

Numerical Calculation of Normalization Factor for the Analysis of Azimuthal Asymmetry Amplitudes Related to Deeply Virtual Compton Scattering

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Received February 21, 2013; revised April 1, 2013; accepted May 2, 2013

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ABSTRACT

Numerical calculation of normalization factor in a maximum likelihood technique providing a bin-free fit in azimuthal angles to the data is presented. It can be used for simultaneous extraction of asymmetry amplitudes with respect to the combined dependence of the cross section of hard exclusive leptonproduction of real photons on transversely polarized protons on beam charge and helicity and target transverse polarization. This method of data processing can reveal previously unseparated contributions from deeply virtual Compton scattering and its interference with the Bethe-Heitler process.

Keywords: GPDs; DVCS; Transversely Polarized Hydrogen Target

1. Deeply Virtual Compton Scattering

Deeply virtual Compton scattering (DVCS) is one of the cleanest processes that used to constraint Generalized Parton Distributions (GPDs) [1-3] via Compton Form Factors (CFFs). Such CFFs are convolutions of corresponding GPDs with hard scattering coefficient functions.

The five-fold differential cross section for the exclusive leptonproduction of real photons from a transversely polarized hydrogen target reads [4]

$$\frac{d^5\sigma}{dx_B dQ^2 dt |d\phi d\phi_S} = \frac{x_B e^6}{32(2\pi)^4 Q^4} \frac{|\mathcal{T}|^2}{\sqrt{1+\varepsilon^2}}. \quad (1.1)$$

Here, e is the elementary charge, $x_B \equiv Q^2/(2p \cdot q)$ is the Bjorken scaling variable, where p is the proton's four momentum and Q^2 is the negative square of the four-momentum of the virtual photon q that mediates the lepton-nucleon scattering process, t is the Mandelstam variable, which is the squared momentum transfer to the target nucleon, $\varepsilon \equiv 2x_B M_p / Q^2$ with M_p the proton mass and \mathcal{T} is the reaction amplitude. Two azimuthal angles ϕ and ϕ_S appear in the cross section in the case of transverse polarization of the target. The angle ϕ is defined as

the angle between the lepton scattering plane and the real photon production plane, while the angle ϕ_S denotes the angle between the lepton scattering plane and the component of the target polarization vector orthogonal to the momentum of the virtual photon

The initial and final states of the DVCS process are indistinguishable from those of the Bethe-Heitler (BH) process, which is elastic lepton-nucleon scattering with the emission of a bremsstrahlung photon by the lepton. Hence the cross section contains the coherent superposition of the BH and DVCS amplitudes:

$$|\mathcal{T}|^2 = |\mathcal{T}_{BH}|^2 + |\mathcal{T}_{DVCS}|^2 + I, \quad (1.2)$$

where $I = \mathcal{T}_{DVCS} \mathcal{T}_{BH}^* + \mathcal{T}_{DVCS}^* \mathcal{T}_{BH}$ denotes the BH-DVCS interference term. The BH amplitude is calculable to leading order in QED using the form factors measured in elastic scattering. The interference term I and the squared DVCS amplitude $|\mathcal{T}_{DVCS}|^2$ in Equation (1.2) provide experimental access to the (complex) DVCS amplitude through measurements of various cross-section asymmetries [4].

Each of the three terms of Equation (1.2) can be decomposed as a Fourier series:

$$|\mathcal{T}_{BH}|^2 = \frac{K_{BH}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ \sum_{n=0}^2 c_{n,ump}^{BH} \cos(n\phi) + \lambda S_{\perp} \left[\sum_{n=0}^1 c_{n,TP}^{BH} \cos(\phi - \phi_S) \cos(n\phi) + s_{1,TP}^{BH} \sin(\phi - \phi_S) \sin(\phi) \right] \right\}, \quad (1.3)$$

$$\begin{aligned}
 |T_{DVCS}|^2 = & K_{DVCS} \left\{ \sum_{n=0}^2 c_{n,unp}^{DVCS} \cos(n\phi) + \lambda s_{1,unp}^{DVCS} \sin(\phi) \right. \\
 & + S_{\perp} \left[\sum_{n=0}^2 c_{n,TP-}^{DVCS} \sin(\phi - \phi_S) \cos(n\phi) + \sum_{n=1}^2 s_{n,TP+}^{DVCS} \cos(\phi - \phi_S) \sin(n\phi) \right] \\
 & \left. + \lambda S_{\perp} \left[\sum_{n=0}^1 c_{n,TP+}^{DVCS} \cos(\phi - \phi_S) \cos(n\phi) + s_{1,TP-}^{DVCS} \sin(\phi - \phi_S) \sin(n\phi) \right] \right\}, \tag{1.4}
 \end{aligned}$$

$$\begin{aligned}
 I = & \frac{K_I e_{\ell}}{\mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ \sum_{n=0}^3 c_{n,unp}^I \cos(n\phi) + \lambda \left[\sum_{n=1}^2 s_{n,unp}^I \sin(n\phi) \right] \right. \\
 & + S_{\perp} \left[\sum_{n=0}^3 c_{n,TP-}^I \sin(\phi - \phi_S) \cos(n\phi) + \sum_{n=1}^3 s_{n,TP+}^I \cos(\phi - \phi_S) \sin(n\phi) \right] \\
 & \left. + \lambda S_{\perp} \left[\sum_{n=0}^2 c_{n,TP+}^I \cos(\phi - \phi_S) \cos(n\phi) + \sum_{n=1}^2 s_{n,TP-}^I \sin(\phi - \phi_S) \sin(n\phi) \right] \right\}. \tag{1.5}
 \end{aligned}$$

The symbols $K_{BH} = 1/\left[x_B^2 t(1 + \varepsilon^2)\right]$, $K_{DVCS} = 1/(Q^2)$ and $K_I = 1/(x_B y t)$ denote kinematic factors, where y is the fraction of the beam energy carried by the virtual photon in the target rest frame and e_{ℓ} stands for the (signed) lepton charge in units of the elementary charge. Also, $\lambda = \pm 1$ and S_{\perp} are respectively the helicity of the lepton beam and the magnitude of the component of the target polarization vector that is orthogonal to the direction of the virtual photon. The subscript ‘‘TP’’ is used for BH terms, while the subscript ‘‘TP+’’ (‘‘TP-’’) is used for DVCS and interference terms containing $\cos(\phi - \phi_S)$ ($\sin(\phi - \phi_S)$). The BH coefficients $c_{n,unp}^{BH}$, $c_{n,TP}^{BH}$ and $s_{1,TP}^{BH}$ in Equation (1.3) depend on electromagnetic elastic form factors of the target, while the DVCS (interference) coefficients $c_{n,unp}^{DVCS}$ ($c_{n,unp}^I$), $s_{1,unp}^{DVCS}$ ($s_{n,unp}^I$), $c_{n,TP+(-)}^{DVCS}$ ($c_{n,TP+(-)}^I$) and $s_{n,TP+(-)}^{DVCS}$ ($s_{n,TP+(-)}^I$) involve various

GDs. The squared BH and interference terms in Equation (1.3) and Equation (1.5) have an additional ϕ dependence in the denominator due to the lepton propagators $\mathcal{P}_1(\phi)$ and $\mathcal{P}_2(\phi)$ [4,5]. The Fourier coefficients appearing in the interference term can be expressed as linear combinations of CFFs [4], while the coefficients from the squared DVCS term are bilinear in the CFFs.

The coefficients of particular interest in Ref. [6] were $c_{n,TP}^{BH}$, $s_{1,TP}^{BH}$, $c_{n,TP+}^{DVCS}$, $s_{1,TP-}^{DVCS}$, $c_{n,TP+}^I$ and $s_{1,TP-}^I$, which relate the double-spin asymmetries involving transverse target polarization. (The dependences of beam-charge and charge-difference or charge-averaged single-spin asymmetry amplitudes on remaining Fourier coefficients in Equations (1.3)-(1.5) were discussed in previously published HERMES papers [7-10]). The leading (twist-two) coefficients $c_{0,TP+}^I$, $c_{1,TP+}^I$ and $s_{1,TP-}^I$ can be approximated as [4]

$$\begin{aligned}
 c_{0,TP+}^I & \approx \frac{8M_p \sqrt{1-y} K}{Q} y \Re e \left\{ \left(\frac{(2-y)^2}{1-y} + 2 \right) C_{TP+}^I + \Delta C_{TP+}^I \right\}, \tag{1.6}
 \end{aligned}$$

$$c_{1,TP+}^I \approx \frac{8M_p \sqrt{1-y}}{Q} y(2-y) \Re e C_{TP+}^I, \tag{1.7}$$

$$s_{1,TP-}^I \approx \frac{8M_p \sqrt{1-y}}{Q} y(2-y) \Re e C_{TP-}^I. \tag{1.8}$$

Here, K is a kinematic factor and the C -functions C_{TP+}^I , C_{TP-}^I and ΔC_{TP+}^I can be expressed as linear combination of four CFFs (\mathcal{H} , \mathcal{E} , $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$) and the Dirac and Pauli electromagnetic form factors F_1 and F_2 , respectively:

$$\begin{aligned}
 C_{TP+}^I = & (F_1 + F_2) \left\{ \frac{x_B^2}{2-x_B} \left(\mathcal{H} + \frac{x_B}{2} \mathcal{E} \right) + \frac{x_B^2}{4M_p^2} \right\} \\
 & - \frac{x_B^2}{2-x_B} F_1 \left(\tilde{\mathcal{H}} + \frac{x_B}{2} \tilde{\mathcal{E}} \right) + \frac{t}{4M_p^2} \left\{ 4 \frac{1-x_B}{2-x_B} F_2 \tilde{\mathcal{H}} \right. \\
 & \left. - \left(x_B F_1 + \frac{x_B^2}{2-x_B} F_2 \right) \tilde{\mathcal{E}} \right\}, \tag{1.9}
 \end{aligned}$$

$$\begin{aligned}
 C_{TP-}^I = & \frac{1}{2-x_B} \left(x_B^2 F_1 - (1-x_B) \frac{t}{M_p^2} F_2 \right) \mathcal{H} \\
 & + \left\{ \frac{t}{4M_p^2} \left((2-x_B) F_1 + \frac{x_B^2}{2-x_B} F_2 \right) + \frac{x_B^2}{2-x_B} F_1 \right\} \mathcal{E} \tag{1.10} \\
 & - \frac{x_B^2}{2-x_B} (F_1 + F_2) \left(\tilde{\mathcal{H}} + \frac{t}{M_p^2} \tilde{\mathcal{E}} \right),
 \end{aligned}$$

$$\Delta C_{TP+}^I = -\frac{t}{M_p^2} \left\{ F_2 \tilde{\mathcal{H}} - \frac{x_B}{2-x_B} \left(F_1 + \frac{x_B}{2} F_2 \right) \tilde{\mathcal{E}} \right\}. \quad (1.11)$$

Note that even if the cross sections were measured for all eight possible combinations of beam charge and helicity and target polarization, at fixed Q^2 it would still be impossible to separate the coefficients $c_{n,TP}^{BH} \left(S_{1,TP}^{BH} \right)$ and $c_{n,TP+}^{DVCS} \left(S_{1,TP+}^{DVCS} \right)$. Nevertheless, the BH coefficients can be calculated from the measured elastic form factors.

2. Azimuthal Asymmetries

The asymmetry in the cross section of a longitudinally polarized electron/positron beam off a transversely polarized hydrogen target, which embodies the essential features of the Fourier coefficients in Equations (1.3)-(1.5), can be defined through

$$\begin{aligned} d\sigma = & d\sigma_{UU} dx_B dQ^2 d|t| d\phi d\phi_S \left[1 + e_\ell A_C(\phi) \right. \\ & + \lambda A_{LU}^{DVCS}(\phi) + S_\perp A_{UT}^{DVCS}(\phi, \phi_S) + e_\ell \lambda A_{LU}^I(\phi) \\ & + e_\ell S_\perp A_{UT}^I(\phi, \phi_S) + \lambda S_\perp A_{LT}^{BH+DVCS}(\phi, \phi_S) \\ & \left. + e_\ell \lambda S_\perp A_{LT}^I(\phi, \phi_S) \right]. \end{aligned} \quad (2.1)$$

Here, $d\sigma_{UU}$ is the differential cross section of production of real photons from scattering of an unpolarized lepton beam on an unpolarized hydrogen target averaged over both beam charges. The seven asymmetries presented in Equation (2.1) are defined as

$$\begin{aligned} A_C = & \frac{1}{d\sigma_{UU}} \left[\left(d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^- \right) \right. \\ & \left. - \left(d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^- + d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\downarrow^+ \right) \right], \end{aligned} \quad (2.2)$$

$$\begin{aligned} A_{LU}^{DVCS} = & \frac{1}{d\sigma_{UU}} \left[\left(d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^- \right) \right. \\ & \left. - \left(d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^- + d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\downarrow^+ \right) \right], \end{aligned} \quad (2.3)$$

$$\begin{aligned} A_{UT}^{DVCS} = & \frac{1}{d\sigma_{UU}} \left[\left(d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\downarrow^- \right) \right. \\ & \left. - \left(d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\downarrow^- + d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\uparrow^- \right) \right], \end{aligned} \quad (2.4)$$

$$\begin{aligned} A_{LU}^I = & \frac{1}{d\sigma_{UU}} \left[\left(d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^- \right) \right. \\ & \left. - \left(d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^- + d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\downarrow^+ \right) \right], \end{aligned} \quad (2.5)$$

$$\begin{aligned} A_{UT}^I = & \frac{1}{d\sigma_{UU}} \left[\left(d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\downarrow^- \right) \right. \\ & \left. - \left(d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\downarrow^- + d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\uparrow^- \right) \right], \end{aligned} \quad (2.6)$$

$$\begin{aligned} A_{LT}^{BH+DVCS} = & \frac{1}{d\sigma_{UU}} \left[\left(d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\downarrow^- \right) \right. \\ & \left. - \left(d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\downarrow^- + d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\uparrow^- \right) \right], \end{aligned} \quad (2.7)$$

$$\begin{aligned} A_{LT}^I = & \frac{1}{d\sigma_{UU}} \left[\left(d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\downarrow^- + d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\downarrow^+ \right) \right. \\ & \left. - \left(d\bar{\sigma}_\downarrow^+ + d\bar{\sigma}_\uparrow^- + d\bar{\sigma}_\uparrow^+ + d\bar{\sigma}_\downarrow^- \right) \right], \end{aligned} \quad (2.8)$$

where the + (−) sign stands for lepton charge and horizontal (vertical) arrows for beam (target) polarization states. The experimentally measured quantities are the trigonometric $\cos(n\phi)$ and $\sin(n\phi)$ moments (amplitudes) of the above defined asymmetries

$$A^{\cos(n\phi)} = \frac{1}{F} \int_0^{2\pi} A \cos(n\phi) d\phi, \quad (2.9)$$

$$A^{\sin(n\phi)} = \frac{1}{F} \int_0^\pi A \sin(n\phi) d\phi. \quad (2.10)$$

Here, F is the normalization factor and is equal to 2π when $n = 0$, and π when $n > 1$.

When extracting the asymmetry amplitudes from the real experiment, one deals with the measured beam and target polarization values, which are not equal to ± 1 . Moreover, they can be different for different data sets. In order to account for these imbalances in the luminosity-averaged beam and target polarization values an appropriate normalization needs to be applied. In this case the distribution in the expectation value of the measured yield can be written through the integrated luminosity \mathcal{L} and values of the beam and target polarization P_ℓ and S_T . The number of events dN in a small time period $d\tau$ and phase-space interval $dx = dQ^2 dx_B d|t| d\phi d\phi_S$ can be written as

$$\begin{aligned} dN(x, \tau) = & \mathcal{L}(\tau) d\tau dx d\sigma_{UU}(x) \left\{ 1 + e_\ell A_C(x) \right. \\ & + P_\ell(\tau) A_{LU}^{DVCS}(x) + S_T(\tau) A_{UT}^{DVCS}(x) \\ & + e_\ell P_\ell(\tau) A_{LU}^I(x) + e_\ell S_T(\tau) A_{UT}^I(x) \\ & + P_\ell(\tau) S_T(\tau) A_{LT}^{BH+DVCS}(x) \\ & \left. + e_\ell P_\ell(\tau) S_T(\tau) A_{LT}^I(x) \right\}. \end{aligned} \quad (2.11)$$

It is convenient to introduce an effective charge P_{eff} as a product of beam charge with beam and target polarization $P_{eff} = e_\ell P_\ell S_T$ and change an integration variable from time τ to effective charge P_{eff} , then integration of Equation (2.11) over P_{eff} for different states gives

$$\begin{aligned} \vec{N}^{\uparrow\uparrow} \equiv & \int_{P_\ell > 0, S_T > 0, e_\ell = +1} \frac{dN(x, P_{eff})}{dx dP_{eff}} dP_{eff} = \vec{L}^{\uparrow\uparrow} d\sigma_{UU}(x) \\ & \left[1 + e_\ell A_C(x) + \vec{\mathbb{P}}^{\uparrow\uparrow} A_{LU}^{DVCS}(x) + \vec{\mathbb{S}}^{\uparrow\uparrow} A_{UT}^{DVCS}(x) \right. \\ & + e_\ell \vec{\mathbb{P}}^{\uparrow\uparrow} A_{LU}^I(x) + e_\ell \vec{\mathbb{S}}^{\uparrow\uparrow} A_{UT}^I(x) \\ & \left. + \vec{\mathbb{P}}^{\uparrow\uparrow} \vec{\mathbb{S}}^{\uparrow\uparrow} A_{LT}^{BH+DVCS}(x) + e_\ell \vec{\mathbb{P}}^{\uparrow\uparrow} \vec{\mathbb{S}}^{\uparrow\uparrow} A_{LT}^I(x) \right], \end{aligned} \quad (2.12)$$

$$\begin{aligned} \bar{N}^{+\downarrow} &\equiv \int_{P_t > 0, S_T < 0, e_t = +1} \frac{dN(x, P_{eff})}{dx dP_{eff}} dP_{eff} = \bar{\mathbb{L}}^{+\downarrow} d\sigma_{UU}(x) \\ &\left[1 + e_t A_C(x) + \bar{\mathbb{P}}^{+\downarrow} A_{LU}^{DVCS}(x) + \bar{\mathbb{S}}^{+\downarrow} A_{UT}^{DVCS}(x) \right. \\ &+ e_t \bar{\mathbb{P}}^{+\downarrow} A_{LU}^I(x) + e_t \bar{\mathbb{S}}^{+\downarrow} A_{UT}^I(x) \\ &\left. + \bar{\mathbb{P}}^{+\downarrow} \bar{\mathbb{S}}^{+\downarrow} A_{LT}^{BH+DVCS}(x) + e_t \bar{\mathbb{P}}^{+\downarrow} \bar{\mathbb{S}}^{+\downarrow} A_{LT}^I(x) \right], \end{aligned} \tag{2.13}$$

$$\begin{aligned} \bar{N}^{+\uparrow} &\equiv \int_{P_t < 0, S_T > 0, e_t = +1} \frac{dN(x, P_{eff})}{dx dP_{eff}} dP_{eff} = \bar{\mathbb{L}}^{+\uparrow} d\sigma_{UU}(x) \\ &\left[1 + e_t A_C(x) + \bar{\mathbb{P}}^{+\uparrow} A_{LU}^{DVCS}(x) + \bar{\mathbb{S}}^{+\uparrow} A_{UT}^{DVCS}(x) \right. \\ &+ e_t \bar{\mathbb{P}}^{+\uparrow} A_{LU}^I(x) + e_t \bar{\mathbb{S}}^{+\uparrow} A_{UT}^I(x) \\ &\left. + \bar{\mathbb{P}}^{+\uparrow} \bar{\mathbb{S}}^{+\uparrow} A_{LT}^{BH+DVCS}(x) + e_t \bar{\mathbb{P}}^{+\uparrow} \bar{\mathbb{S}}^{+\uparrow} A_{LT}^I(x) \right], \end{aligned} \tag{2.14}$$

$$\begin{aligned} \bar{N}^{+\downarrow} &\equiv \int_{P_t < 0, S_T < 0, e_t = +1} \frac{dN(x, P_{eff})}{dx dP_{eff}} dP_{eff} = \bar{\mathbb{L}}^{+\downarrow} d\sigma_{UU}(x) \\ &\left[1 + e_t A_C(x) + \bar{\mathbb{P}}^{+\downarrow} A_{LU}^{DVCS}(x) + \bar{\mathbb{S}}^{+\downarrow} A_{UT}^{DVCS}(x) \right. \\ &+ e_t \bar{\mathbb{P}}^{+\downarrow} A_{LU}^I(x) + e_t \bar{\mathbb{S}}^{+\downarrow} A_{UT}^I(x) \\ &\left. + \bar{\mathbb{P}}^{+\downarrow} \bar{\mathbb{S}}^{+\downarrow} A_{LT}^{BH+DVCS}(x) + e_t \bar{\mathbb{P}}^{+\downarrow} \bar{\mathbb{S}}^{+\downarrow} A_{LT}^I(x) \right], \end{aligned} \tag{2.15}$$

$$\begin{aligned} \bar{N}^{-\uparrow} &\equiv \int_{P_t > 0, S_T > 0, e_t = -1} \frac{dN(x, P_{eff})}{dx dP_{eff}} dP_{eff} = \bar{\mathbb{L}}^{-\uparrow} d\sigma_{UU}(x) \\ &\left[1 + e_t A_C(x) + \bar{\mathbb{P}}^{-\uparrow} A_{LU}^{DVCS}(x) + \bar{\mathbb{S}}^{-\uparrow} A_{UT}^{DVCS}(x) \right. \\ &+ e_t \bar{\mathbb{P}}^{-\uparrow} A_{LU}^I(x) + e_t \bar{\mathbb{S}}^{-\uparrow} A_{UT}^I(x) \\ &\left. + \bar{\mathbb{P}}^{-\uparrow} \bar{\mathbb{S}}^{-\uparrow} A_{LT}^{BH+DVCS}(x) + e_t \bar{\mathbb{P}}^{-\uparrow} \bar{\mathbb{S}}^{-\uparrow} A_{LT}^I(x) \right], \end{aligned} \tag{2.16}$$

$$\begin{aligned} \bar{N}^{-\downarrow} &\equiv \int_{P_t > 0, S_T < 0, e_t = -1} \frac{dN(x, P_{eff})}{dx dP_{eff}} dP_{eff} = \bar{\mathbb{L}}^{-\downarrow} d\sigma_{UU}(x) \\ &\left[1 + e_t A_C(x) + \bar{\mathbb{P}}^{-\downarrow} A_{LU}^{DVCS}(x) + \bar{\mathbb{S}}^{-\downarrow} A_{UT}^{DVCS}(x) \right. \\ &+ e_t \bar{\mathbb{P}}^{-\downarrow} A_{LU}^I(x) + e_t \bar{\mathbb{S}}^{-\downarrow} A_{UT}^I(x) \\ &\left. + \bar{\mathbb{P}}^{-\downarrow} \bar{\mathbb{S}}^{-\downarrow} A_{LT}^{BH+DVCS}(x) + e_t \bar{\mathbb{P}}^{-\downarrow} \bar{\mathbb{S}}^{-\downarrow} A_{LT}^I(x) \right], \end{aligned} \tag{2.17}$$

$$\begin{aligned} \bar{N}^{-\uparrow} &\equiv \int_{P_t < 0, S_T > 0, e_t = -1} \frac{dN(x, P_{eff})}{dx dP_{eff}} dP_{eff} = \bar{\mathbb{L}}^{-\uparrow} d\sigma_{UU}(x) \\ &\left[1 + e_t A_C(x) + \bar{\mathbb{P}}^{-\uparrow} A_{LU}^{DVCS}(x) + \bar{\mathbb{S}}^{-\uparrow} A_{UT}^{DVCS}(x) \right. \\ &+ e_t \bar{\mathbb{P}}^{-\uparrow} A_{LU}^I(x) + e_t \bar{\mathbb{S}}^{-\uparrow} A_{UT}^I(x) \\ &\left. + \bar{\mathbb{P}}^{-\uparrow} \bar{\mathbb{S}}^{-\uparrow} A_{LT}^{BH+DVCS}(x) + e_t \bar{\mathbb{P}}^{-\uparrow} \bar{\mathbb{S}}^{-\uparrow} A_{LT}^I(x) \right], \end{aligned} \tag{2.18}$$

$$\begin{aligned} \bar{N}^{-\downarrow} &\equiv \int_{P_t < 0, S_T < 0, e_t = -1} \frac{dN(x, P_{eff})}{dx dP_{eff}} dP_{eff} = \bar{\mathbb{L}}^{-\downarrow} d\sigma_{UU}(x) \\ &\left[1 + e_t A_C(x) + \bar{\mathbb{P}}^{-\downarrow} A_{LU}^{DVCS}(x) + \bar{\mathbb{S}}^{-\downarrow} A_{UT}^{DVCS}(x) \right. \\ &+ e_t \bar{\mathbb{P}}^{-\downarrow} A_{LU}^I(x) + e_t \bar{\mathbb{S}}^{-\downarrow} A_{UT}^I(x) \\ &\left. + \bar{\mathbb{P}}^{-\downarrow} \bar{\mathbb{S}}^{-\downarrow} A_{LT}^{BH+DVCS}(x) + e_t \bar{\mathbb{P}}^{-\downarrow} \bar{\mathbb{S}}^{-\downarrow} A_{LT}^I(x) \right], \end{aligned} \tag{2.19}$$

where \mathbb{L} is the integrated luminosity for the given state (configuration of beam charge and beam (target) polarization), \mathbb{N} is the corresponding yield in a small phase-space interval, and \mathbb{P} and \mathbb{S} respectively are luminosity-averaged beam and target polarizations for given states. Since in the analyzed data sample the polarization value was not varying to much the averaged value $|\mathbb{S}| = 0.721$ was used for all states.

Resolving the system of Equations (2.12)-(2.19), one can represent the unpolarized cross section $d\sigma_{UU}$ and seven asymmetries through the luminosity normalized yields $n = \mathbb{N}/\mathbb{L}$ for each state and polarization values \mathbb{P} and \mathbb{S} . The analytical solution can be approximated by numerical methods using the measured values for the beam/target polarizations for the various helicity states as input. For example, taking into account the measured values of the beam and target polarization for data used in the analysis of DVCS process on transversely polarized hydrogen target at HERMES (see **Table 1**) one can get

$$\begin{aligned} d\sigma_{UU} &= 0.138405 \bar{n}^{+\uparrow}(x) + 0.138431 \bar{n}^{+\downarrow}(x) \\ &+ 0.111595 \bar{n}^{-\uparrow}(x) + 0.111569 \bar{n}^{+\downarrow}(x) \\ &+ 0.114496 \bar{n}^{-\uparrow}(x) + 0.114567 \bar{n}^{-\downarrow}(x) \\ &+ 0.135504 \bar{n}^{-\uparrow}(x) + 0.135433 \bar{n}^{-\downarrow}(x), \\ A_C(x) &= 1/d\sigma_{UU} \left[0.138405 \bar{n}^{+\uparrow}(x) \right. \\ &+ 0.138431 \bar{n}^{+\downarrow}(x) + 0.111595 \bar{n}^{-\uparrow}(x) \\ &+ 0.111569 \bar{n}^{+\downarrow}(x) - 0.114496 \bar{n}^{-\uparrow}(x) \\ &- 0.114567 \bar{n}^{-\downarrow}(x) - 0.135504 \bar{n}^{-\uparrow}(x) \\ &\left. - 0.135433 \bar{n}^{-\downarrow}(x) \right], \end{aligned} \tag{2.20}$$

$$\begin{aligned} A_{LU}^{DVCS}(x) &= 1/d\sigma_{UU} \left[0.345123 \bar{n}^{+\uparrow}(x) \right. \\ &+ 0.345207 \bar{n}^{+\downarrow}(x) - 0.345123 \bar{n}^{-\uparrow}(x) \\ &- 0.345207 \bar{n}^{+\downarrow}(x) + 0.400390 \bar{n}^{-\uparrow}(x) \\ &+ 0.400740 \bar{n}^{-\downarrow}(x) - 0.400390 \bar{n}^{-\uparrow}(x) \\ &\left. - 0.400740 \bar{n}^{-\downarrow}(x) \right], \end{aligned} \tag{2.22}$$

Table 1. The type of the beam particle, the luminosity-averaged beam and target polarizations and the integrated luminosity of the data sets used for the extraction of the various asymmetry amplitudes on a transversely polarized hydrogen target at HERMES experiment [6]. The data were taken with an e^+ beam during the years 2003 (6.9 pb^{-1}) and 2004 (51.7 pb^{-1}) and an e^- beam during 2005 (98.7 pb^{-1}). The uncertainties for the beam and target polarizations are 2.2% and 8.3%, respectively.

Lepton Type	Longitudinal Beam Polarization		Transverse Target Polarization	Luminosity [pb^{-1}]	
	←	→		←	→
e^-	-0.286	+0.338	+0.721	29.1	20.1
e^-	-0.286	+0.338	-0.721	28.9	20.6
e^+	-0.401	+0.323	+0.721	11.8	17.5
e^+	-0.401	+0.323	-0.721	11.7	17.6
Total				81.5	75.8

$$A_{UT}^{DVCS}(x) = 1/d\sigma_{UU} \left[0.191962\bar{n}^{+\uparrow}(x) - 0.191999\bar{n}^{+\downarrow}(x) + 0.154778\bar{n}^{+\uparrow}(x) - 0.154742\bar{n}^{+\downarrow}(x) + 0.158801\bar{n}^{-\uparrow}(x) - 0.158900\bar{n}^{-\downarrow}(x) + 0.187939\bar{n}^{-\uparrow}(x) - 0.178841\bar{n}^{-\downarrow}(x) \right], \quad (2.23)$$

$$A_{LU}^I(x) = 1/d\sigma_{UU} \left[0.345123\bar{n}^{+\uparrow}(x) + 0.345207\bar{n}^{+\downarrow}(x) - 0.345123\bar{n}^{+\uparrow}(x) - 0.345207\bar{n}^{+\downarrow}(x) - 0.400390\bar{n}^{-\uparrow}(x) - 0.400740\bar{n}^{-\downarrow}(x) + 0.400390\bar{n}^{-\uparrow}(x) + 0.400740\bar{n}^{-\downarrow}(x) \right], \quad (2.24)$$

$$A_{UT}^I(x) = 1/d\sigma_{UU} \left[0.191962\bar{n}^{+\uparrow}(x) - 0.191999\bar{n}^{+\downarrow}(x) + 0.154778\bar{n}^{+\uparrow}(x) - 0.154742\bar{n}^{+\downarrow}(x) - 0.158801\bar{n}^{-\uparrow}(x) + 0.158900\bar{n}^{-\downarrow}(x) - 0.187939\bar{n}^{-\uparrow}(x) + 0.178841\bar{n}^{-\downarrow}(x) \right], \quad (2.25)$$

$$A_{LT}^{BH+DVCS}(x) = 1/d\sigma_{UU} \left[0.478673\bar{n}^{+\uparrow}(x) - 0.478790\bar{n}^{+\downarrow}(x) - 0.478673\bar{n}^{+\uparrow}(x) + 0.478790\bar{n}^{+\downarrow}(x) + 0.555326\bar{n}^{-\uparrow}(x) - 0.555811\bar{n}^{-\downarrow}(x) - 0.555326\bar{n}^{-\uparrow}(x) + 0.555811\bar{n}^{-\downarrow}(x) \right], \quad (2.26)$$

$$A_{LT}^I(x) = 1/d\sigma_{UU} \left[0.478673\bar{n}^{+\uparrow}(x) - 0.478790\bar{n}^{+\downarrow}(x) - 0.478673\bar{n}^{+\uparrow}(x) + 0.478790\bar{n}^{+\downarrow}(x) - 0.555326\bar{n}^{-\uparrow}(x) + 0.555811\bar{n}^{-\downarrow}(x) + 0.555326\bar{n}^{-\uparrow}(x) - 0.555811\bar{n}^{-\downarrow}(x) \right]. \quad (2.27)$$

$$\int \bar{N}(x) dx \approx N_{obs} = \sum_{i=1}^{N_{obs}} 1, \quad (2.28)$$

$$\int \bar{N}(x) A(x) dx = \sum_{i=1}^{N_{obs}} A(x_i) \quad (2.29)$$

will give the total numbers of events or the normalization

$$N = \int \left[\bar{N}^{+\uparrow}(x) + \bar{N}^{+\downarrow}(x) + \bar{N}^{-\uparrow}(x) + \bar{N}^{-\downarrow}(x) + \bar{N}^{-\uparrow}(x) + \bar{N}^{-\downarrow}(x) + \bar{N}^{-\uparrow}(x) + \bar{N}^{-\downarrow}(x) \right] dx \\ \approx \sum_{i=1}^{N_{obs}} K_i(e_\ell, P_\ell, S_T) \left[M_1 + M_2 A_C(x_i) + M_3 A_{LU}^{DVCS}(x_i) + M_4 A_{UT}^{DVCS}(x_i) + M_5 A_{LU}^I(x_i) + M_6 A_{UT}^I(x_i) + M_7 A_{LT}^{BH+DVCS}(x_i) + M_8 A_{LT}^I(x_i) \right]. \quad (2.30)$$

Here,

The above given expressions for the asymmetries can be used for a least square fit without applying any helicity balancing cuts, since the coefficients in front of the each normalized yield account for the polarization imbalances in data sets. In order to obtain the normalization of the maximum likelihood function Equation (2.20) can be substituted into the system of Equations (2.12)-(2.19) and an integration over the whole phase-space x according to

$$K(e_\ell, P_\ell, S_T) = \begin{cases} 0.138405/\bar{\mathbb{L}}^{+\uparrow}(e_\ell = +1, P_\ell > 0, S_T > 0) \\ 0.138431/\bar{\mathbb{L}}^{+\downarrow}(e_\ell = +1, P_\ell > 0, S_T < 0) \\ 0.111595/\bar{\mathbb{L}}^{+\uparrow}(e_\ell = +1, P_\ell < 0, S_T > 0) \\ 0.111569/\bar{\mathbb{L}}^{+\downarrow}(e_\ell = +1, P_\ell < 0, S_T < 0) \\ 0.114496/\bar{\mathbb{L}}^{-\uparrow}(e_\ell = -1, P_\ell > 0, S_T > 0) \\ 0.114567/\bar{\mathbb{L}}^{-\downarrow}(e_\ell = -1, P_\ell > 0, S_T < 0) \\ 0.135504/\bar{\mathbb{L}}^{-\uparrow}(e_\ell = -1, P_\ell < 0, S_T > 0) \\ 0.135433/\bar{\mathbb{L}}^{-\downarrow}(e_\ell = -1, P_\ell < 0, S_T < 0) \end{cases} \quad (2.31)$$

and

$$M_1 = \bar{\mathbb{L}}^{+\uparrow} + \bar{\mathbb{L}}^{+\downarrow} + \bar{\mathbb{L}}^{-\uparrow} + \bar{\mathbb{L}}^{-\downarrow} + \bar{\mathbb{L}}^{-\uparrow} + \bar{\mathbb{L}}^{-\downarrow} + \bar{\mathbb{L}}^{-\uparrow} + \bar{\mathbb{L}}^{-\downarrow}, \quad (2.32)$$

$$M_2 = \bar{L}^{+\uparrow} + \bar{L}^{+\downarrow} + \bar{L}^{-\uparrow} + \bar{L}^{-\downarrow} - \bar{L}^{-\uparrow} - \bar{L}^{-\downarrow} - \bar{L}^{-\uparrow} - \bar{L}^{-\downarrow}, \quad (2.33)$$

$$M_3 = \bar{L}^{+\uparrow}\bar{P}^{+\uparrow} + \bar{L}^{+\downarrow}\bar{P}^{+\downarrow} + \bar{L}^{+\uparrow}\bar{P}^{+\downarrow} + \bar{L}^{+\downarrow}\bar{P}^{+\uparrow} + \bar{L}^{-\uparrow}\bar{P}^{-\uparrow} + \bar{L}^{-\downarrow}\bar{P}^{-\downarrow} + \bar{L}^{-\uparrow}\bar{P}^{-\downarrow} + \bar{L}^{-\downarrow}\bar{P}^{-\uparrow}, \quad (2.34)$$

$$M_4 = \left[\bar{L}^{+\uparrow} - \bar{L}^{+\downarrow} + \bar{L}^{-\uparrow} - \bar{L}^{-\downarrow} + \bar{L}^{-\uparrow} - \bar{L}^{-\downarrow} \right] |S|, \quad (2.35)$$

$$M_5 = \bar{L}^{+\uparrow}\bar{P}^{+\uparrow} + \bar{L}^{+\downarrow}\bar{P}^{+\downarrow} + \bar{L}^{+\uparrow}\bar{P}^{+\downarrow} + \bar{L}^{+\downarrow}\bar{P}^{+\uparrow} - \bar{L}^{-\uparrow}\bar{P}^{-\uparrow} - \bar{L}^{-\downarrow}\bar{P}^{-\downarrow} - \bar{L}^{-\uparrow}\bar{P}^{-\downarrow} - \bar{L}^{-\downarrow}\bar{P}^{-\uparrow}, \quad (2.36)$$

$$M_6 = \left[\bar{L}^{+\uparrow} - \bar{L}^{+\downarrow} + \bar{L}^{-\uparrow} - \bar{L}^{-\downarrow} - \bar{L}^{-\uparrow} + \bar{L}^{-\downarrow} - \bar{L}^{-\uparrow} + \bar{L}^{-\downarrow} \right] |S|, \quad (2.37)$$

$$M_7 = \left[\bar{L}^{+\uparrow}\bar{P}^{+\uparrow} - \bar{L}^{+\downarrow}\bar{P}^{+\downarrow} + \bar{L}^{+\uparrow}\bar{P}^{+\downarrow} - \bar{L}^{+\downarrow}\bar{P}^{+\uparrow} + \bar{L}^{-\uparrow}\bar{P}^{-\uparrow} - \bar{L}^{-\downarrow}\bar{P}^{-\downarrow} + \bar{L}^{-\uparrow}\bar{P}^{-\downarrow} - \bar{L}^{-\downarrow}\bar{P}^{-\uparrow} \right] |S|, \quad (2.38)$$

$$M_8 = \left[\bar{L}^{+\uparrow}\bar{P}^{+\uparrow} - \bar{L}^{+\downarrow}\bar{P}^{+\downarrow} + \bar{L}^{+\uparrow}\bar{P}^{+\downarrow} - \bar{L}^{+\downarrow}\bar{P}^{+\uparrow} - \bar{L}^{-\uparrow}\bar{P}^{-\uparrow} + \bar{L}^{-\downarrow}\bar{P}^{-\downarrow} - \bar{L}^{-\uparrow}\bar{P}^{-\downarrow} + \bar{L}^{-\downarrow}\bar{P}^{-\uparrow} \right] |S|. \quad (2.39)$$

As an example, in **Figure 1** the results of asymmetry amplitudes of particular interest extracted in a single bin over all kinematic variables, obtained from HERMES data on DVCS from transversely polarized hydrogen target [6,9], are shown with their statistical and total uncertainties.

The correlation among all fitted asymmetry amplitudes is presented in **Figure 2**.

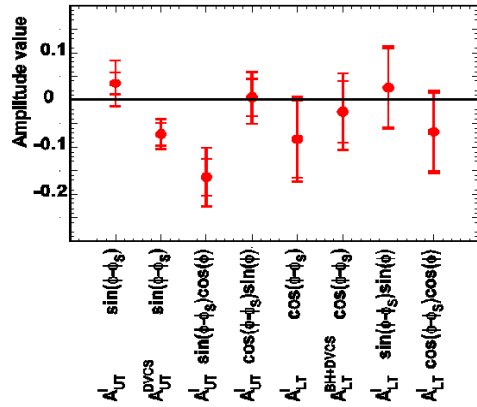


Figure 1. Azimuthal asymmetry amplitudes of particular interest measured at HERMES with transversely polarized proton [6,9], given at the average kinematics.

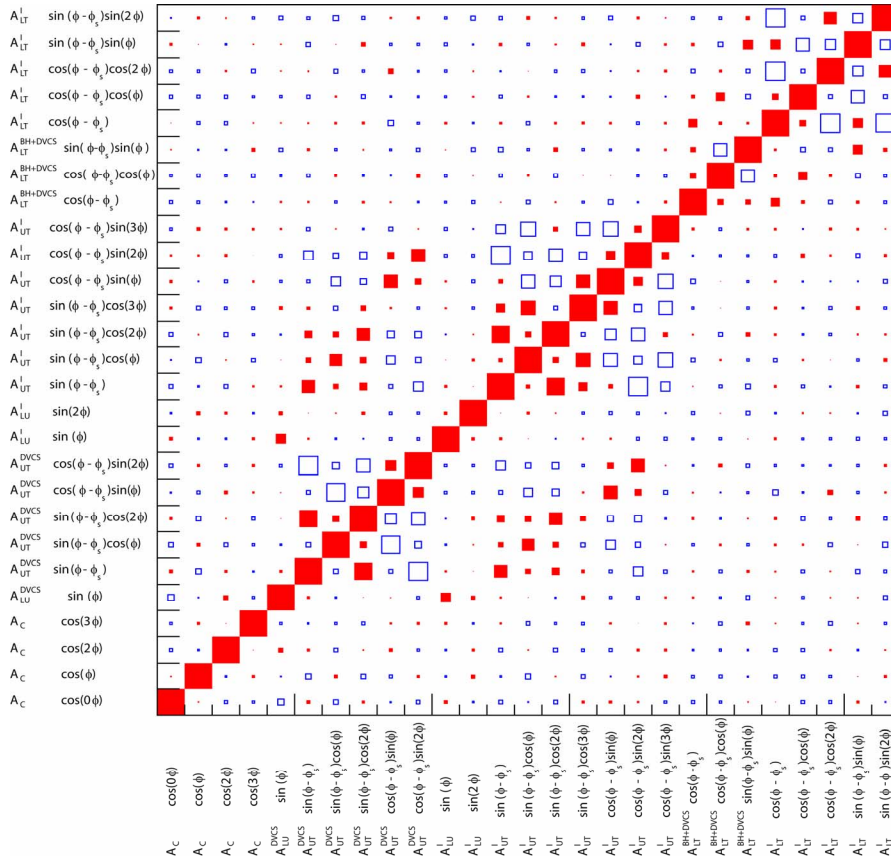


Figure 2. Correlation matrix for all fitted asymmetry amplitudes. The closed symbols represent positive values, while the open ones are for negative values. The area of the symbols represents the size of the correlation [6].

The above described method of simultaneous extraction of the “full set” of asymmetry amplitudes from data obtained from hard exclusive leptonproduction of real photons from transversely polarized protons is applicable for any other experiments (for example the planned COMPASS II program on DVCS at CERN), where measurements with both polarization states of the beam and target can be realized.

3. Acknowledgements

The author would like to thank the HERMES collaboration for support, G. Schell and C. Van Hulse for very useful discussions, comments. The author is also grateful to A. Movsisyan for his significant effort and participation in this work.

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