Journal of Modern Physics, **2013, 4, 32-37**
http://dx.doi.org/10.4236/jmp.2013.44A006 Published Online April 2013 (http://www.scirp.org/journal/jmp)

Zariski 3-Algebra Model of M-Theory

Matsuo Sato
Department of Natural Science, Faculty of Education, Hirosaki University, Hirosaki, Japan
Email: msato@cc.hirosaki-u.ac.jp

Received January 15, 2013; revised February 20, 2013; accepted March 3, 2013

ABSTRACT

We review on Zariski 3-algebra model of M-theory. The model is obtained by Zariski quantization of a semi-light-cone supermembrane action. The model has manifest $\mathcal{N}=1$ supersymmetry in eleven dimensions and its relation to the supermembrane action is clear.

Keywords: M-Theory; 3-Algebra; Matrix Model; String Theory

1. Introduction

Recently, structures of 3-algebras [1-3] were found in the effective actions of the multiple M2-branes [4-12] and 3-algebras have been intensively studied [13-29]. It had been expected that structures of 3-algebras play more fundamental roles in M-theory than the accidental structures in the effective descriptions, and 3-algebra models of M-theory were proposed [30-34].

In this paper, we review one of the models, called Zariski 3-algebra model of M-theory. This model has manifest $\mathcal{N}=1$ supersymmetry in eleven dimensions and the relation to the supermembrane action is clear. We start with the fact found in [32] that the supermembrane action in a semi-light-cone gauge is a gauge theory based on a 3-algebra that is generated by the Nambu-Poisson bracket [13,14]. The gauge theory's thirty-two supersymmetries form the $\mathcal{N}=1$ supersymmetry algebra in eleven dimensions. By performing the Zariski quantization, the action is second quantized and we obtain Zariski 3-algebra model of M-theory.

2. Supermembrane Action in a Semi-Light-Cone Gauge

In this section, we review the fact that the supermembrane action in a semi-light-cone gauge can be described by Nambu bracket, where structures of 3-algebra are manifest. The 3-algebra models of M-theory are defined based on the semi-light-cone supermembrane action.

The fundamental degrees of freedom in M-theory are supermembranes. The covariant supermembrane action in M-theory [35] is given by

$$S_{M2}=\int \mathrm{d}^3\sigma\Bigg(\sqrt{-G}+\frac{i}{4}\epsilon^{\alpha\beta\gamma}\bar{\Psi}\Gamma_{MN}\partial_\alpha\Psi\bigg(\Pi^M_\beta\Pi^N_\gamma +\frac{i}{2}\Pi^M_\beta\bar{\Psi}\Gamma^N\partial_\gamma\Psi-\frac{1}{12}\bar{\Psi}\Gamma^M\partial_\beta\Psi\bar{\Psi}\Gamma^N\partial_\gamma\Psi\bigg)\Bigg) \quad (1)$$

where

$$M,N=0,\cdots,10,\alpha,\beta,\gamma=0,1,2,G_{\alpha\beta}=\Pi^M_\alpha\Pi_{\beta M}$$

and

$$\Pi^M_\alpha=\partial_\alpha X^M-\frac{i}{2}\bar{\Psi}\Gamma^M\partial_\alpha\Psi .$$

Ψ is a $SO(1,10)$ Majorana fermion.

This action is invariant under dynamical supertransformations,

$$\begin{aligned}\delta\Psi&=\epsilon,,\\ \delta X^M&=-i\bar{\Psi}\Gamma^M\epsilon.\end{aligned} \quad (2)$$

These transformations form the $\mathcal{N}=1$ supersymmetry algebra in eleven dimensions,

$$\begin{aligned}\left[\delta_1,\delta_2\right]X^M&=-2i\epsilon_1\Gamma^M\epsilon_2,\\ \left[\delta_1,\delta_2\right]\Psi&=0.\end{aligned} \quad (3)$$

The action is also invariant under the κ-symmetry transformations,

$$\begin{aligned}\delta\Psi&=(1+\Gamma)\kappa(\sigma),\\ \delta X^M&=i\bar{\Psi}\Gamma^M(1+\Gamma)\kappa(\sigma),\end{aligned} \quad (4)$$

where

$$\Gamma=\frac{1}{3!\sqrt{-G}}\epsilon^{\alpha\beta\gamma}\Pi^L_\alpha\Pi^M_\beta\Pi^N_\gamma\Gamma_{LMN}. \quad (5)$$

If we fix the κ-symmetry (4) of the action by taking a semi-light-cone gauge [32]

$$\Gamma^{012}\Psi = -\Psi, \tag{6}$$

we obtain a semi-light-cone supermembrane action,

$$S_{M2} = \int d^3\sigma \left(\sqrt{-G} + \frac{i}{4}\epsilon^{\alpha\beta\gamma} \left(\bar{\Psi}\Gamma_{\mu\nu}\partial_\alpha\Psi \left(\Pi^\mu_\beta \Pi^\nu_\gamma + \frac{i}{2}\Pi^\mu_\beta \bar{\Psi}\Gamma^\nu\partial_\gamma\Psi - \frac{1}{12}\bar{\Psi}\Gamma^\mu\partial_\beta\Psi\bar{\Psi}\Gamma^\nu\partial_\gamma\Psi \right) + \bar{\Psi}\Gamma_{IJ}\partial_\alpha\Psi\partial_\beta X^I \partial_\gamma X^J \right)\right), \tag{7}$$

where

$$G_{\alpha\beta} = h_{\alpha\beta} + \Pi^\mu_\alpha \Pi_{\beta\mu}, \Pi^\mu_\alpha = \partial_\alpha X^\mu - \frac{i}{2}\bar{\Psi}\Gamma^\mu\partial_\alpha\Psi,$$

and $h_{\alpha\beta} = \partial_\alpha X^I \partial_\beta X_I$.

In [32], it is shown under an approximation up to the quadratic order in $\partial_\alpha X^\mu$ and $\partial_\alpha \Psi$ but exactly in X^I, that this action is equivalent to

$$\begin{aligned} S_{cl} = \int d^3\sigma \sqrt{-g} \Bigg(& -\frac{1}{12}\left\{X^I, X^J, X^K\right\}^2 - \frac{1}{2}\left(A_{\mu ab}\left\{\varphi^a, \varphi^b, X^I\right\}\right)^2 + \frac{1}{2}\Lambda - \frac{1}{3}E^{\mu\nu\lambda}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\left\{\varphi^a, \varphi^c, \varphi^d\right\}\left\{\varphi^b, \varphi^e, \varphi^f\right\} \\ & -\frac{i}{2}\bar{\Psi}\Gamma^\mu A_{\mu ab}\left\{\varphi^a, \varphi^b, \Psi\right\} + \frac{i}{4}\bar{\Psi}\Gamma_{IJ}\left\{X^I, X^J, \Psi\right\}\Bigg), \end{aligned} \tag{8}$$

where $I, J, K = 3, \cdots, 10$ and

$$\left\{\varphi^a, \varphi^b, \varphi^c\right\} = \epsilon^{\alpha\beta\gamma}\partial_\alpha\varphi^a\partial_\beta\varphi^b\partial_\gamma\varphi^c$$

is the Nambu-Poisson bracket. X^I is a scalar and Ψ is a $SO(1,2)\times SO(8)$ Majorana-Weyl fermion satisfying (6). $E^{\mu\nu\lambda}$ is a Levi-Civita symbol in three dimensions and Λ is a cosmological constant.

(8) is invariant under 16 dynamical supersymmetry transformations,

$$\begin{aligned} &\delta X^I = i\bar{\epsilon}\Gamma^I\Psi, \delta A_\mu(\sigma,\sigma') = \frac{i}{2}\bar{\epsilon}\Gamma_\mu\Gamma_I\left(X^I(\sigma)\Psi(\sigma') - X^I(\sigma')\Psi(\sigma)\right), \\ &\delta\Psi = -A_{\mu ab}\left\{\varphi^a, \varphi^b, X^I\right\}\Gamma^\mu\Gamma_I\epsilon - \frac{1}{6}\left\{X^I, X^J, X^K\right\}\Gamma_{IJK}\epsilon, \end{aligned} \tag{9}$$

where $\Gamma_{012}\epsilon = -\epsilon$. These supersymmetries close into gauge transformations on-shell,

$$\begin{aligned} &\left[\delta_1, \delta_2\right]X^I = \Lambda_{cd}\left\{\varphi^c, \varphi^d, X^I\right\}, \left[\delta_1, \delta_2\right]A_{\mu ab}\left\{\varphi^a, \varphi^b, \right\} \\ &= \Lambda_{ab}\left\{\varphi^a, \varphi^b, A_{\mu cd}\left\{\varphi^c, \varphi^d, \right\}\right\} - A_{\mu ab}\left\{\varphi^a, \varphi^b, \Lambda_{cd}\left\{\varphi^c, \varphi^d, \right\}\right\} + 2i\bar{\epsilon}_2\Gamma^\nu\epsilon_1 O^A_{\mu\nu}, \\ &\left[\delta_1, \delta_2\right]\Psi = \Lambda_{cd}\left\{\varphi^c, \varphi^d, \Psi\right\} + \left(i\bar{\epsilon}_2\Gamma^\mu\epsilon_1\Gamma_\mu - \frac{i}{4}\bar{\epsilon}_2\Gamma^{KL}\epsilon_1\Gamma_{KL}\right)O^\Psi, \end{aligned} \tag{10}$$

where gauge parameters are given by

$$\Lambda_{ab} = 2i\bar{\epsilon}_2\Gamma^\mu\epsilon_1 A_{\mu ab} - i\bar{\epsilon}_2\Gamma_{JK}\epsilon_1 X^J_a X^K_b.$$

$O^A_{\mu\nu} = 0$ and $O^\Psi = 0$ are equations of motions of $A_{\mu\nu}$ and Ψ, respectively, where

$$\begin{aligned} O^A_{\mu\nu} &= A_{\mu ab}\{\varphi^a, \varphi^b, A_{\nu cd}\left\{\varphi^c, \varphi^d, \right\}\} - A_{\nu ab}\left\{\varphi^a, \varphi^b, A_{\mu cd}\left\{\varphi^c, \varphi^d, \right\}\right\} \\ &\quad + E_{\mu\nu\lambda}\left(-\left\{X^I, A^\lambda_{ab}\left\{\varphi^a, \varphi^b, X_I\right\}, \right\} + \frac{i}{2}\left\{\bar{\Psi}, \Gamma^\lambda\Psi, \right\}\right), \\ O^\Psi &= -\Gamma^\mu A_{\mu ab}\left\{\varphi^a, \varphi^b, \Psi\right\} + \frac{1}{2}\Gamma_{IJ}\left\{X^I, X^J, \Psi\right\}. \end{aligned} \tag{11}$$

(10) implies that a commutation relation between the dynamical supersymmetry transformations is

$$\delta_2\delta_1 - \delta_1\delta_2 = 0 \tag{12}$$

up to the equations of motions and the gauge transformations.

This action is invariant under a translation,

$$\delta X^I(\sigma) = \eta^I, \delta A^\mu(\sigma,\sigma') = \eta^\mu(\sigma) - \eta^\mu(\sigma'), \tag{13}$$

where η^I are constants.

The action is also invariant under 16 kinematical supersymmetry transformations

$$\tilde{\delta}\Psi = \tilde{\epsilon}, \tag{14}$$

and the other fields are not transformed. $\tilde{\epsilon}$ is a constant and satisfy $\Gamma_{012}\tilde{\epsilon} = \tilde{\epsilon}$. $\tilde{\epsilon}$ and ϵ should come from sixteen components of thirty-two $\mathcal{N} = 1$ supersymmetry parameters in eleven dimensions, corresponding to eigen values ± 1 of Γ_{012}, respectively. This $\mathcal{N} = 1$ supersymmetry consists of remaining 16 target-space supersymmetries and transmuted 16 κ-symmetries in the semi-light-cone gauge [32,36,37].

A commutation relation between the kinematical supersymmetry transformations is given by

$$\tilde{\delta}_2\tilde{\delta}_1 - \tilde{\delta}_1\tilde{\delta}_2 = 0. \tag{15}$$

A commutator of dynamical supersymmetry transformations and kinematical ones acts as

$$\left(\tilde{\delta}_2\delta_1 - \delta_1\tilde{\delta}_2\right)X^I(\sigma) = i\bar{\epsilon}_1\Gamma^I\tilde{\epsilon}_2 \equiv \eta_0^I, \left(\tilde{\delta}_2\delta_1 - \delta_1\tilde{\delta}_2\right)A^\mu(\sigma,\sigma') = \frac{i}{2}\bar{\epsilon}_1\Gamma^\mu\Gamma_I\left(X^I(\sigma) - X^I(\sigma')\right)\tilde{\epsilon}_2 \equiv \eta_0^\mu(\sigma) - \eta_0^\mu(\sigma'), \tag{16}$$

where the commutator that acts on the other fields vanishes. Thus, the commutation relation is given by

$$\tilde{\delta}_2\delta_1 - \delta_1\tilde{\delta}_2 = \delta_\eta, \tag{17}$$

where δ_η is a translation.

If we change a basis of the supersymmetry transformations as

$$\delta' = \delta + \tilde{\delta}, \tilde{\delta}' = i\left(\delta - \tilde{\delta}\right), \tag{18}$$

We obtain

$$\begin{aligned}
&\delta_2'\delta_1' - \delta_1'\delta_2' = \delta_\eta, \\
&\tilde{\delta}_2'\tilde{\delta}_1' - \tilde{\delta}_1'\tilde{\delta}_2' = \delta_\eta, \\
&\tilde{\delta}_2'\delta_1' - \delta_1'\tilde{\delta}_2' = 0.
\end{aligned} \tag{19}$$

These thirty-two supersymmetry transformations are summarised as $\Delta = \left(\delta', \tilde{\delta}'\right)$ and (19) implies the $\mathcal{N} = 1$ supersymmetry algebra in eleven dimensions,

$$\Delta_2\Delta_1 - \Delta_1\Delta_2 = \delta_\eta. \tag{20}$$

3. Zariski Quantization

In this section, we review the Zariski Quantization and apply it for the semi-light-cone supermembrane action (8). In [34], it is shown that the Zariski quantization is a second quantization and the Zariski quantized action reduces to the supermembrane action if the fields are restricted to one-body states.

First, we define elements of linear spaces $\mathcal{M}_\hbar$ by

$$\mathbf{X}_\hbar = \sum_{r=0}^{\infty}\left(\sqrt{\hbar}\right)^r \sum_{u_r} Y_{u_r}^r(\sigma) Z_{u_r} \in \mathcal{M}_\hbar, \tag{21}$$

where the basis Z_u are labeled by polynomials $u = u(x_1, x_2)$ in the valuables x_1, x_2 with real or complex coefficients. The summation is taken over all the polynomials of two valuables $\{u(x_1, x_2)\} \cdot Z_u$ satisfies $Z_{au} = aZ_u$ where a is a real (complex) number. The coefficients $Y_u(\sigma)$ are functions over 3-dimensional spaces. Summation is defined naturally as linear spaces.

The quantum Zariski product $\bullet_\hbar$ is defined as

$$\begin{aligned}
\mathbf{X}_\hbar \bullet_\hbar \mathbf{X}'_\hbar &= \left(\sum_{r=0}^{\infty}\left(\sqrt{\hbar}\right)^r \sum_{u_r} Y_{u_r}^r(\sigma) Z_{u_r}\right) \bullet_\hbar \left(\sum_{s=0}^{\infty}\left(\sqrt{\hbar}\right)^s \sum_{v_s} Y'^s_{v_s}(\sigma) Z_{v_s}\right) \\
&= \left(\sum_{u_0} Y_{u_0}^0(\sigma) Z_{u_0}\right) \bullet_\hbar \left(\sum_{v_0} Y'^0_{v_0}(\sigma) Z_{v_0}\right) = \sum_{u_0 \cdot v_0} Y_{u_0}^0(\sigma) Y'^0_{v_0}(\sigma) Z_{u_0} \bullet_\hbar Z_{v_0}.
\end{aligned} \tag{22}$$

Any polynomial can be decomposed uniquely as $u = au_1u_2\cdots u_M$, where a is a real (complex) number and u_i are irreducible normalized polynomials. $Z_u \bullet_\hbar Z_v$ is defined by

$$Z_u \bullet_\hbar Z_v = ab\zeta\left(\left(u_1u_2\cdots u_M\right) \times_\hbar \left(u_{M+1}u_{M+2}\cdots u_N\right)\right), \tag{23}$$

where $v = bu_{M+1}u_{M+2}\cdots u_N \cdot \times_\hbar$ is defined by

$$\left(u_1u_2\cdots u_M\right) \times_\hbar \left(u_{M+1}u_{M+2}\cdots u_N\right) := \frac{1}{N!}\sum_{\sigma \in S_N} u_{\sigma_1} * u_{\sigma_2} * \cdots * u_{\sigma_N}, \tag{24}$$

where S_N is the permutation group of $\{1, 2, \cdots, N\}$ $*$ is the Moyal product defined by

$$f * g = \sum_{r=0}^{\infty}\frac{\left(\sqrt{\hbar}\right)^r}{r!}\epsilon^{i_1j_1}\epsilon^{i_2j_2}\cdots\epsilon^{i_rj_r}\frac{\partial}{\partial x_{i_1}}\frac{\partial}{\partial x_{i_2}}\cdots\frac{\partial}{\partial x_{i_r}} f \frac{\partial}{\partial x_{j_1}}\frac{\partial}{\partial x_{j_2}}\cdots\frac{\partial}{\partial x_{j_r}} g, \tag{25}$$

where i_r and j_r run from 1 to 2. ζ is defined by

$$\zeta\left(\sum_{r=0}^{\infty}\left(\sqrt{\hbar}\right)^r u_r\right)=\sum_{r=0}^{\infty}\left(\sqrt{\hbar}\right)^r Z_{u_r}. \qquad (26)$$

We define derivatives on $\mathcal{M}_\hbar$ by derivatives with respect to $\sigma^i\ (i=1,2,\cdots,p)$ as

$$\frac{\partial}{\partial\sigma^i}\mathbf{X}_\hbar=\sum_{r=0}^{\infty}\left(\sqrt{\hbar}\right)^r\sum_{u_r}\frac{\partial}{\partial\sigma^i}Y_{u_r}^r(\sigma)Z_{u_r}. \qquad (27)$$

One can show that the quantum Zariski product is Abelian, associative and distributive, and the derivative is commutative and satisfies the Leibniz rule [34].

We define the Zariski quantized Nambu-Poisson bracket by

$$\begin{aligned}\left[\mathbf{X}_\hbar,\mathbf{X}'_\hbar,\mathbf{X}''_\hbar\right]_{\bullet_\hbar}&:=\epsilon^{ijk}\frac{\partial}{\partial\sigma^i}\mathbf{X}_\hbar\bullet_\hbar\frac{\partial}{\partial\sigma^j}\mathbf{X}'_\hbar\bullet_\hbar\frac{\partial}{\partial\sigma^k}\mathbf{X}''_\hbar\\&=\sum_{u_0,v_0,w_0}\epsilon^{ijk}\frac{\partial}{\partial\sigma^i}Y_{u_0}^0(\sigma)\frac{\partial}{\partial\sigma^j}Y_{v_0}'^0(\sigma)\frac{\partial}{\partial\sigma^k}Y_{w_0}''^0(\sigma)Z_{u_0}\bullet_\hbar Z_{v_0}\bullet_\hbar Z_{w_0},\end{aligned} \qquad (28)$$

where $i,j,k=1,2,3$. By definition, the bracket is skew-symmetric. By using the above properties, one can show that it satisfies the Leibniz rule and the fundamental identity;

$$\left[A,B,\left[X,Y,Z\right]_{\bullet_\hbar}\right]_{\bullet_\hbar}=\left[\left[A,B,X\right]_{\bullet_\hbar},Y,Z\right]_{\bullet_\hbar}+\left[X,\left[A,B,Y\right]_{\bullet_\hbar},Z\right]_{\bullet_\hbar}+\left[X,Y,\left[A,B,Z\right]_{\bullet_\hbar}\right]_{\bullet_\hbar}, \qquad (29)$$

for any $A,B,X,Y,Z\in\mathcal{M}_\hbar$. Thus, the Zariski quantized Nambu-Poisson bracket has the same Nambu-Poisson structure as the original Nambu-Poison bracket.

We define a metric for $X_\hbar, X'_\hbar\in\mathcal{M}_\hbar$ by

$$\begin{aligned}\left\langle\mathbf{X}_\hbar,\mathbf{X}'_\hbar\right\rangle&=\left\langle\mathbf{X}_\hbar\bullet_\hbar\mathbf{X}'_\hbar\right\rangle=\int\mathrm{d}^p\sigma\left\langle\left\langle\mathbf{X}_\hbar\bullet_\hbar\mathbf{X}'_\hbar\right\rangle\right\rangle=\sum_{u_0,v_0}\int\mathrm{d}^p\sigma Y_{u_0}^0(\sigma)Y_{v_0}'^0(\sigma)\left\langle\left\langle Z_{u_0}\bullet_\hbar Z_{v_0}\right\rangle\right\rangle\\&=\sum_{u_0,v_0}\int\mathrm{d}^p\sigma Y_{u_0}^0(\sigma)Y_{v_0}'^0(\sigma)\sum_{r=0}^{\infty}\alpha^r\sum_{w_r}\left\langle\left\langle Z_{w_r}\right\rangle\right\rangle,\end{aligned}$$

where $\left\langle\left\langle Z_w\right\rangle\right\rangle$ is defined by

$$\left\langle\left\langle Z_w\right\rangle\right\rangle=a \text{ if } w=az^2,\ \text{otherwise}\left\langle\left\langle Z_w\right\rangle\right\rangle=0, \qquad (30)$$

where a is a real (complex) number and z is a normalized polynomial, whose monomial of the highest total degree has coefficient 1.

This metric is invariant under a gauge transformation generated by the Zariski quantized Nambu-Poisson bracket [34] as

$$\left\langle\left[\mathbf{X}_\hbar^3,\mathbf{X}_\hbar^4,\mathbf{X}_\hbar^1\right]_{\bullet_\hbar},\mathbf{X}_\hbar^2\right\rangle+\left\langle\mathbf{X}_\hbar^1,\left[\mathbf{X}_\hbar^3,\mathbf{X}_\hbar^4,\mathbf{X}_\hbar^2\right]_{\bullet_\hbar}\right\rangle=0. \qquad (31)$$

By performing the Zariski quantization of the supermembrane action in a semi-light-cone gauge (8), we obtain

$$\begin{aligned}S_{3\mathrm{alg}M}=\Big\langle&-\frac{1}{12}\left[\mathbf{X}^I,\mathbf{X}^J,\mathbf{X}^K\right]_{\bullet_\hbar}^2-\frac{1}{2}\left(\mathbf{A}_{\alpha ab}^u\left[\varphi_u^a,\varphi_u^b,\mathbf{X}^I\right]_{\bullet_\hbar}\right)^2-\frac{1}{3}E^{\alpha\beta\gamma}\mathbf{A}_{\alpha ab}^u\mathbf{A}_{\beta cd}^v\mathbf{A}_{\gamma ef}^w\left[\varphi_u^a,\varphi_v^c,\varphi_v^d\right]_{\bullet_\hbar}\left[\varphi_u^b,\varphi_w^e,\varphi_w^f\right]_{\bullet_\hbar}\\&-\frac{i}{2}\bar{\Psi}\Gamma^\alpha\mathbf{A}_{\alpha ab}^u\left[\varphi_u^a,\varphi_u^b,\Psi\right]_{\bullet_\hbar}+\frac{i}{4}\Psi\Gamma_{IJ}\left[\mathbf{X}^I,\mathbf{X}^J,\Psi\right]_{\bullet_\hbar}\Big\rangle.\end{aligned} \qquad (32)$$

The Zariski quantization preserves the supersymmetries of the semi-light-cone supermembrane theory, because the quantum Zariski product is Abelian, associative and distributive, and admits a commutative derivative satisfying the Leibniz rule.

4. Conclusion

Zariski 3-algebra model of M-theory has manifest $\mathcal{N}=1$ supersymmetry in eleven dimensions because Zariski quantization preserves the supersymmetry of the supermembrane action in the semi-light-cone gauge. The relation between the model and the supermembrane action is clear: If the fields are restricted to one-body states, the model reduces to the supermembrane action.

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