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Out-of-Equilibrium Dissipative *ac*—Susceptibility in Quantum Ising Spin Glass

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ABSTRACT

The imaginary part of the non-equilibrium magnetic susceptibility of Ising spin glass in a transverse field under time-dependent longitudinal external magnetic field has been calculated at very low temperature on the basis of quantum droplet model and quantum linear response theory. Quantum and aging effects on the low temperature dynamics of the model are discussed. A comparison with recent theoretical and experimental data in spin glass is made.

Keywords: Spin Glasses; Non-Equilibrium Dynamics; Droplet Theory

1. Introduction

Over the last two decades there is a great deal of research on the experimental and theoretical description of disordered magnetic materials. The understanding of the interplay between disorder, quantum and thermal fluctuations remains among the most relevant problem of condensed matter physics. In strongly disordered systems the dynamics becomes very slow which is characteristic of glassy state and aging scenario [1-8]. Aging phenomena and non-equilibrium slow dynamics have been investigated during last years in many materials with glassy properties such as spin glasses [1-8], polymer glasses [9,10], gels [11] and other areas like neural networks, information processing, optimization problems [12]. Despite a great progress towards the understanding of non-equilibrium dynamics, some problems remain open. One of them is an investigation of a very low temperature non-equilibrium dynamics in quantum spin glasses, namely the nature of quantum channels of relaxation, the behavior of quantum glassy system subjected to periodic driving force, aging at very low temperatures. The natural basis for the interpretation of aging is based on coarsening ideas of a slow domain growth of a spin-glass type ordered phase [8,13,14]. A large attention in the last decade was paid to the spin glasses representing a model systems for study of non-equilibrium dynamics providing a measure of processes causing the aging: the magnetic susceptibility [15-22].

In classical spin glasses in the *ac* susceptibility measurements the magnetic response of the system to a small oscillating magnetic field applied after quenching exhibits aging effects. This response depends on its thermal history and the time interval the system has been kept at a constant temperature in the glass phase [1-8,15-17].

It is assumed that isothermal aging of a d-dimensional spin glass is a coarsening process of domain walls, and the temporal *ac* susceptibility (real part χ' andimaginary part χ'') at a given frequency of *ac* magnetic field ω at time t after the quenching scales as [23-25]

$$\frac{\chi''(\omega,t)-\chi''_{eq}(\omega)}{\chi''(\omega,t)}\sim\left[\frac{L(1/\omega)}{R(t)}\right]^{d-\theta}, \tag{1}$$

$$\frac{\chi'(\omega,t)-\chi'_{eq}(\omega)}{\chi'(\omega,t)}\sim\left[\frac{L(1/\omega)}{R(t)}\right]^{d-\theta}, \tag{2}$$

for $|\ln\omega|\ll\ln t$, if $L(1/\omega)$ is proportional to $\ln\omega^{-1}$ and $R(t)$ is proportional to $\ln t$; $\theta\le(d-1)/2$. Here $L(1/\omega)$ is size of the droplet being polarized by oscillating field and $R(t)$ is the typical domain size, L and R may scale according logarithmic growth law or algebraic one [23-25]. $\left[L(1/\omega)/R(t)\right]$ is proportional to $\left[\ln(\omega/\omega_0)/\ln(t/t_0)\right]^{1/\psi}$ if droplet theory is used, ψ is some exponent, $0\le\psi\le d-1$, and t_0 is a certain-microscopic characteristic time [26]. The logarithmic growth law (like the algebraic one) is supported by recent experiments [23-25]. The expressions (1) and (2) are found when relaxation is governed by thermal activation over a free energy barrier B. It is supposed that barriers for annihilation and creation of the droplet excitations scale as $B \sim \Delta L^{\psi}$, where Δ is a barrier energy at temperature $T \ll T_g$ (T_g is the spin glass transition temperature). The barriers have a broad distribution of energies. A droplet with barrier B lasts for a time t of order of $t \sim t_0 \exp\left[B/(k_B T)\right]$ where k_B is Boltzmann's constant; here t is a rate of classic activation over energy barrier B. After a time t after quenching the domain size in the system grows as

$$R(t) \sim \left[\left(k_B T/\Delta(T)\right)\ln\left(t/t_0\right)\right]^{1/\psi}.$$

In the *ac* susceptibility measurements, the *ac* field excites droplets of length scales L up to

$$L(1/\omega) \sim \left[\left(k_B T/\Delta(T)\right)\left|\ln\left(\omega/\omega_0\right)\right|\right]^{1/\psi}.$$

Because in aging experiments the time t spent after quench is $t \geq \omega^{-1}$ [23-26] one has $L(1/\omega) < R(t)$. These droplets have walls which partly coincide with walls of the domain of size R.

In this paper we investigate the real time non-equilibrium dynamics in *d*-dimensional Ising spin glass in a transverse field in terms of droplet model at very low temperatures [26]. We calculate the dissipative component of the *ac* susceptibility as a function of the time elapsedsince a thermal quench and frequency of driven field.

2. Model Hamiltonian

The droplet model describing the low-dimensional short-range Ising spin glass is based on renormalization group arguments [26,28]. In dimensions above the lower critical dimension d_l (usually in spin glass $2 \leq d_l < 3$) the droplet model finds a low temperature spin-glass phase in zero magnetic field. This phase differs essentially from the spin-glass phase in the mean-field approximation of the Sherrington-Kirkpatrick infinite-range spin-glass model [13]. In the droplet model there are only two pure thermodynamical states related to each other by a global spin flip. In magnetic field there is no phase transition. A droplet is an excited cluster in an ordered state where all the spins are inverted. The natural scaling ansatz for droplet free energy ε_L, which are considered to be independent random variables, is $\varepsilon_L \sim L^{\theta}$, $L \geq \zeta(T)$; ζ is the correlation length, L is the length scale of droplet and θ is the zero temperature thermal exponent. The droplet excitations have a broad distribution of their free energies at scale L for large L in a scaling form [4]

$$P_L(\varepsilon_L)d\varepsilon_L = \frac{d\varepsilon_L}{\gamma(T)L^{\theta}}P\left(\frac{\varepsilon_L}{\gamma(T)L^{\theta}}\right),\ L \to \infty \qquad (3)$$

It is assumed that $P_L(x \to 0) > 0$, $P(0) - P(x) \sim x^{\phi}$, $x \to 0$, $\phi \leq 1$. $\gamma(T)$ is a generalized temperature dependent stiffness modulus which is of order of characteristic exchange $J = \overline{\left(J_{ij}^2\right)}^{1/2}$ at $T = 0$ and vanishes for $T \geq T_g$. One droplet consists of order L^d spins. Below d_l, $\theta < 0$; above d_l one has $\theta > 0$. The droplet model of classical Ising spin glass was considered by D.S. Fisher and D.A. Huse [26,27].

In this paper we use a phenomenological quantum droplet model of spin glass theory [26-29] (which does not use the mean-field approximation) in order to describe the non equilibrium behavior of the magnetic dynamical susceptibility at very low (but finite) temperatures T.

We consider the following model hamiltonian of *d*-dimensional Ising spin glass in a transverse field,

$$H = -\sum_{\langle ij\rangle} J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x \qquad (4)$$

where σ_i^x and σ_i^z are the Pauli matrices, Γ is the strength of the transverse field and the nearest neighbor interactions J_{ij} are independent random variables of mean zero and the sum in Equation (4) is performed over nearest neighbors.

The quantum spin glass transition in a dilute dipole coupled magnet $\mathrm{LiHo}_x\mathrm{Y}_{1-x}\mathrm{F}_4$ is described by this model hamiltonian [4,13,20]. It is supposed that this model may also represent, for example, the physics of deuteron glass such as $\mathrm{Rb}_{1-x}(\mathrm{ND}_4)_x\mathrm{D}_2\mathrm{PO}_4$ and mixed betaine phosphate-phosphite [28]. The transverse field in [28] is interpreted as the frequency of the proton tunneling.

In ref. [26,27], M. J. Thill and D. A. Huse have shown that for enough low T the quantum hamiltonian Equation (4) can be represented as independent quantum two-level systems (low energy droplets) with the hamiltonian,

$$H = \frac{1}{2}\sum_{\tilde{L}}\sum_{D_L}\left(\varepsilon_{D_L}\sigma_{D_L}^z + \Gamma_L\sigma_{D_L}^x\right) \qquad (5)$$

where $\sigma_{D_L}^x$ and $\sigma_{D_L}^z$ are the Pauli matrices representing the two states of the droplet D_L; the sum is over all droplets D_L at length scale L and over all length scales L, and

$$\sum_{\tilde{L}} \sim \int_{L_0}^{\infty}\frac{\mathrm{d}L}{L}$$

where L_0 is a short-distance cutoff; ε_{D_L} is the droplet energy which is independent random variable with scaling ansatz $\varepsilon_{D_L} \sim L^{\theta}$. The droplet length scale L is more or of order of the correlation length.The value $\Gamma_L = \Gamma_0\exp\left(-\delta L^d\right)$ which regulates the strength of quantum fluctuations ($\Gamma_L \to 0$ corresponds to the classical limit) is the tunneling rate for a droplet of linear size L and δ is a coefficient which is approximately the same for all droplets. Γ_0 is a microscopic tunneling rate and, finally, we assume Γ_L is the same for all droplets of scale L [26,27].

In the quantum droplet model of Thill and Huse [26,27] the relative reduction of the Edwards-Anderson order parameter $q_{EA}(T)$ from its zero temperature value $q_{EA}(0)$ for $\theta > 0$ is given by,

$$1-\frac{q_{EA}(T)}{q_{EA}(0)} \sim \frac{k_B T}{\gamma L^{*\theta}(T)} = \frac{k_B T}{\gamma} \frac{\sigma^{\frac{\theta}{d}}}{\left(\ln \frac{\Gamma_0}{k_B T}\right)^{\frac{\theta}{d}}}, \quad T \to 0$$

Here γ is the stiffness modulus, and

$$L^*(T) = \left(\frac{1}{\sigma} \ln \frac{\Gamma_0}{k_B T}\right)^{\frac{1}{d}}$$

is the classical-to-quantum crossover length scale defined by $\Gamma_{L^*(T)} = k_B T$. For droplets with $L \ll L^*(T)$ and $\Gamma_L \gg k_B T$ (quantum regime [26-28]), the excitation energy $\sqrt{\varepsilon_L^2 + \Gamma_L^2}$ is always greater than $k_B T$ and thermal fluctuations are irrelevant. These droplets behave quantum mechanically. The larger droplets $(L \gg L^*(T))$ have $\Gamma_L \ll k_B T$ and behave classically. In quantum regime the length growth is due to quantum fluctuations connected with droplet are quantum-mechanically active for $T \to 0$ is proportional to Γ_L.

3. Dynamic Nonequilibrium Magnetic Susceptibility

We consider the time dependent Hamiltonian $\hat{H}$ of the quantum system in the form [34]

$$\hat{H} = \hat{H}_0 + \hat{H}'(t) = \hat{H}_0 - \hat{A}h(t) \tag{6}$$

where $\hat{H}_0$ is the Hamiltonian of the unperturbed system and describes the equilibrium system.We suppose that the external perturbation $\hat{H}'(t)$ is in some sense small. $\hat{A}$ is a linear operator which connects external time-dependent force $h(t)$ with the system. We shall use the quantum-mechanical equations for the system dynamical response $\Delta\hat{B}(t) \equiv \langle \hat{B}(t) \rangle - \langle \hat{B} \rangle_0$ to the force $h(t)$ in terms of the time-evolution operator $\hat{U}(t,t')$; $\hat{B}(t)$ is an Heisenberg operator, $\hat{B}(t) = \hat{U}^+(t,t')\hat{B}(t_0)\hat{U}(t,t')$, $\langle \hat{B} \rangle_0$ is the average value of $\hat{B}$ in equilibrium; the sign + means conjugate value. It is necessary to approximate $\hat{U}(t,t')$ using the well-known perturbation expansion through first order in h. We have

$$\begin{aligned}&\hat{U}(t,t')\\ &\approx \hat{U}_0(t,t')\left\{\hat{1} - \frac{i}{\hbar}\int_{t'}^{t} \mathrm{d}t_1 \hat{U}_0^+(t_1,t')\hat{H}'(t_1)\hat{U}_0(t_1,t')\right\}\end{aligned} \tag{7}$$

where $\hat{U}_0(t,t') = \exp\left[-(i/\hbar)\left((t-t')\hat{H}_0\right)\right]$.

We consider the functional of the dynamic response of the form [34] $\Phi(t,t';t_0) \equiv \frac{1}{i\hbar}\left\langle \left[\hat{A}(t_0), \hat{B}(t,t')\right]\right\rangle_0$, where $\langle \cdots \rangle_0$ means thermal average with a density matrix $\hat{\rho}_0 = \hat{\rho}(t_0)$, t_0 is the time moment when the perturbating field is turned on,

$$\hat{\rho}_0 = \left(Tr \exp\left[-\beta \hat{H}_0\right]\right)^{-1} \exp\left[-\beta \hat{H}_0\right].$$

Now we apply the aforementioned expressions for dynamic response to a magnetic droplet system. The response $\langle \hat{B}(t) \rangle$ is then the induced magnetization $M(t)$ of the system. $\langle \hat{B} \rangle_0$ is the equilibrium magnetization M_0. Let a small magnetic oscillating field $h(t) = h\exp(i\omega t)$ be applied in z-direction. Here h and ω are the amplitude and frequency of the *ac* field.

When one measures the *ac* susceptibility in spin glasses in the external magnetic oscillating field it is observed an aging effect: magnetic response of the system to the weak external field depends on the thermal history of the sample, on the time during which the system was kept in a spin glass phase. The sample is quenched in zero magnetic field from temperature $T \gg T_g$ to the temperature $T_1 < T_g$ which is reached at time $t = 0$. At this moment a very small external magnetic oscillating field $h(t)$ is applied to measure the *ac* susceptibility of the sample. The evolution continues in isothermal conditions, χ_{ac} is measured as a function of the time t elapsed since the sample reached the temperature T_1 at fixed frequency ω.

The system is probed at the time t after quench end ("the age"). Using the linear response theory the magnetization of magnetic system is [35]

$$\begin{aligned}M(t) - M_0 &= \int_0^t \chi(t,t_1)h(t_1)\mathrm{d}t_1\\ &= \int_0^t \chi(t,t-t')h(t-t')\mathrm{d}t'\end{aligned} \tag{8}$$

where $\chi(t,t-t')$ is the magnetic dynamic susceptibility and defines the magnetic response at moment t to a unit pulse of magnetic field at moment $(t-t')$. The nonequilibrium processes are investigated by means of low-frequency susceptibility measurements. The frequency dependent *ac* susceptibility is measured by means of applied *ac* magnetic field $h(t)$ at time $t = 0$. Then one can find $\chi(\omega,t)$ by the Fourier-transform of the magnetization over the time interval $t_m (t_m \sim 2\pi/\omega)$ centered on t [15,35],

$$\chi(\omega,t) = \frac{1}{t_m}\int_{t-\frac{t_m}{2}}^{t+\frac{t_m}{2}} \mathrm{d}t'' \mathrm{e}^{-i\omega t''} \int_0^{t''} \mathrm{d}t' \chi(t'',t''-t')\mathrm{e}^{i\omega(t''-t')} \tag{9}$$

If magnetic response function slightly changes over the time segment t_m then the susceptibility $\chi(\omega,t)$

will be equal to [15,35]

$$\chi(\omega,t)=\int_0^t \mathrm{d}t'\chi(t,t-t')\mathrm{e}^{-i\omega t'} \tag{10}$$

We consider the behavior of the magnetic droplet system described by the Hamiltonian $\hat{H}$ and under *ac* field $h(t)$ in quantum regime $(\Gamma_L \gg k_B T)$. In our calculations, we suppose that $\theta > 0(d > d_l)$. There is a complicated crossover between classic and quantum behaviors of the droplets which depends on temperature, *ac* field frequency and length scale L. According to [26,27] the dynamical crossover length is determined from the condition $\Gamma_L^{-1}=t$, *i.e.* $L^*_{dyn}(T)\sim\left(\frac{\sigma}{\Delta}k_B T\right)^{\frac{1}{\psi-d}}$. The system behaves presumably classically or quantum mechanically when the dominant length scale L is above or below L^*_{dyn} for fixed frequency ω.

Following to aforementioned quantum droplet theory with model Hamiltonian (4) and domain growth ideas, we calculate the magnetic dynamic susceptibility using the dynamical response functional which includes first and second order linear response functions. The contribution of a single droplet to the *ac* susceptibility up to some factor $\sim q_{EA}^2 L^{2d}$ is,

$$\begin{aligned}\chi_{D_L}(\omega,t)-\chi_{D_L}(\omega=0)\sim &-\tanh\left(\frac{\beta a_L}{2}\right)\sin^2\varphi\frac{a_L}{a_L^2-\omega^2}\\ &-h\tanh\left(\frac{\beta a_L}{2}\right)\cos\varphi\sin^2\varphi\left(\left\{\frac{\left(7a_L^2-10\omega^2\right)\cos(\omega t)}{\left(a_L^2-\omega^2\right)\left(a_L^2-4\omega^2\right)}-\frac{3a_L\sin(\omega t)\sin(a_L t)+6\omega\cos(\omega t)\cos(a_L t)}{\omega\left(a_L^2-4\omega^2\right)}-\frac{\cos(a_L t)}{a_L^2-\omega^2}\right\}\right.\\ &\left.+i\left\{\frac{3a_L}{}\frac{3\left(a_L^2+2\omega^2\right)\sin(\omega t)}{\left(a_L^2-\omega^2\right)\left(a_L^2-4\omega^2\right)}-\frac{3\cos(\omega t)\sin(a_L t)-6\omega\sin(\omega t)\cos(a_L t)}{\omega\left(a_L^2-4\omega^2\right)}+\frac{3\sin(a_L t)}{\omega(a_L^2-\omega^2)}\right\}\right)\end{aligned} \tag{11}$$

where q_{EA} is the Edwards-Anderson order parameter, $a_L=\sqrt{\varepsilon_L^2+\Gamma_L^2}$, $\sin\varphi=\Gamma_L/a_L$, $\cos\varphi=\varepsilon_L/a_L$, $\chi_{D_L}(\omega=0)$ is the static susceptibility of the droplet D_L. The expression (11) is obtained for low frequencies under the condition $\omega t\geq 1$ (and $\Gamma_L<\omega$), because this condition is used to observe non-stationary dynamics in susceptibility measurements [5]. Now we have to average the susceptibility (11) over droplet energies ε_L and over droplet length scales L. We use the droplet energy distribution $P_L(\varepsilon_L)$ given by Equation (3). Here we assume $\phi=0$. While integrating over L, we note that the susceptibility is dominated by droplets of length scale $L_1\sim\left[(1/\sigma)\ln(\Gamma_0/\omega)\right]^{1/d}$; L_1 is the natural length scale of the problem when $\Gamma_L\sim\omega$ and it is the low limit ofthe integration over L. The upper limit is $L_2\sim\left[(1/\sigma)\ln(t_0\Gamma_0)\right]^{1/d}$. After integration over droplet energies and length scales we obtain the following expression for imaginary part of susceptibility of the droplet system:

$$\begin{aligned}\chi''(\omega,t)\sim\frac{h\Gamma_0^2}{\gamma\omega^3}\Bigg\{&-\frac{1}{d}\left[\sigma\left(\zeta^{d-\theta}E\left[\frac{\theta}{d},2A\right]-M^{d-\theta}E\left[\frac{\theta}{d},2B\right]\right)\sin(\omega t)\right]\\ &+\frac{1}{d}\left[\sigma^{\frac{\theta}{d}}\left(G\left[-\frac{\theta}{d},A\right]-G\left[-\frac{\theta}{d},B\right]\right)\left(\frac{\cos(\omega t)}{a}-\frac{\hbar\omega\sin(\omega t)}{\Gamma_0}\right)\right]\\ &+\frac{1}{d}\Bigg[\left(\zeta^{-\theta}E\left[\frac{d+\theta}{d},2A\right]-M^{-\theta}E\left[\frac{d+\theta}{d},2B\right]\right)\Bigg(-\frac{g+2j\left(t^2+\hbar^2\beta^2\right)}{64x}-\frac{1}{\omega t}\\ &-\frac{4t\left(\beta j\hbar+\omega t^2\right)}{x}-\frac{\cos(\omega t)}{2\omega t}+\frac{1}{8}\cos(\omega t)\times\left(\pi-\frac{2\hbar\beta t}{x}-\frac{2\left(-3\cos(3\omega t)+\cos(5\omega t)\right)}{\omega t}\right)\\ &+\frac{1}{6}\mathrm{e}^j\left(20E[1,j]+\mathrm{e}^j\left(-3E[1,2j]+E[1,3j]\right)\right)+\ln 2-\frac{\ln 3}{4}-\ln\left(\frac{\Gamma_0}{\hbar\omega}\right)+4\mathrm{e}^j\sin(\omega t)\\ &+\frac{\sin^2(\omega t)}{\omega t}+\frac{1}{x}\left(\mathrm{e}^{2j}\omega\left(-j\left(g+2t^2\right)\cos(2\omega t)+t\left(-2j\beta\hbar+g\omega\right)\sin(2\omega t)\right)\right)\Bigg)\Bigg]\Bigg\}\end{aligned} \tag{12}$$

where $G[\alpha,\beta]$ is the incomplete gamma-function, $E[\alpha,\beta]$ is the exponential integral function,

$$\zeta=\sigma^{-1/d}\left|\ln\left(\frac{\Gamma_0}{\omega\hbar}\right)\right|^{1/d},\ M=\sigma^{-1/d}\left|\ln\left(\frac{t\Gamma_0}{\hbar}\right)\right|^{1/d},\ A=\sigma\zeta^d,$$

$$B=\sigma M^d,\ a=\frac{t\Gamma_0}{\hbar},\ g=\hbar^2\beta^2-t^2,\ x=\omega^2\left(\hbar^2\beta^2+t^2\right)^2,$$

$j=\beta\hbar\omega$.

During the calculations we have used the following approximations: $si(z)\approx\frac{\pi}{2}-\frac{\cos(z)}{z}$, $ci(z)\approx\frac{\sin(z)}{z}$.

The Equation (12) is the main result of this paper.

As we see, the susceptibility $\chi''(\omega,t)$ depends on many parameters of the droplet system and the external *ac* magnetic field: on the form of droplet energies distribution $P_L(\varepsilon_L)$, on the droplet microscopic tunneling rate Γ_0, on the temperature, on the system "age" t, on the *ac* field frequency and amplitude. Furthermore, we note that the expression (12) consists of terms which are time independent which describe oscillations with frequency, and terms which depend on t and define non-stationary non-equilibrium dynamics of the droplet system. Thus the imaginary part of susceptibility can be represented as a sum of stationary part (χ''_{ST}) and non-stationary part (χ''_{NST}):

$$\chi''(\omega,t)\cong\chi''_{ST}+\chi''_{NST} \qquad (13)$$

For a numerical calculations of the expression (12), we take the following values of the parameters: $d=3$, $\theta=0.5$, $\gamma=10^{-15}$, $\Gamma_0=10^{-16}$, $\hbar=10^{-27}$, $h=1$, $\sigma=10^{-15}$, $t=1\div50$, $\omega=0.05,0.1$.

In **Figures 1-3** it is shown the t-dependence of the imaginary part $\chi''(\omega,t)$ at different fixed T (**Figure 1**), Γ_0 (**Figure 2**) and ω (**Figure 3**). The susceptibility quickly goes down and then slowly decays to some value with oscillations. Then we observe the stationary behavior of susceptibility. In particular, in **Figure 2** we observe as, on longer times, the quantum fluctuations (Γ_0 dependence) becomes irrelevant.

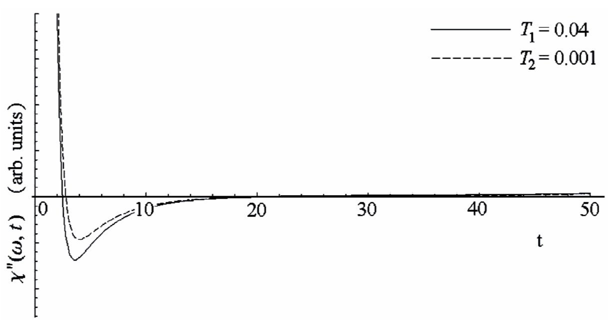

Figure 1. Imaginary part $\chi''(\omega,t)$ as function of time t at $\Gamma_0=10^{-16}$ and $\omega=0.05$ for different temperatures $T_1=0.04$ and $T_2=0.001$.

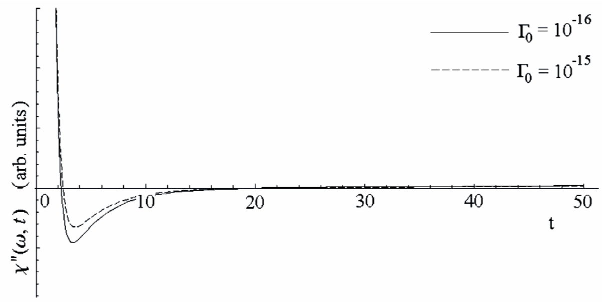

Figure 2. Imaginary part $\chi''(\omega,t)$ as function of time $t=1\div50$ at $T=0.01$ and $\omega=0.05$ for different values of quantum parameter $\Gamma_0=10^{-16}$ and $\Gamma_0=10^{-15}$.

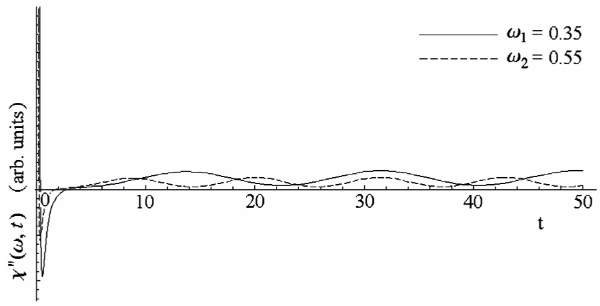

Figure 3. Imaginary part $\chi''(\omega,t)$ as function of time $t=1\div50$ at $T=0.01$ and $\Gamma_0=10^{-16}$ for different frequencies $\omega_1=0.35$ and $\omega_2=0.55$.

4. Discussion and Conclusions

In this paper we have investigated the low temperature non-equilibrium dynamic behavior of magnetic susceptibility in *d*-dimensional short-range Ising spin glass in a transverse field in terms of phenomenological droplet model taking into account quantum fluctuations. In particular we calculated the imaginary part of low-frequency susceptibility $\chi''(\omega,t)$ as function of time t (elapsed from the quench to measurement moment) and frequency ω of the *ac* magnetic field. It has been shown that the imaginary part of $\chi''(\omega,t)$ of the droplet system at low temperatures (quantum regime) has two time regions where its time behavior has different nature. On short times t we observe quickly non-equilibrium dynamic decay of $\chi''(\omega,t)$. On long times the susceptibility curve is a periodical function oscillating near some constant value (stationary process). We find temperature dependence of imaginary part of susceptibility and show how the quantum fluctuations influence the dynamic susceptibility of the droplet system at very low temperatures. If the *ac* field frequency increases then the non-equilibrium dynamics is suppressed. Thus the droplet system response to an external perturbing field depends on its thermal history.

In [17] it is shown that the behavior of response function $R(t,t_w)$ confirms the existence of two time regimes in spin glass: stationary and aging regimes in quantum systems. The theoretical curve (Figure 2 in [17]) of $R(t,t_w)$ was given as function of $\tau(\tau = t - t_w)$ for $\tau \in [0,50]$, and $t_w = 2.5, 5, 10, 20$ and 40 (t_w—waiting time). For $\tau \le \tau_{char}$ (τ_{char} is some characteristic time) a stationary regime was found, whereas for $\tau > \tau_{char}$ the dynamics is non-stationary. In [17] it is shown that quantum fluctuations in quantum glassy systems depress the phase transition temperature, in a glassy phase the aging effect survives the quantum fluctuations, and because of quantum fluctuations, the fluctuation-dissipation theorem is modified. In reference [19-21] it is shown that all terms in the dynamical equations governing the time-evolution of spin response and correlation function which are due to quantum effects, are irrelevant at long times. Quantum effects enter only through the renormalization of parameters in dynamical equations [19-21]. The behavior of spin response as function of τ in [36] is similar to behavior of dynamical susceptibility in our paper. In [36] it is shown that quantum fluctuation slightly influences the aging regime and the quantum system behavior is approximately classic.

As far as we know there are no experiments on quantum spin glasses. In papers [17,23] there are experimental data for dynamic susceptibility in classic spin glasses. P. Svedlindh *et al.* [35] have investigated the behavior of $\chi''(\omega,t)$ and have found that decay is close to a logarithmic one. Shins *et al.* [22] also show that susceptibility decays with time in a nearly logarithmic way.

In our quantum system at very low temperature, we cannot find agreement with these data because classical and quantum spin glass has, in general, different behavior. We may compare our results with experimental data only approximately because these experimental data are on classical spin glasses and we consider here low-temperature dynamics of quantum spin glass. We observe a qualitatively similar behavior in the range of the small times elapsed since the quench.

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