, too. R_0 and K_0 is the foundational source of every physical phenomenon.

The change of Radius *R* (or Curvature *K*) will surely bring the change of distribution in field-matter-ball, thus will help us to deeply understand the ontology of wave function.

3.3. Describe of the Field-Matter-Ball

1) In coordinate complex space,

$$z = x + iy = re^{i\alpha} \tag{11}$$

$r = r(x, y) = \sqrt{x^2 + y^2}$ describes a standard great circle of a spherical coordinate.

If the modulus of the complex number *r* is defined by Curvature Radius *R*, then the above-mentioned coordinate becomes into a field-matter-ball's spherical coordinate,

$$R = R(x, y) = \sqrt{x^2 + y^2} \tag{12}$$

For static micro-object: $R = R_0 = \hbar/m_0c$.

For kinetic micro-object: $R = R_1 = \hbar/mc$.

For mapping in three-dimension space $R = R_i = \hbar/mv_i$. Field-matter-ball is described in coordinate complex space.

2) In curvature complex space,

$$w=\frac{1}{z^*}=u(x,y)+iv(x,y)=\frac{1}{re^{-i\alpha}}=\frac{1}{r}e^{i\alpha} \tag{13}$$

where w is mapping space of z, describing a spherical coordinate whose modulus is $k(x,y)=\frac{1}{r}$. Similarly, if k is defined by Curvature $K=\frac{1}{R}$, then Curvature complex space w describes field-matter-ball's Curvature coordinate.

For static micro-object: $K=K_0=1/R_0=m_0c/\hbar$.

For kinetic micro-object: $K=K_1=1/R_1=mc/\hbar$.

For mapping in three-dimension space $K=K_i=1/R_i=mv_i/\hbar$.

In z space, matter-wave-field is in, and the outer in empty, thus it can be simply treated as a mass point under the large scale. In mapping space w, matter-wave-field is out of the ball by mapping, and the inner is empty. For field-matter-ball, z space and w space are mapping to each other, describing the same thing.

In real time and space, we describe the motion or the probability by mass point model. While in complex space, we use Curvature Radius or Curvature. And this is the physical significance why micro-object does not have orbital motion.

Curvature $\boldsymbol{K}$ is an expansion of the wave number $\boldsymbol{k}$, geometry of a micro-object, indication of the hidden degrees of freedom by mass point model.

The movement of matter-wave-field has something to do with the interaction of a micro-object.

4. Matter-Wave and Phase Space

4.1. How to Generate the Matter-Wave and Wave Function

The rotation of field-matter can be written as

$$w=\frac{1}{z^*}=u+iv=ke^{i\omega t} \tag{14}$$

where ω is angular frequency. Set the coordinate of static micro-object itself as $k'(x',t')$, the equation of rotation is

$$\psi_0=A_0e^{i\omega_0 t'} \tag{15}$$

where $\omega_0=m_0c^2/\hbar$.

Suppose the K' coordinate has a uniform velocity v in the x-direction, in the observer's system, using Lorentz transformation, $t'=\frac{t-vx/c^2}{\sqrt{1-(v/c)^2}}$ we can rewrite Equation (15) as:

$$\psi=Ae^{i\omega_0\frac{t-vx/c^2}{\sqrt{1-(v/c)^2}}}=Ae^{i\omega\left(t-vx/c^2\right)}=Ae^{-\frac{i}{\hbar}(px-Et)} \tag{16}$$

where $\omega=\frac{\omega_0}{\sqrt{1-v^2/c^2}}$ and $\omega t=Et/\hbar$.

Function (16) is just the wave function in Quantum Mechanics. So the intrinsic property of matter-wave is field-matter's rotation. For free micro-object, A is normalization constant.

The deduction of matter-wave can at least solve the following questions:

1) Can ensure that it is harmonic with traditional Quantum Mechanics, such as the assumption of energy, frequency, momentum and wavelength, without increasing any new assumption.

2) Can uncover the mysteries of Quantum Mechanics wave-function. Because we can see that matter-wave is the real propagation of physical wave, and it is not an assumption or a thought.

3) Can unify the Quantum Mechanics and the theory of relativity in logic foundation, also can support the assumption that wave function is continuous field in quantum field theory.

4) Prompt us to find the internal relation between matter-wave function and probability function.

4.2. Schrodinger Equation and Dirac Equation

If we treat Equation (16) as a complex description of field-matter-ball by observe system and take no account of its phase, phase space, the physical relationship between field-matter-ball and its motion ,and the physical significance, then we can get Dirac equation by using Dirac operator. Thereby, this theory system is conform to the relativistic Quantum Mechanics' rule. When $m = m_0$ and $E = \frac{m_0 v^2}{2}$, Equation (16) is the classical limit of relativistic Quantum Mechanics, namely, the nonrelativistic Quantum Mechanics wave function. Using Schrodinger operator, we can get the Schrodinger equation, and it is conform to the nonrelativistic Quantum Mechanics' rule [9]. Using the Klein-Gordon operator, we can get the Klein-Gordon equation. But now we take no consider of the rotation vector effect of the matter-field-ball, so it is a relativistic scalar matter-field. Consequently, Schrodinger equation and Dirac equation are not assumptions, but the motion equation generated by using mathematical methods in real matter-wave function.

5. Phase Space of Wave Function

In fact, the phase space of wave function has a specific physical significance when we take the field-matter-ball model into account. Wave function will present the change of a micro-object's spatial structure in Dual 4-dimension complex phase space

$$\psi = Ae^{-\frac{i}{\hbar}(\boldsymbol{p}\cdot\boldsymbol{x}-Et)} = Ae^{-i\left(\frac{\boldsymbol{p}}{\hbar}\cdot\boldsymbol{x}-\frac{mc}{\hbar}ct\right)} = Ae^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-k_1x_1)}$$

So

$$\psi = Ae^{-i(k_\mu x_\mu)} \tag{17}$$

where $\mu = 1,2,3,4$ $k_1 = \frac{mc}{\hbar}$ $x_1 = ct$.

The phase angle is function of k_μ and x_μ and it has no dimension. k_μ is mapping of the kinetic field-matter-ball Curvature in 3-dimension space. x_μ and k_μ is the real part and imaginary part in Dual 4-dimension complex phase space, respectively. Hence, the phase space has a specific physical significance and the matter-wave is just described in Dual 4-dimension complex phase space.

The phase space demonstrates that there are hidden degrees of freedom by Particle Model in traditional Quantum Mechanics. It has a direct relationship with the motion state of a field-matter-ball and the observer system, too.

1) $k_{2,3,4}$ $x_{2,3,4}$ indicates the movement in our three-dimension space has something to do with the phase angle of a micro-object (R_0).

2) $\frac{Et}{\hbar} = \frac{mc}{\hbar}ct = k_1x_1$ indicates the rotation of a micro-object's influence on phase angle. k_1 is mapping of micro-object R_0 on the Light Cone surface.

3) The rotation of itself and the motion all have reflections in the phase and the matter-wave Ψ consists of both. Obviously, the rotation of a micro-object added into a free object is the reason why there will be a so called "the High frequency random vibration" [10]. If the motion is independent of time, the state is stationary.

6. The Dual 4-Dimension Complex Phase Space

6.1. Sphere of Complex Numbers

The wave function is formed from the rotation and motion and is a function in Dual 4-dimension complex phase space. Plural can be defined on the plural ball [11]. See **Figure 1**.

Complex number can be defined by a sphere of complex numbers.

$$w = \frac{1}{z^*} = Ae^{i\theta} = u + iv \tag{18}$$

$$w^* = \frac{1}{z} = Ae^{-i\theta} = u - iv \tag{19}$$

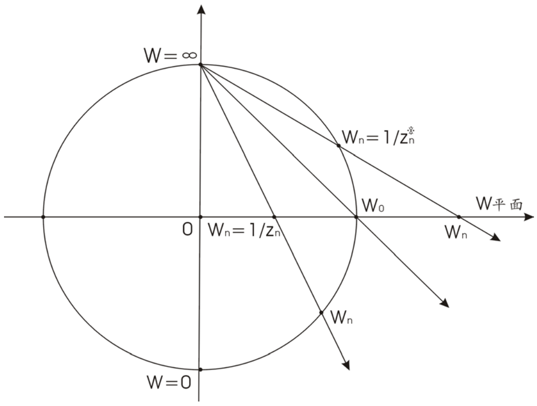

Figure 1. The plural ball-Riemannian ball.

$$A^2 = u^2 + v^2 \tag{20}$$

Suppose $z = x + iy = re^{i\theta}$, where $r = R$, then $A = \frac{1}{r} = \frac{1}{R} = k$. R is Curvature Radius and K is Curvature.

1) In complex number $w = \frac{1}{z^*} = Ae^{i\theta} = u + iv$, when $r \to 0$ and $w \to \infty$, it represents the north pole point; when $r = 0$, $\left|z^*\right| = 0$, w has no definition; in other cases, the point on the sphere can be mapped out and becomes a geometric point.

2) The curvature:

When $R \to 0$, $k \to \infty$, it becomes a mass point; the Curvature model becomes the same as the particle model.

When $0 < R \le R_0$, $K_0 \le K < \infty$, it represents a matter-wave and this matter-wave will be mapped into a real space the same as a probability wave does.

3) The significance of the mapping from internal to external.

From Dual 4-dimension complex phase space to the 4-dimension real time and space, a micro-object's *k*-space will be compacted into zero-dimension space by $K \to \infty$, thus the hidden degrees will be hidden again and becomes a mass point (R = 0). In that case, the wave motion of a micro-object will turn into a particle motion by orbits or by probability. Then the Quantum Mechanics return to traditional.

6.2. The Dual 4-Dimension Complex Phase Space

6.2.1. The Dual Quaternions Complex Space

Set $z_\mu = x_\mu + iy_\mu$ and $z_\mu^* = x_\mu - iy_\mu$ $w(x,y) = \frac{1}{z^*} = u(x,y) + iv(x,y) = Ae^{i\alpha}$

$$\text{Wave function } \Psi(x,y) = u(x,y) + iv(x,y) = A(x,y)e^{-iy_\mu x_\mu} \tag{21-1}$$

6.2.2. The Dual 4-Dimension Complex Phase Space

The Dual 4-dimension complex phase space comes from the Dual 4-dimension complex number $z_\mu = x_\mu + iy_\mu$ and $z_\mu^* = x_\mu - iy_\mu$. $x_\mu = (x_1, -x_2, -x_3, -x_4)$ is the real part and $y_\mu = (y_1, -y_2, -y_3, -y_4)$ is the imaginary part. Then $z_\mu = x_\mu + iy_\mu$ can be treated as a Dual 4-dimension complex phase space came from vector ***x***, ***y***.

$$\text{Wave function } \Psi(x,y) = u(x,y) + iv(x,y) = A(x,y)e^{-iy_\mu x_\mu} \tag{21-2}$$

(21-2) is a function defined in a complex phase space. Though the function is completely the same as (21-1), x_μ, y_μ are components while ***x***, ***y*** are vectors. Also, the phase angle is dimensionless.

6.2.3. Matter-Wave in Complex Phase Space

Set ***y*** = ***k***, namely, $y_\mu = k_\mu = k_1, -k_2, -k_3, -k_4$. Vector ***y*** becomes into a Lorentz vector ***k***, therefore we will get a new 4-dimension complex phase space $w'_\mu = x_\mu + ik_\mu$. $x_\mu = (x_1, -x_2, -x_3, -x_4)$ and $k_\mu = (k_1, -k_2, -k_3, -k_4)$.

$$\text{Wave function}\quad \Psi(x,k)=u(x,k)+iv(x,k)=A(x,k)\mathrm{e}^{-ik_\mu x_\mu} \tag{21-3}$$

Actually, this is the Equation (17).

Using the position vector ***X*** and Curvature vector ***K***, we get a Dual 4-dimension complex phase space $w'_\mu = x_\mu + ik_\mu$. The matter-wave is just in it and the phase angle $k_\mu x_\mu$ has no dimension.

Generally we call a Quantum Mechanical wave function phase space mathematical space. We can differ that the micro-object is in a complex space. When we observe it, it will automatically come into being a real physical space we call Dual 4-dimension complex time and space.

$$\{\text{Field}-\text{matter}-\text{ball}+\text{Complex space}\}+\text{Observe System}=\text{Matter}-\text{wave}+\text{Complex phase space.}$$

It must be pointed that, the position complex space $z(x,y)$ where field-matter-ball (noumenon) is in and the Minkowski space where observer is in and the Dual 4-dimension complex phase space $w'(x,k)$ is not a space. They can convert to each other by $k=\dfrac{1}{\sqrt{x^2+y^2}}$. Also, their quantities are not equal; the Dual 4-dimension complex phase space $w'(x,k)$ allows a velocity faster-than-light. The further discussion will be our next work.

The metric tensor of $w'(x,k)$ is

$$g^{\mu,\nu}=\mathrm{diag}(1,-1,-1,-1)$$

$$x^2=x_\mu g^{\mu\nu}x_\nu=x_1^2-x_2^2-x_3^2-x_4^2$$

$$k^2=k_\mu g^{\mu\nu}k_\nu=k_1^2-k_2^2-k_3^2-k_4^2$$

k and $z^2=zz^*=x^2+k^2$ are both Lorentz invariant quantity.

7. The Matter-Wave Differential Function

In Dual 4-dimension complex space, the wave function must be analytic.

$$\Psi(x,k)=u(x,k)+iv(x,k)=A(x,k)\mathrm{e}^{-ik_\mu x_\mu} \tag{22}$$

The real part and imaginary part should satisfy the Cauchy-Reimam conditions

$$\nabla_x u(x,y)=\nabla_y v(x,y)$$
$$\nabla_y u(x,y)=-\nabla_x v(x,y)$$

The amplitude is $A(x,y)=\sqrt{u^2+v^2}$.

Set ***y*** = ***k***, we can get the matter-wave-function described in Dual 4-dimension complex phase space:

$$\psi(x,k)=A(x,k)\mathrm{e}^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-Et/\hbar)}=A(x,k)\mathrm{e}^{-ik_\mu x_\mu} \tag{23}$$

$A(x,y)$ is only concerned with the space coordinates and Curvature coordinates.

$A(x,k)=A(\boldsymbol{x},\boldsymbol{k})$, ***K*** is associated with the potential function $V(x)$ so the inner motion of a micro-object is associated with the interaction it has felt. The amplitude of the matter-wave is function of *k*, which satisfy the following differential function:

$$H(\boldsymbol{x},\boldsymbol{k})\mathrm{e}^{\frac{i}{2}\left(\overleftarrow{\partial}_x\vec{\partial}_k-\overleftarrow{\partial}_k\vec{\partial}_x\right)}A(\boldsymbol{x},\boldsymbol{k})=H(\boldsymbol{x},\boldsymbol{k})*A(\boldsymbol{x},\boldsymbol{k}) \tag{24}$$

where $H(\boldsymbol{x},\boldsymbol{y})$ is the Hamiltonian function, * represents the Moyal product rule.

Moyal product rule:

$$f(\boldsymbol{x},\boldsymbol{y})*g(\boldsymbol{x},\boldsymbol{y})=f(\boldsymbol{x},\boldsymbol{y})\mathrm{e}^{\frac{i}{2}\left(\overleftarrow{\partial}_x\vec{\partial}_y-\overleftarrow{\partial}_y\vec{\partial}_x\right)}g(\boldsymbol{x},\boldsymbol{k})$$

The average value of a physical quantity $F(\boldsymbol{x},\boldsymbol{y})$ in this stationary state is

$$\langle F\rangle=\int_{-\infty}^{+\infty}F(\boldsymbol{x},\boldsymbol{k})A(\boldsymbol{x},\boldsymbol{k})\mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{k} \tag{25}$$

The wavelength of a system can be defined by general De Broglie wavelength.

8. The Relationship of Wave Function $\Psi(x,k)$ Probability Distribution Function $\psi(x)$, and Field Matter Distribution Function $\varphi(k)$

The point (***x***, ***k***) in Dual 4-dimension complex phase space can be explained as follows: $K(k_1,k_2,k_3,k_4)$ represents the spatial structure of a micro-object, presenting the existing form, while $x(x_1,x_2,x_3,x_4)$ represents the position of a micro-object in real space.

8.1. $\Psi(x,k)$, $\psi(x)$ and $\phi(k)$

The micro-object described in Dual 4-dimension complex space, shows its extension in *K* space and position in *x* space. As to the wave-particle duality, it can be confirmed by the mapping from the wave function in Dual 4-dimension complex space to the probability in real 4-dimension space and time.

It has been proved that, the wave function

$$\psi(x,k)=A(x,k)\mathrm{e}^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-Et/\hbar)}=A(x,k)\mathrm{e}^{-ik_\mu x_\mu}$$

is a matter-wave function in Dual 4-dimension complex space. The amplitude $A(x,k)$ is a function of position ***x*** and Curvature ***k*** which is related to the density distribution. So it is also a function of the density distribution.

The micro-object's motion satisfies the Dirac equation (or Schrodinger equation). And it has a relationship with $A(\boldsymbol{x},\boldsymbol{k})$ [12]:

$$A(\boldsymbol{x},\boldsymbol{k})=\int_{-\infty}^{+\infty}\mathrm{d}\xi\mathrm{e}^{-i\xi\boldsymbol{k}}\psi^*\left(\boldsymbol{x}-\frac{1}{2}\xi\right)\psi\left(\boldsymbol{x}+\frac{1}{2}\xi\right)$$

$$A(\boldsymbol{x},\boldsymbol{k})=\int_{-\infty}^{+\infty}\mathrm{d}\xi\mathrm{e}^{-i\xi\boldsymbol{x}}\varphi^*\left(\boldsymbol{k}-\frac{1}{2}\xi\right)\varphi\left(\boldsymbol{k}+\frac{1}{2}\xi\right)$$

For the two integral operations

1) Eliminating the variable *k*, then the amplitude $A(\boldsymbol{x},\boldsymbol{y})$ is mapped into 4-dimension real space. We can get the probability density distribution function $\rho(\boldsymbol{x})$

$$\int_{-\infty}^{+\infty}A(\boldsymbol{x},\boldsymbol{k})\mathrm{d}\boldsymbol{k}=\left|\psi(\boldsymbol{x})\right|^2=\rho(\boldsymbol{x}) \tag{26}$$

where the normalization is $\int_\tau c\rho(\boldsymbol{x})\mathrm{d}\tau=\int_\tau c\left|\psi(\boldsymbol{x})\right|^2\mathrm{d}\tau=1$.

2) Eliminating the variable *x*, then $A(\boldsymbol{x},\boldsymbol{k})$ is mapped into the imaginary part. We can get the matter-field density distribution function $\eta(k)$

$$\int_{-\infty}^{+\infty}A(\boldsymbol{x},\boldsymbol{k})\mathrm{d}\boldsymbol{x}=\left|\varphi(\boldsymbol{k})\right|^2=\eta(\boldsymbol{k}) \tag{27}$$

where the normalization is $\int_v c\eta(\boldsymbol{k})\mathrm{d}v=\int_v c\left|\varphi(\boldsymbol{k})\right|^2\mathrm{d}v=\int_v \mathrm{d}v/v=1$.

3) $\psi(\boldsymbol{x})$ and $\varphi(\boldsymbol{k})$ has a Fourier transform relationship, so it does between the probability density distribution and the matter-field density distribution. Thus we can say the Fourier transform relationship between $\psi(\boldsymbol{x})$ and $\varphi(\boldsymbol{k})$ has a new physical meaning and the probability density distribution and the matter-field density distribution can translate to each other.

Now let us talk something about the point ***x*** (and its neighborhood):

1) when *k* is common: the larger *ρ* the larger *η* and the smaller *ρ* the smaller η $\eta\propto\rho$; 2) when *ρ* is common, the larger *k* the larger *η* and the smaller *k* the smaller η, $\eta\propto k$. So, the higher the matter-field density it is, the higher probability density it is. And vice versa.

We can make an intuitive understanding that we get the information of the probability density distribution from the matter-field density distribution. Where there is a higher distribution of matter-field density, where there is a higher distribution of probability density .They have a positive correlation.

Based on this, the sphere model and the particle model or the *η* description and the *ρ* description can make sense: for a micro-object, when the volume is smaller, field-matter density *η* will become larger, so the probability of it can appear in the ball as a particle is larger. When the volume becomes larger, it goes to the opposite result. $V=\infty$, $\eta=0$, $\rho=0$, $V=0$ (mass point), $\eta=\infty$, $\rho=1$ (*δ* function). The *η* description and the *ρ* description are actually equivalent while *η* is small scale space description and *ρ* is large scale space description.

Discussion:

1) The matter-field density has a uniform distribution in large scale, so is the probability distribution of where the micro-object would occur. If the distribution was not uniform, the probability would be large in the places where the density is large.

2) For free micro-object, the density is uniform. The matter-field density η and the probability density ρ in unit volume are both an average value. If the scale was smaller than the Radius scale R, the average η and ρ had no definition. The reason why we define the η and ρ both in a sphere with a Radius R is that there is an equivalency between the matter-field density η and the probability density ρ.

3) When $k(2,3,4)=0$, $k_1 = k_0 = \frac{m_0}{\hbar} = \text{const}$, it represents the noumenon of a Micro-object. In fact, Ψ_0 is the spinor wave function in Quantum Mechanics.

4) k_0 is independent of position and existing in vacuum space. Also, it is described in Curvature space. k_0 represents the high frequency vibration and is source of the probability [13].

5) The uncertainly of position is because of the size of a micro-object (Compton wavelength)

8.2. Wave Function, Eigen State and Eigen Value

The wave function $\psi(x,k) = A(x,k)\mathrm{e}^{-i(\boldsymbol{k}\cdot\boldsymbol{x}-Et/\hbar)} = A(x,k)\mathrm{e}^{-ik_\mu x_\mu}$ can be de-composited by general Fourier decomposition.

For stationary state wave function:

$$H(\boldsymbol{x},\boldsymbol{k}) * \psi_n(\boldsymbol{x},\boldsymbol{k}) = E_n \psi_n(\boldsymbol{x},\boldsymbol{k}) \quad \text{and} \quad A(\boldsymbol{x},\boldsymbol{k}) = \sum_0^\infty c_n \psi_n(\boldsymbol{x},\boldsymbol{k})$$

$$\iint \psi_n^*(\boldsymbol{x},\boldsymbol{k}) \psi_m(\boldsymbol{x},\boldsymbol{k}) \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{k} = \delta_{nm} \tag{28}$$

The modular square of the decomposition coefficient is the probability of finding an eigen state in some place. Before measuring, the wave function is in the Dual 4-dimension complex phase space .while after measuring, it is collapsed in real space. Measurement contains the transformation of space to describe and the corresponding experiment values. Measurement will have a new connotation in our new theory, and we will discuss it in another article.

The eigen states of an atom is non-continuous so that the energy band in E-k space will have corresponding energy states. With the decrease of the energy gap, the energy will become continuous. Then the Hilbert space will be useful.

Discussion:

1) When $k_1 = \frac{mc}{\hbar} = \text{const}$, $v = \text{const}$, $\omega = \frac{mc^2}{\hbar} = \text{const}$, k and k_1 are both independent of time, the amplitude $A(x,k) = A(\boldsymbol{x},\boldsymbol{k})$, it is a stationary state. Each constant velocity v corresponds to an eigenstate and an eigenvalue.

2) When $k_1 = \frac{mc}{\hbar} \neq \text{const}$, $v \neq \text{const}$, $\omega = \frac{mc^2}{\hbar} \neq \text{const}$, the motion state of a micro-object is changeable, the spin and the energy will change, too.

a) To increase the frequency, the micro-object will speed up absorbing energy and transiting to higher energy state.

b) To decrease the frequency, the micro-object will slow down releasing energy and transiting to lower energy state

a) and b) are both breaking down to the stationary state, and we call them stimulated and de-stimulated, respectively. Wave function becomes a function of time. By absorbing or releasing energy, electron transit from one energy state to another. The generation or annihilation of a particle in quantum field theory is transition actually. The observe system we use in the derivation of the wave function is a corresponding quantum transition in Dual 4-dimension complex space. To solve the specific quantum transition and take the interaction time into account, we can use time-dependent Schrodinger equation or Heisenberg equation.

3) Every single static stable micro-object has standing wavelength. For uniform motion, the standing wave-

length is. For non-uniform motion, the wavelength is changing and the original stable condition is broken down, thus causing the radiation, until the new stable state come into being.

4) There are no two different states at the same time and the electron energy emission in transition is one by one (photon). The change of energy is a sudden change and do not need time if neglecting the interaction process. This will provide a physical foundation of the discontinuity of eigenvalues or eigenstates which come from the wave function operated by an operator.

The confusing phenomenon that there are different probabilities in different eigen states of a micro-object is deeply hidden in the mathematical property. "Existing at the same time" is not a real physical process but a derivative feature from physics to math caused by the sudden change of energy state. The noumenon can never be observed, what we have seen is the phenomenon [9] [14] and what can "Existing at the same time" is phenomenon.

9. Experiments Evidence on Particle Radius

9.1. Hofstadter's Particle Radius Experiments

Using the $R_0 = \frac{\hbar}{m_0 c} = \frac{\lambda_0}{2\pi}$ (or $R_1 = \frac{\hbar}{mc}$) as the Radius of a micro-object [15].

We can see the theoretical value and the experimental value are very close (**Table 1**).

The micro-object is not a mass point. It has its spatial distribution and the Radius will decrease when the velocity increases. It is reasonable we use Compton wavelength to describe as the micro-object's Radius in extension.

9.2. The Landau Uncertain Property of a Single Mechanical Quantity

Actually, Landau had an explanation [16] on m_0c in 1930. He said, when measuring a single mechanic quantity, the uncertainty of position is $\Delta x_0 = \hbar / m_0 c$ and the uncertainty of momentum is $\Delta p_0 = m_0 c$. While the uncertainty of position is equal to the theoretical value of Radius $R_0 = \hbar / m_0 c$. Landau thought the Δx_0 may come from the particle model.

Electron is not a mass point, and it has its real Radius where it distributes in. If we treat an electron as a point, it must be in the size R_0. The size of the electron R_0 becomes the range an electron point can reach. We cannot have a clearer knowledge about where is the electron smaller than R_0. So we have recognition of the uncertainty of position. This is an important theoretical foundation what Landau had provided for the non-point model.

10. Conclusions

1) When observing a micro-object in a macro large scale condition, the micro-object can be simply treated as a mass point ($h \to 0$, $R \to 0$). It has a certain geometric coordinate and can be described completely and clearly in 4-dimension real space. However, when observing it in a micro condition, the micro-object itself is a rotating field-matter-ball. We have to recover the hidden degrees of freedom and it would never be taken as a mass point.

2) We have known, the matter-wave carries the information of the structure and the field density of the micro-object. The rotation and motion of either charged particles or uncharged particles will spread the matter-wave. So people can use it in communication science.

3) We should have new understanding of the five basic assumptions in Quantum Mechanics. Matter-wave and probability-wave can convert to each other.

Table 1. Radius of electrons, protons and neutrons experimental value compared with the theoretical value Compton wavelengths (static—R0, move—R1).

Type	Proton (R_0) static	Neutron (R_0) static	Electron (R_0) static	Electron (R_0) 20 Gev	Electron (R_0) 60 Gev
Theoretical value	0.2×10^{-15} m	0.2×10^{-15} m	3.8×10^{-11} cm	0.9×10^{-15} cm	3.1×10^{-16} cm
Experimental value	1.1×10^{-15} m	1.1×10^{-15} m	Same order	$<10^{-15}$ cm	$<10^{-16}$ cm

4) We construct a Dual 4-dimension complex space for Quantum Mechanics from the vector (***x***, ***K***) to describe the movement of a micro-object. The static ones are in coordinate complex space, while the kinetic or matter-waves are in Dual 4-dimension complex space.

5) Matter-wave may be the source of the whole Quantum Mechanics.

6) The field-matter-ball model is in keeping with the ring current model for electron by Dirac [17]. We will calculate the Bohr magneton, the spin magnetic moment of electron, the nuclear magneton, the proton magneton and the neutron magneton accurately in supplement

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Supplement

1.1. Field-Matter-Ball Model for Electron

Spin radius:

$$\begin{aligned}&\text{for static electron } R_0 = \hbar/m_0c\\&\text{for kinetic electron } R_1 = \hbar/mc\end{aligned} \tag{1}$$

Spin frequency:

$$\begin{aligned}&\text{for static electron } \nu_0 = m_0c^2/h\\&\text{for kinetic electron } \nu_1 = mc^2/h\end{aligned} \tag{2}$$

Spin period:

$$\begin{aligned}&\text{static electron } T_0 = 1/\nu_0 = h/m_0c^2\\&\text{kinetic electron } T_1 = 1/\nu_1 = h/mc^2\end{aligned}$$

Current:

$$\begin{aligned}&\text{static electron } I_0 = e/T_0 = em_0c^2/h\\&\text{kinetic electron } I_1 = e/T_1 = emc^2/h\end{aligned} \tag{3}$$

1.2. The Electron Magnetic Moment

1.2.1. Static Electron

$$P_{0m} = IS = \left(em_0c^2/h\right)\pi\left(\hbar/m_0c\right)^2 = e\hbar/2m_0 \tag{4}$$

The spin magnetic moment of electron is equal to Bohr Magnetron.

1.2.2. Kinetic Electron

$$P_m = IdS = I\pi R_1^2 = eh/4\pi m = P_{0m}\left(1-v^2/c^2\right)^{1/2} \tag{5}$$

So the kinetic electron magnetic moment will decrease as the velocity increase.

1.2.3. Nuclear Magneton and Proton Magneton

For nuclear magneton: $R_P = \dfrac{\lambda_P}{2\pi} = \dfrac{h}{2\pi m_p c}$, $\nu_p = m_pc^2/h$, $I = \dfrac{e}{T_p} = \dfrac{em_pc^2}{h}$.

Set $P_{\text{NM}} = IS = I\pi R_p^2 = \dfrac{e\hbar}{2m_p}$. For proton magneton, we know that $m_p = 3m_u = 3m_d$, where m_u and m_d is the mass of up quark and down quark, respectively.

The spin magnetic moment of up quark is $P_u = I_uS_u = I_u\pi R_u^2 = \dfrac{2}{3}\dfrac{em_uc^2h^2}{4hm_u^2c^2} = \dfrac{2}{3}\dfrac{e\hbar}{2m_u} = \dfrac{e\hbar}{m_p}$;

The spin magnetic moment of down quark is $P_d = I_dS_d = I_d\pi R_d^2 = -\dfrac{1}{3}\dfrac{em_dc^2h^2}{4\pi hm_d^2c^2} = -\dfrac{1}{3}\dfrac{e\hbar}{2m_d} = -\dfrac{e\hbar}{2m_p}$.

So the magnetic moment of proton: $P_p = 2P_u + P_d = \dfrac{3e\hbar}{2m_p} = 3P_{\text{NM}}$.

But this theory result is quite different from the experiment result. We can rewrite the magnetic moment of proton like that $P_p = P_u + P_u + P_d = P_{\text{NM}}\left(\cos\alpha + 2\right)$, where α is the angle between the two up quark.

When $\alpha = 37^\circ$, $P_p = 2.793P_{\text{NM}}$. The results were in agreement with experimental data well.

It must be pointed that, quark cannot be directly observed. So the magnetic angle assumption has no contradiction with the fact. And it is the result after we have considered all the interactions in micro field. In spite of the inequality of the mass between the up quark and the down quark, we can adjust the angle to make sense and make it equal to the experiment results. The magnetic angle assumption is general applicable. The correspondence between the angle and spin magnetic moment in experiment is just like the correspondence between eigenstate and eigenvalue.

1.2.4. Neutron Magneton

If the up quark is anti-parallel to the two down quarks, then

$$P_n = P_u + 2P_d = 0$$

While in fact the magnetic moment of neutron is not 0, the condition is wrong.

Adjusting the angle α, then

$$P_p = P_u + P_d + P_d = P_{\mathrm{NM}}\left(2\cos\alpha - 1\right)$$

So the value is negative.

If the mass of up and down quark is not equal, by adjusting the angle, we can make the theory value and the experiment value exactly match.

We are looking forward to the experimental physicists' work.