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Phenomenological and Semi-microscopic Analysis for the Elastic Scattering of Protons from ^{12}C Nuclei at Different Energies

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ABSTRACT

Analysis of the elastic scattering of protons from ^{12}C nuclei had been performed within the framework of both the optical model and single folding model at different proton energies; 17, 30.3, 40, 49.48 and 61.4 MeV. We have obtained the global potential parameters which could fairly reproduce the experimental data for p+^{12}C elastic scattering at the aforementioned energies. The radial and energy dependence of the real and imaginary parts of the potential were calculated. Good agreement between experimental data and theoretical predictions in the whole angular range was obtained using both phenomenological approach (Optical Model), and semi-microscopic approach (Single Folding). In single folding calculations, the real part of the potential was calculated from a more fundamental basis by the folding method in which the *NN* interaction $V_{NN}(r)$, is folded into the density of the target nuclei and supplemented with a phenomenological imaginary potential. The obtained normalization factor N_r is in the range of 0.75 - 0.9.

Keywords: Elastic Scattering; Optical Model; Nuclear Structure; Single Folding Model

1. Introduction

Elastic scattering of nucleon-nucleus data at intermediate energies is a useful tool for testing and analyzing nuclear structure models and intermediate energy reaction theories [1-10]. The elastic scattering of protons from ^{12}C was analyzed at different energies (17, 30.3, 40, 49.48 and 61.4 MeV) from literature [11-15]. The analysis of the experimental data was performed either by using Wood-Saxons (WS) forms for both real and imaginary parts of the potential, or by obtaining the real part from the folding procedure [16,17] and using it with a WS term for the imaginary part of the potential, in addition to spin orbit potential which has been introduced due to the 0.5 spin of protons.

The folding model which is a powerful tool for the microscopic analysis of nuclear reactions has been used for years to calculate the nucleon-nucleus optical potential and inelastic form factors. It can be seen from the basic folding formulas that this model generates the first-order term of the microscopic optical potential that is derived from Feshbach's theory of nuclear reactions. The success of this approach in describing the observed nucleon-nucleus elastic scattering data for many targets suggests that the first-order term of the microscopic optical potential is indeed the dominant part of the nucleon optical potential. A popular choice for the effective *NN* interaction has been one of the M3Y interactions. Although these density independent M3Y interactions were originally developed for using in the Distorted Wave Born Approximation (DWBA) for the analysis of (p,p′) reaction, they have been used much more often in the double folding calculation of the heavy ion interaction potential at low and medium energies. The elastic scattering of proton nucleus has been analyzed in order to determine ground state matter densities empirically for comparison with Hartree–Fock predictions [18-20]. In single folding calculations, the real part of potential obtained from the folding model was supplemented by a phenomenological imaginary potential, and during the fitting process the real potential was normalized and the imaginary potential optimized. The basic inputs for a single folding calculation of the nucleon-nucleus potential are the nuclear densities of the target and the effective nucleon-nucleon (NN) interaction. The folding model is a very useful approach to check the target nuclear densities [21].

2. The Nuclear Optical Model and Single Folding Model

Optical model analysis of proton scattering data have

been carried out for a wide range of incident proton energies, and a few attempts have been made to empirically determine the energy dependence of the optical model potential. In practice it is required to obtain the potential from the experimental data, and this may be done by systematically varying the parameters of the potential to optimize the overall fit to the data, using appropriate computer programs. To do the best fitting, we kept the product $V_0 r_0^2$ constant, the same we have done for the imaginary part. In the energy region below 61 MeV, extensive proton elastic scattering data exist. These have, in general been analyzed in terms of an optical model in which the interaction is represented as the scattering of a point particle (proton) by a potential of form:

$$U_{op}(r) = V_C(r) - V(r) - iW(r) - V_{so}(r) \quad (1)$$

The Coulomb potential was assumed to be as two uniform charge distributions with radii consistent with electron scattering [22].

$$V_C(r) = \frac{Z_p Z_t e^2}{2R_C}\left(3 - r^2/R_C^2\right) \quad \text{for} \quad r \le R_C \quad (2)$$

$$V_C(r) = \frac{Z_p Z_t e^2}{r} \quad \text{for} \quad r > R_C$$

with radius

$$R_i = r_i (A_T^{1/3}), \quad i = V, W, C$$

Real volume part has the following form:

$$V(r) f(r, r_V, a_V) = V_0 \left[1 + \exp\left(\frac{r - r_V}{a_V}\right)\right]^{-1} \quad (3)$$

Imaginary volume part has the following form:

$$W_V(r) f(r, r_w, a_w) = -W_0 \left[1 + \exp\left(\frac{r - r_W}{a_W}\right)\right]^{-1}, \quad (4)$$

Real spin-orbit part has the following form:

$$V_{so}(r) = \frac{2}{h^2 r} V_{so} \left[-\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{1 + \exp\left(\frac{r - r_{so}A^{1/3}}{a_{so}}\right)}\right]\right] \quad (5)$$

The spin-orbit term $U_{so}(r) = V_{so}(r) + iW_{so}(r)$, it is usual to take $W_{so}(r) = 0$, leaving the three parameters V_{so}, $r_{so}$and a_W .The model thus involves nine parameters although several analysis have been performed using more restricted sets by equating some of the geometrical parameters and/or neglecting one the imaginary terms. So, the interaction potential can be rewritten as:

$$U(r) = V_C(r) - V_0 f(r, r_V, a_V) - iW_0 f(r, r_W, a_W) + \frac{2}{h^2 r} V_{so} \frac{\mathrm{d}}{\mathrm{d}r} f(r, r_{so}, a_{so}) \quad (6)$$

Many such analysis of nucleon scattering have now been made and is found that the potentials are quite similar for all nuclei and vary other slowly with the incident energy.

In practice it is frequently found that many sets of parameters give equally good fits to the data, and the question then arises whether any one of these is more physical than the others and if so which is to be preferred. These parameter ambiguities, as they are called, are of two main types, discrete and continuous. The existence of these ambiguities means that it is not possible to establish the optical potential by phenomenological analyses alone and it is necessary to derive the potential using more microscopic method such as double folding. The real part of the optical potential for the nucleon–nucleus elastic scattering is given within the framework of single folding model, in the following form:

$$V^{SF}(R) = N_r \int \rho_1(r_1) V_{NN}(|R - r_1|) d^3 r_1 \quad (7)$$

where, $\rho_1(r_1)$ is the matter density distribution of the target nucleus (^{12}C), V_{NN} is the effective NN-interaction. In the present calculation the effective NN-interaction is taken according to [23] in the form of M3Y-interaction

$$V(R) = 7999\frac{\exp(-4R)}{4R} - 2134\frac{\exp(-2.5R)}{2.5R} - 276\left(1 - \frac{0.005E}{A}\right)\delta(R) \quad (8)$$

The nuclear density distribution for ^{12}C was calculated using Three-parameter Fermi model (3PF), where $\rho(r)$ was calculated from the following formula

$$\rho(r) = \rho_0 (1 + \frac{wr^2}{c^2}) / (1 + \exp((r - c)/z)) \quad (9)$$

with w = - 0.149, z = 0.5224 and c = 2.355.

3. Results and Discussion

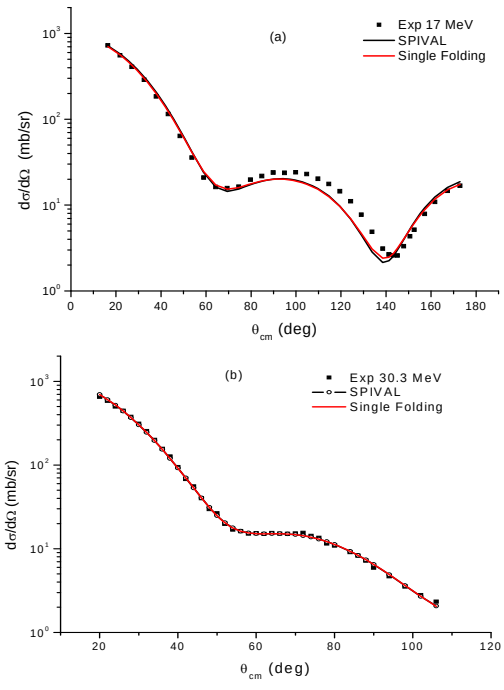
The comparison between the experimental data and the theoretical predictions within the framework of both; the optical model and single folding model at energies 17.0, 30.3, 40.0, 49.48 and 61.4 MeV is shown in **Figure 1**. The optimal optical potential parameters obtained from (SPIVAL code) [24], and also those from single folding model (DFPOT code) [25] using Three parameter Fermi model for calculating ^{12}C density distributions are shown in **Table 1**. The obtained normalization factor (Nr) is in the range 0.9 - 0.75. We investigated the energy dependence on the values of V_0 and W_0 for ^{12}C(p,p)^{12}C (**Figures 2** and **3**), which showed that, with increasing energy, the value of the real potential depth decreases and can be approximated by the formula: V = 66.39 - 0.5997 E, and the imaginary potential depth increases and can be approximated by the formula: W = 14.856 + 0.0887 E. The

radii of the real and imaginary parts of the potential were fixed at r_V = 1.15 fm and r_W = 1.25 fm, Coulomb radius parameter was fixed at 1.25 fm and the radius parameter r_{so} for spin orbit potential was fixed at 1.1 fm. The radial dependences of the real and imaginary parts of the potentials are shown in **Figures 4** and **5** respectively.

Table 1. Optimal potential parameters from SPIVAL code, and also those from single folding model.

E (MeV)	V_0 (MeV)	a_V (fm)	N_r	W_0 (MeV)	a_W (fm)	V_{so} (MeV)	a_{so} (fm)
17.0 OM	53.08	0.825	0.9	16.11	0.765	1.42	0.234
SF				16.09	0.764	1.42	0.334
30.3 OM	50.97	0. 84	0.91	17.88	0.701	1.42	0.234
SF				21.21	0.753	1.42	0.334
40.0 OM	44.79	0. 78	0.9	18.48	0.923	6.34	0.566
SF				10.1	1.33	6.34	0.666
49.5 OM	36.86	0.88	0.78	19.18	0.826	7.75	0.655
SF				22.13	0.788	7.75	0.655
61.4 OM	27.41	0.84	0.75	20.21	1.06	9.5	0.528
SF				17.15	1.01	9.5	0.528

E (MeV)	J_V (MeV.fm^3)	J_W (MeV.fm^3)
17.0 OM	276.20	93.68
SF	275.94	93.51
30.3 OM	271.68	124.66
SF	267.36	121.90
40.0 OM	222.93	126.94
SF	184.38	106.75
49.5 OM	204.83	118.93
SF	215.53	131.86
61.4 OM	145.5	137.43
SF	155.76	129.36



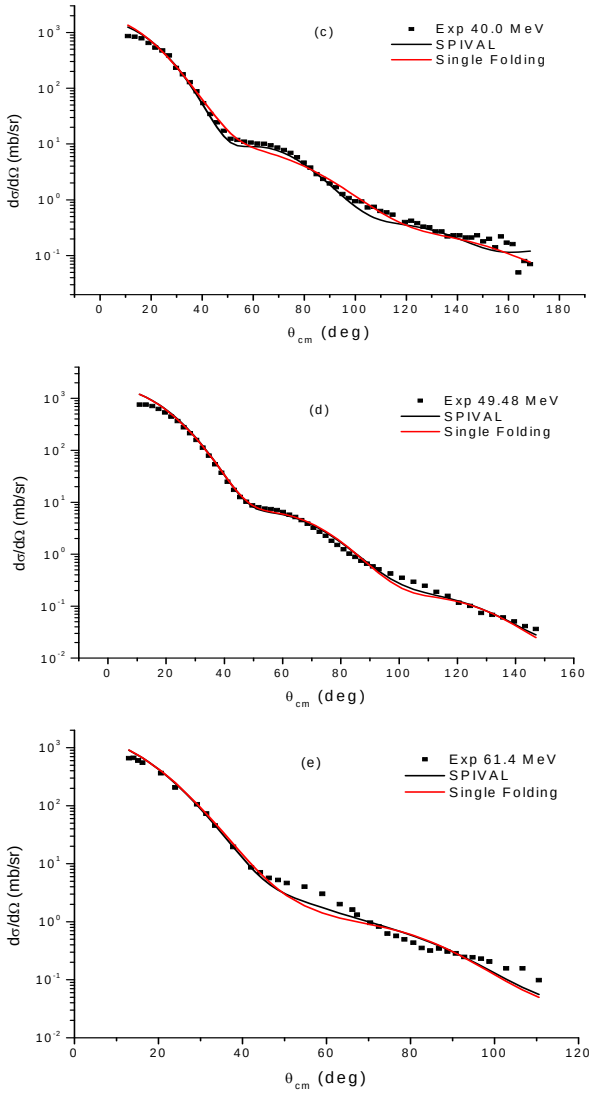


Figure 1. The comparison between the experimental data for p+^{12}C elastic scattering and the theoretical predictions using both optical and single folding model at energies (a) 17 MeV, (b) 30.3, (c) 40 MeV, (d) 49.48 MeV and (e) 61.4 MeV.

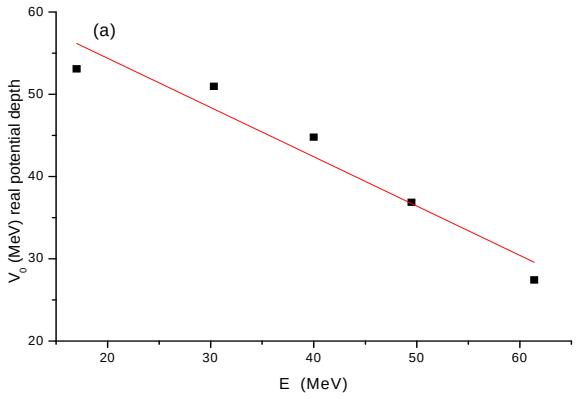


Figure 2. The relation between the real potential depth (V_0) and energy (E).

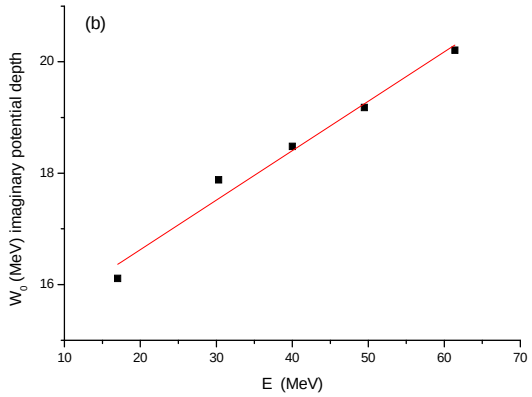


Figure 3. The relation between the imaginary potential depth (W_0) and energy (E).

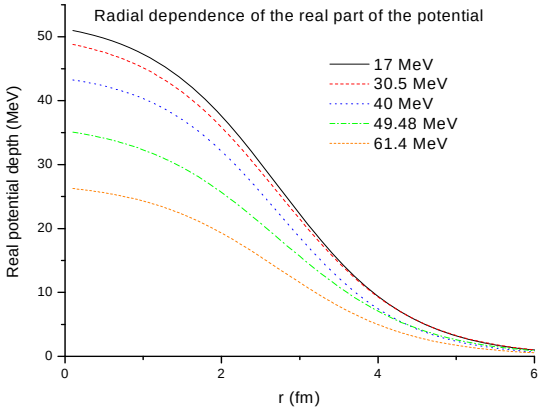


Figure 4. The radial dependences for the real part of the potentials.

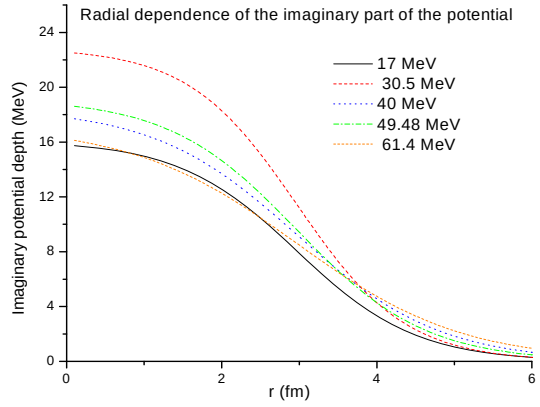


Figure 5. The radial dependences for the imaginary part of the potentials.

4. Summary

The analysis of the elastic scattering of protons from ^{12}C at energies 17, 30.3, 40, 49.48 and 61.4 Mev was performed within the framework of two approaches: an optical code SPIVAL and single folding potential using DFPOT code. Both approaches give satisfactory results. The normalization factor **Nr** was calculated and found to be in the range 0.75-0.9. A good agreement in the whole energy range was found using the two previous discussed approaches with reliable values for the real and imaginary volume integral J_V, J_W.

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