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Pair Production in Non-Perturbative QCD

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Abstract

In this paper, a method to calculate the vacuum to vacuum transition amplitude in the presence of a non-abelian background field is introduced. The number of non-perturbative quark-antiquark produced per unit time, per unit volume and per unit transverse momentum from a given constant chromo-electric field is calculated and its application to quark-gluon plasma is presented.

Keywords

Pair Production, Non-Perturbative QCD

1. Introduction

Lattice QCD predicts a phase transition from Hadrons gaz (HG) to quark-gluon plasma (QGP) at deconfinement temperature, T ~ 170 MeV. It is believed that QGP has been produced in relativistic heavy ions collision [1]-[4] where in the initial pre-equilibrium stage of QGP about half the total center-of-mass energy, E_{cm}, goes into the production of a semi-classical gluon field [5]-[17]. Therefeore, to study the production of a QGP from a classical chromo field, it is necessary to know how quarks and gluons are formed from the latter. The production rate of quark-antiquark from a given constant chromo-electric field E^a has been derived in Ref. [18] and the integrated p_T distribution has been obtained in [19]-[22] (for a review see [23]).

In this short technical note, we will extend the results of Ref. [18] to a general constant background field. The method presented here may simplify the complexity found in the Non-perturbative QCD calculations. Also, the obtained p_T distribution for quark (antiquark) production can be used in the analysis of the experimental results at the RHIC and the LHC colliders.

The paper is organized as follows: in the next section, we will calculate the one loop effective action needed in the evaluation of the p_T distribution of the quark (antiquark) production. In Section 3 the p_T distribution is presented. Finally, in Section 4, an application to heavy ion collision is given.

2. The One Loop Effective Action

As described in the above section, we will evaluate here the one loop effective action in the presence of a

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constant chromo-field. For this purpose, we start from the QCD Lagrangian density for a quark in a non-abelian background field A_μ^a which is given by

$$\mathcal{L} = \bar{\psi}\left[\left(\not{\hat{p}} - gT^a \not{A}^a\right) - m\right]\psi = \bar{\psi}\, D[A]\psi, \tag{1}$$

Then the vacuum to vacuum transition amplitude is given by

$$\langle 0|0\rangle = \frac{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \bar{\psi}\, D[A]\psi}}{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \bar{\psi}\, D[0]\psi}} = \mathrm{Det}\left[D[A]\right] / \mathrm{Det}\left[D[0]\right]. \tag{2}$$

And the one loop effective action can be written in this form

$$S = -\mathrm{i}\ln\langle 0|0\rangle = -\mathrm{i}\,\mathrm{Tr}\ln\left[\frac{\left(\not{\hat{p}} - gT^a \not{A}^a\right) - m}{\not{\hat{p}} - m}\right]. \tag{3}$$

Thus, using the invariance of trace under transposition and the following relation

$$\ln\frac{a}{b} = \int_0^\infty \frac{\mathrm{d}s}{s}\left[\mathrm{e}^{\mathrm{i}s(b+\mathrm{i}\epsilon)} - \mathrm{e}^{\mathrm{i}s(a+\mathrm{i}\epsilon)}\right], \tag{4}$$

we obtain the following expression[1]

$$2S = \mathrm{i}\mathrm{Tr}\int_0^\infty \frac{\mathrm{d}s}{s}\left[\exp \mathrm{i}s\left[\left(\hat{p} - gT^a A^a\right)^2 + \frac{g}{2}\sigma_{\mu\nu}T^a F^{a\mu\nu} - m^2 + \mathrm{i}\epsilon\right] - \exp \mathrm{i}s\left[\hat{p}^2 - m^2 + \mathrm{i}\epsilon\right]\right]. \tag{5}$$

The quickest way to calculate the effective action is to work in a basis $|\Psi\rangle$ that are the eigenstates of $\hat{H}$ defined by:

$$\hat{H} = \left(\hat{p} - gT^a A^a\right)^2 + \frac{g}{2}\sigma_{\mu\nu}T^a F^{a\mu\nu}. \tag{6}$$

which is a part of the one loop effective action S of Equation (5).

As an application to this idea, we first consider the case of a constant electric field in the z direction (direction of the beam in the heavy ion collision). In this case, we choose a gauge such that we can take $A_z^a = E^a t$. Thus the second part of Equation (6) can be written in this form

$$\frac{g}{2}\sigma_{\mu\nu}T^a F^{a\mu\nu} = \mathrm{i}gE^a T^a \sigma_3 \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{7}$$

The Hamiltonian becomes

$$\hat{H} = \hat{p}_t^2 - \hat{p}_x^2 - \hat{p}_y^2 - \left(\hat{p}_z - gT^a E^a t\right)^2 + \frac{g}{2}\sigma_{\mu\nu}T^a F^{a\mu\nu}. \tag{8}$$

After a straightforward algebra one can find the following eigenvalues of the Hamiltonian $\hat{H}$

$$E_n^{p_x,p_y,p_z,\Lambda_i,\lambda_j} = -\hat{p}_T^2 - g\lambda_j(2n+1) + \mathrm{i}g\Lambda_i\lambda_j. \tag{9}$$

where Λ_i are the eigenvalues over the Dirac matrices such that $\Lambda_1 = \Lambda_3 = 1$, and $\Lambda_2 = \Lambda_4 = -1$. And λ_j, with $j = 1,2,3$, are the eigenvalue for $\lambda = T^a E^a$ over the group space and are given by [18].

$$\lambda_1 = \sqrt{\frac{C_1}{3}}\cos\theta,\quad \lambda_2 = \sqrt{\frac{C_1}{3}}\cos(2\pi/3 - \theta),\quad \lambda_3 = \sqrt{\frac{C_1}{3}}\cos(2\pi/3 + \theta), \tag{10}$$

[1]see Ref. [18] and reference therein.

with θ given by

$$0 \le \cos^2(3\theta) = 3C_2/C_1^3 \le 1. \tag{11}$$

where

$$C_1 = E^a E^a, C_2 = \left[d_{abc}E^a E^b E^c\right]^2. \tag{12}$$

Using the obtained eigenvalues of the Hamiltonian $\hat{H}$, the effective action becomes

$$2S = \mathrm{i}\int_0^\infty \frac{\mathrm{d}s}{s}\sum_{i=1}^{4}\sum_{j=1}^{3}\frac{1}{(2\pi)^3}\int \mathrm{d}^4x \int \mathrm{d}^2 p_T\, \mathrm{e}^{-\mathrm{i}s\left(p_T^2+m^2\right)-s\epsilon}\left[\sum_{n=0}^{\infty}\left|g\lambda_j\right| \mathrm{e}^{sg\lambda_j(2n+1)-sg\Lambda_i\lambda_j} - \frac{1}{2s}\right]. \tag{13}$$

Performing the i and n summations we found

$$2S = \mathrm{i}\int_0^\infty \frac{\mathrm{d}s}{s}\sum_{j=1}^{3}\frac{1}{4\pi^3}\int \mathrm{d}^4x \int \mathrm{d}^2 p_T\, \mathrm{e}^{-\mathrm{i}s\left(p_T^2+m^2\right)-s\epsilon}\left[\left|g\lambda_j\right|\frac{\cosh sg\lambda_j}{\sinh s\left|g\lambda_j\right|} - \frac{1}{s}\right]. \tag{14}$$

which is the same results as Ref. [18]. Clearly, the one loop magnetic effective action can be found upon the following substitution $E^a \to -\mathrm{i}B^a$. Therefore

$$2S^{(m)} = \mathrm{i}\int_0^\infty \frac{\mathrm{d}s}{s}\sum_{j=1}^{3}\frac{1}{4\pi^3}\int \mathrm{d}^4x \int \mathrm{d}^2 p_T\, \mathrm{e}^{-\mathrm{i}s\left(p_T^2+m^2\right)-s\epsilon}\left[\left|g\lambda_j\right|\frac{\cos sg\lambda_j}{\sin s\left|g\lambda_j\right|} - \frac{1}{s}\right]. \tag{15}$$

3. Pair Production in Non-Perturbative QCD

Now, in the same manner as in Ref. [18] we may derive the non-perturbative quarks (antiquarks) production per unit time, per unit volume and per unit transverse momentum from a given constant chromo-electric field E^a. Thus as done in Ref. [18] we can find that

$$\frac{\mathrm{d}N_{q,\bar{q}}}{\mathrm{d}t\mathrm{d}^3x\mathrm{d}^2 p_T} = -\frac{1}{4\pi^3}\sum_{j=1}^{3}\left|g\lambda_j\right|\ln\left\{1-\exp\left[-\frac{\pi\left(p_T^2+m^2\right)}{\left|g\lambda_j\right|}\right]\right\}, \tag{16}$$

where m is the effective mass of the quark and the eigenvalues λ_j are given above.

4. Application to Heavy Ion Collisions

Let's consider the situation of two relativistic heavy nuclei colliding and leaving behind a semi-classical gluon field which then non-perturbatively produces gluon and quark-antiquark pairs via the Schwinger mechanism [19]. As estimated in Ref. [24] for Au-Au collision at RHIC collider with $R \approx 10$ fm and center-of-mass energy ≈ 200 GeV per nucleon, the initial energy density is $\rho \approx 100$ GeV^4 and $C_1 \sim 100$ GeV^4. For our analysis we take $\theta = 0$ which can be justified by the sensitivity check that has been made in Ref. [24] where it has been found that the production rate is not very sensitive to C_2.

In **Figure 1** we plot the rate of quark production as a function of the transverse momentum for two values of $\alpha_s = 0.3$ (used in [25]) and $\alpha_s = 0.4$ with initial energy density $\rho \approx 100$ GeV^4. Clearly seen from this figure that the production rate decrease with p_T and becomes negligible at $p_T \sim 3$ GeV. The obtained p_T distribution for quark (antiquark) production can be used to fix the initial conditions for the QGP in heavy ion collision at the RHIC and the LHC colliders.

5. Conclusion

In this note we have proposed a method for calculating the vacuum to vacuum transition amplitude in the presence of the non-abelian background field. The method can be applied to a general background field and it can be updated to study the non-perturbative soft gluon production [26]. Also, we have evaluated the rate for

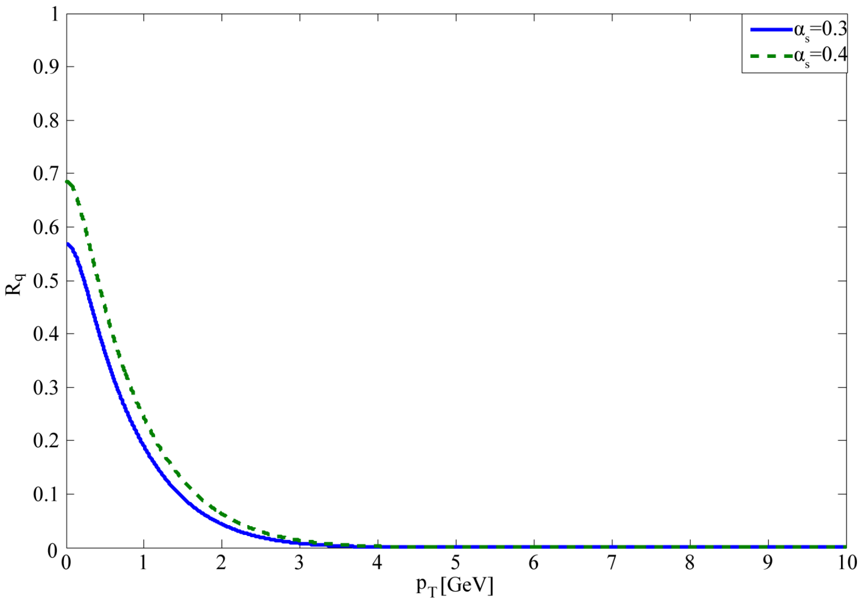

Figure 1. (Color online) Transverse production rate for quarks for $C_1 = 100\ \text{GeV}^4$ for $\alpha_s = 0.3, 0.4$, as a function of p_T. For simplicity we denote here the quark production rate given in Equation (16) by R_q. We take $\theta = 0$, $m = m_q \approx 1/3$ GeV.

quark (antiquark) production in a constant chromo-electric field E^a. These results are used to determine the quark (antiquark) production rate in heavy ion collision.

References

[1] Collins, J.C. and Perry, M.J. (1975) *Physical Review Letters*, **34**, 1353. http://dx.doi.org/10.1103/PhysRevLett.34.1353

[2] Karsch, F., Laermann, E. and Peikert, A. (2000) *Physics Letters B*, **478**, 447-455. http://dx.doi.org/10.1016/S0370-2693(00)00292-6

[3] Hamieh, S., Letessier, J. and Rafelski, J. (2003) *Physical Review A*, **67**, Article ID: 014301. http://dx.doi.org/10.1103/PhysRevA.67.014301

[4] Hamieh, S., Redlich, K. and Tounsi, A. (2004) *Journal of Physics G*, **30**, 481, http://dx.doi.org/10.1088/0954-3899/30/4/007

[5] Baym, G. (1984) *Physics Letters B*, **138**, 18-22. http://dx.doi.org/10.1016/0370-2693(84)91863-X

[6] Kajantie, K. and Matsui, T. (1985) *Physics Letters B*, **164**, 373-378. http://dx.doi.org/10.1016/0370-2693(85)90343-0

[7] Eskola, K.J. and Gyulassy, M. (1993) *Physical Review C*, **47**, 2329. http://dx.doi.org/10.1103/PhysRevC.47.2329

[8] Nayak, G.C. and Ravishankar, V. (1997) *Physical Review D*, **55**, 6877. http://dx.doi.org/10.1103/PhysRevD.55.6877

[9] Nayak, G.C. and Ravishankar, V. (1998) *Physical Review C*, **58**, 356. http://dx.doi.org/10.1103/PhysRevC.58.356

[10] Bhalerao, R.S. and Nayak, G.C. (2000) *Physical Review C*, **61**, Article ID: 054907. http://dx.doi.org/10.1103/PhysRevC.61.054907

[11] Kluger, Y., Eisenberg, J.M., Svetitsky, B., Cooper, F. and Mottola, E. (1991) *Physical Review Letters*, **67**, 2427; http://dx.doi.org/10.1103/PhysRevLett.67.2427

[12] Kluger, Y., Eisenberg, J.M., Svetitsky, B., Cooper, F. and Mottola, E. (1992) *Physical Review D*, **45**, 4659. http://dx.doi.org/10.1103/PhysRevD.45.4659

[13] Kharzeev, D. and Tuchin, K. hep-ph/0501234.

[14] McLerran, L. and Venugopalan, R. (1994) *Physical Review D*, **50**, 2225. http://dx.doi.org/10.1103/PhysRevD.50.2225

[15] Dietrich, D.D., Nayak, G.C. and Greiner, W. (2001) *Physical Review D*, **64**, Article ID: 074006. http://dx.doi.org/10.1103/PhysRevD.64.074006

[16] Gyulassy, M. and McLerran, L. (1997) *Physical Review C*, **56**, 2219. http://dx.doi.org/10.1103/PhysRevC.56.2219

[17] Dietrich, D.D. *Physical Review D*, **7**.

[18] Nayak, G.C. and Van Nieuwenhuizen, P. (2005) *Physical Review D*, **72**, 125010. http://dx.doi.org/10.1103/PhysRevD.72.125010

[19] Schwinger, J. (1951) *Physical Review*, **82**, 664. http://dx.doi.org/10.1103/PhysRev.82.664

[20] Claudson, M., Yildiz, A. and Cox, P.H. (1980) *Physical Review D*, **22**, 2022. http://dx.doi.org/10.1103/PhysRevD.22.2022

[21] Casher, A., Neuberger, H. and Nussinov, S. (1979) *Physical Review D*, **20**, 179. http://dx.doi.org/10.1103/PhysRevD.20.179

[22] Glendenning, N.K. and Matsui, T. (1983) *Physical Review D*, **28**, 2890. http://dx.doi.org/10.1103/PhysRevD.28.2890

[23] Dunne, G.V. (2004) Heisenberg-Euler Effective Lagrangians: Basics and Extensions, hep-th/0406216. In: Shifman, M., *et al.*, Eds., *From Fields to Strings*, Vol. 1, 445-522.

[24] Cooper, F., Dawson, J.F. and Mihaila, B. (2008) *Physical Review D*, **78**, 117901. http://dx.doi.org/10.1103/PhysRevD.78.117901

[25] Aurenche, P. and Zakharov, B.G. arXiv:1205.6462 [hep-ph].

[26] work in progress.