

Lévy Flights, $1/f$ Noise and Self Organized Criticality in a Traveling Agent Model

Oliver López Corona¹, Pablo Padilla², Oscar Escolero³, Alejandro Frank⁴, Ruben Fossion⁵

¹Posgrado en Ciencias de la Tierra, Instituto de Geología, Universidad Nacional Autónoma de México, Ciudad de México, México

²IIMAS, Universidad Nacional Autónoma de México, Ciudad de México, México

³Instituto de Geología, Universidad Nacional Autónoma de México, Ciudad de México, México

⁴Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Ciudad de México, México

⁵Instituto Nacional de Geratría, México

Email: oliverlc@geologia.unam.mx

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ABSTRACT

A new analysis of a previously studied traveling agent model, showed that there is a relation between the degree of homogeneity of the medium where the agents move, agent motion patterns, and the noise generated from their displacements. We proved that for a particular value of homogeneity, the system self organizes in a state where the agents carry out Lévy walks and the displacement signal corresponds to $1/f$ noise. Using probabilistic arguments, we conjectured that $1/f$ noise is a fingerprint of a statistical phase transition, from randomness (disorder) to predictability (order), and that it emerges from the contextuality nature of the system which generates it.

Keywords: Lévy Flights; $1/f$ Noise; Self Organized Criticality; Agents Modelling; Complexity

1. Introduction

Phenomena with Lévy like distributions have been widely reported, e.g. in experiments with CTAB mielles dissolved in salted water [1], chaotic transport in a laminar fluid flow of a water-glycerol mixture in a rapidly rotating annulus [2], subrecoil laser cooling [3], conservative motion in a two dimensional periodic potential [4], transport in heterogeneous catalysis [5], reactions and in polymer systems under conformational motion [6], in the description of behavior of dynamical systems [7], economic time series [8] and in the motion of gold nanoclusters in graphite [9].

The foraging patterns of a variety of animals [10-15] involve many spatio-temporal scales and are sometimes well described by Lévy walks. A good review on the subject may be found in [16]. This statistical behavior has also been reported in human movement patterns [17].

In this context, Boyer and López-Corona [18] introduced a model of traveling agents (e.g. frugivorous animals) who feed on randomly located vegetation patches and disperse their seeds. The foraging agents use a deterministic strategy with memory, that makes them visit the largest possible patches (most food content) accessible within minimal traveling distances. If the patches have a small initial size, the vegetation total mass increases with time and reaches a maximum corresponding to a self organized critical state with power-law distrib-

uted patch sizes and Lévy-like movement patterns for the foragers. They proved that the distribution of resources is not held fixed and spatial heterogeneities self organize spontaneously under the influence of positive feedback loops in the system dynamics. Particularly, at low plant competition, the power spectrum of biomass time series correspond to a $1/f$ noise.

Pink or $1/f$ noise (sometimes also called Flicker noise) is a signal or process with a frequency spectrum such that the power spectral density is inversely proportional to the frequency [19]. This statistical behavior appears in such diverse phenomena as Quantum Mechanics [20-23], Biology [18,24,25], Medicine [26] and Astronomy [27], among other fields. The frequent occurrence of pink noise in such a seemingly unrelated set of physical systems, has prompted an extensive search for common underlying physical principles [28].

It is clear that $1/f$ noise is part of a wider set of scale invariant signals defined by an inverse power law power spectrum of the form

$$S(f) \sim 1/f^\lambda, \quad (1)$$

where λ is the spectral density exponent, which classifies the signals depending of its value: $\lambda = 0$ for white noise, $\lambda = 1$ for pink and $\lambda = 2$ for brown noise. These tree types of noise exhibit quite different statistical characteristics.

Let's define (following [29]) the autocorrelation func-

tion as the inverse Fourier transform of the power spectrum of the signal

$$C(\tau) = F^{-1}(S(f)). \tag{2}$$

If the power spectrum obeys the power law of Equation (1), and we apply a scale transformation in the time domain, $\tau \rightarrow \tau' = a\tau$ then

$$C(a\tau) = a^{-\lambda-1}C(\tau), \tag{3}$$

and the general solution of Equation (3) is also a power law. In this way, one possible complexity measure is the memory of the time series, as expressed by the autocorrelation function which measures how quickly the time series loses similarity with a copy of itself displaced with some delay time τ . The correlations are zero for white noise ($\lambda = 0$), large for brown noise ($\lambda = 2$), and infinite for a periodic series ($\lambda \rightarrow \infty$). When ($\lambda = 1$) corresponds to a very special case, as we show below. We shall see that the parameter λ gives a measure of the correlation strength and may be used as a control parameter for complexity.

A complex system [30] is a special type of dynamical composite system [31], where under critical circumstances new collective behavior emerges from the short-range interactions between the constituent parts. Intuitively, complexity is inversely related to a simple behavior, *i.e.*, the more we are able to provide a simple description of a phenomenon, the less complex it is. Uncorrelated random signals (white noise) may be highly unpredictable even in cases where the past history is fully known but, at a global level, they admit a very simple description and, therefore, are not really “complex”. From a complex-systems approach, random uncorrelated series are among the least complex signals, and those with long-range correlations are among the most complex [29].

The autocorrelation function of white noise follows a power law with ($\lambda = 0$); strongly correlated Brownian noise, although composed mostly of low-frequency and thus large-period waves, has only local, short-term correlations between neighboring points [32]; finally the autocorrelation function for $1/f$ noise is a logarithmic function, which decays more slowly than any power law. Thus $1/f$ noise has the largest possible memory for a scale invariant signal and therefore, under our definition, is the most complex signal too.

In recent works Eliazar and Klafter [33,34] showed that both Lévy walks and $1/f$ are the result of systems which superimpose the transmissions of infinite independent stochastic signals.

With this in mind, we proceeded to investigate under which conditions, if any, the power spectra of the agents motion (Lévy walks) follow a $1/f$ dynamics. We found a non trivial relationship between the media’s homogeneity coefficient β , the agents motion and the noise type ob-

served. These results (Figure 1 and Table 1) are new and constitute our main result.

2. Method: A Traveling Agent Model

Let’s consider a two-dimensional square domain of unit area with N fixed, point-like food patches randomly and uniformly distributed. Each patch contains a k amount of food.

Initially, an agent is located on a patch chosen at random. Then the following deterministic foraging rules are iteratively applied at every time step:

- 1) The agent located at patch i feeds on that patch, the fruit content decreasing by one unit: $k_i \rightarrow k_i - 1$.
- 2) When k_i has reached the value 0, the agent chooses another patch, j , such that k_j/d_{ij} is maximal over all the allowed patches $j = i$ in the system, where k_j is the food content of patch j and d_{ij} the Euclidean distance between patches i and j . With this rule, the next visited patch (the “best” patch) has a large food content and/or is at a short distance from i . It is assumed that the travel from i to j takes one time unit.
- 3) The agent does not revisit previously visited patches.

The model exhibits some remarkable properties. Let’s define the agents displacement $R(t)$ as $|R(t+t_0) - R(t_0)|$ with $R(t)$ the agent position at time t . When averages are taken, different times t_0 and independent realizations are considered. If the patch size k is taken from an inverse power-law distribution $P(k) = ck^{-\beta}$, where c is an arbitrary constant and β is a coefficient that represents the medium homogeneity. When β is high ($\beta \gg 1$) the medium is very homogeneous, meaning that all patches have similar food content values. On the contrary, when β is low ($\beta \approx 1$) the medium is very heterogeneous, meaning that patches with high food content are numerous. The intermediate case ($\beta \approx 3$), corresponds to a complex medium where patches with high and low food contents are present.

This model produces complex trajectories that have been studied in detail in refs. [35,36] and discussed in connection with spider monkeys foraging patterns ob-

Table 1. Relation between media homogeneity coefficient β , type of medium, agents motion, and the noise type observed.

β	Medium	Motion	Noise	Dynamic
2	Inhomogenous (spatial disorder)	Random confined	White	Randomness-no correlation (dynamic disorder)
3	Transition point (spatial complexity)	Lévy	Pink: $1/f$	Transition point (criticality)
5	Homogeneous (spatial order)	Brownian	Brown	Predictability-strong correlation (dynamic order)

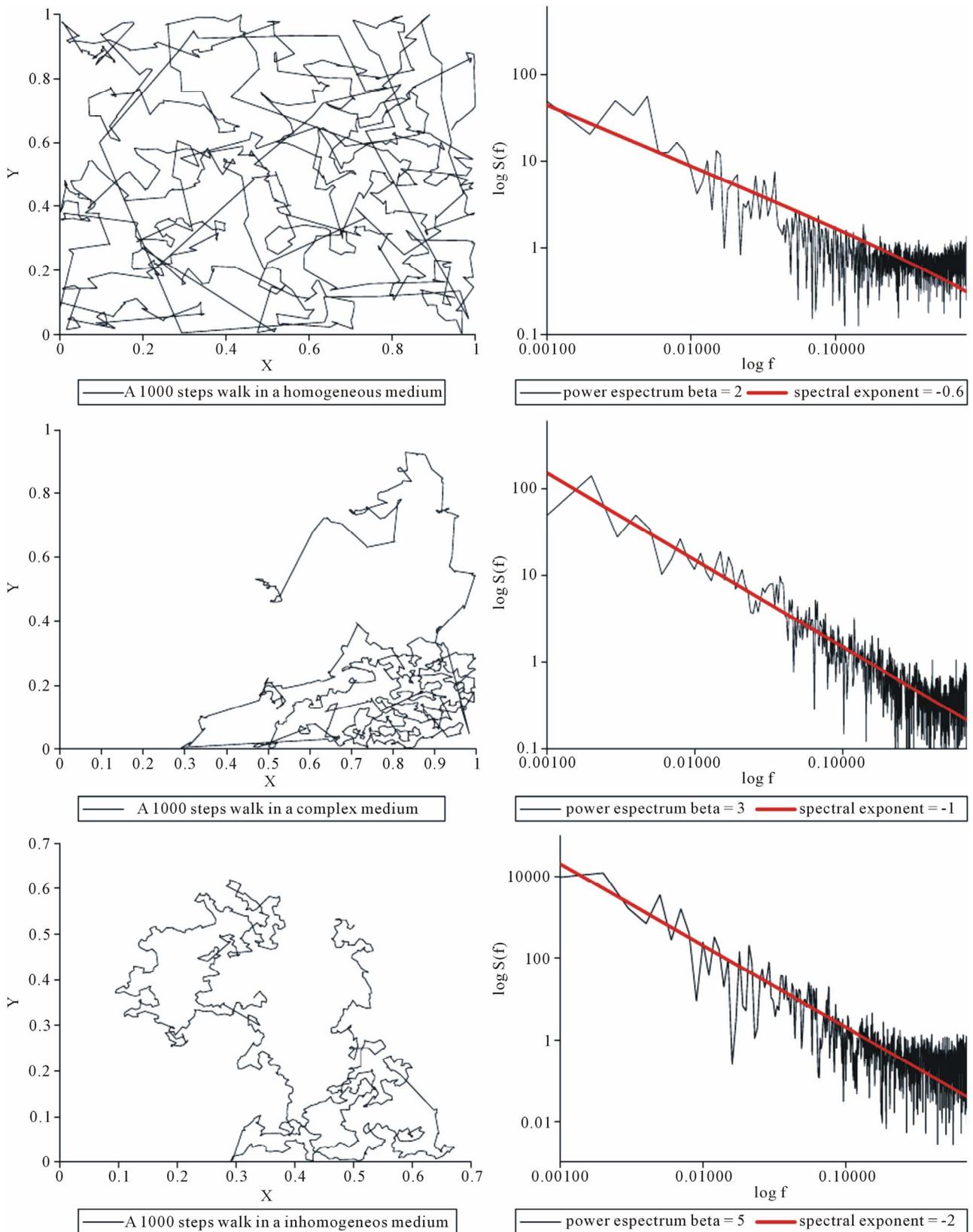


Figure 1. Examples of agent walk (first column) and the corresponding power spectrum (second column) for three values of the medium homogeneity coefficient $\beta = 2$ (homogeneous medium), $\beta = 3$ (complex medium) and $\beta = 5$ (inhomogeneous medium).

served in the field [11]. The different media types produce distinctive motion patterns summarized in **Figure 1**.

3. Results

In **Figure 1**, three values of $\beta = \{2, 3, 5\}$ corresponding to disordered, complex and ordered media were used to construct time series for $R(t)$. The power spectrum defined by $S(f) \equiv |\tilde{R}(f)\tilde{R}(-f)|$ with $\tilde{R}(f)$ the Fourier transformation of the agent displacement, was calculated for each independent run using the fast Fourier transform (fft) technique. In **Table 1**, 50 power spectra were averaged and fitted by an inverse power law $S(f) \sim f^{-\lambda}$. White noise corresponds to a $\lambda = 0$, pink to a $\lambda = 1$, and brown to a $\lambda = 2$.

Figure 2 shows the spectral exponent λ as a function of the medium homogeneity coefficient β . Statistics were taken from the same 50 independent realization used for **Table 1**.

The analysis of time as a function of the step number (**Figures 3 and 4**), reveals that the systems transit from a nonstationary to a stationary regimen. A second derivative parameter

$$\begin{aligned} \xi &= \left| d^2T(n)/dn^2 \right|_{n=3} \\ &= \left| T(n-1) + T(n-1) - 2T(n) \right|_{n=3} \end{aligned}$$

was calculated in the third step of the walk in order to capture the transitory part of the signal.

- For $\beta = 3.0$ the system is highly non-stationary (ξ big) reaching the stationarity only for very big times
- For $3.0 \leq \beta \leq 4.0$ the system is near the stationarity (ξ small).
- For $\beta \geq 4.0$ the system is highly stationary (ξ tends to zero)

The system experiments a sudden transition from non-stationarity to stationarity in $\beta = 3.0$ which may be seen as a phase transition fingerprint.

These results suggests that the emergence of pink

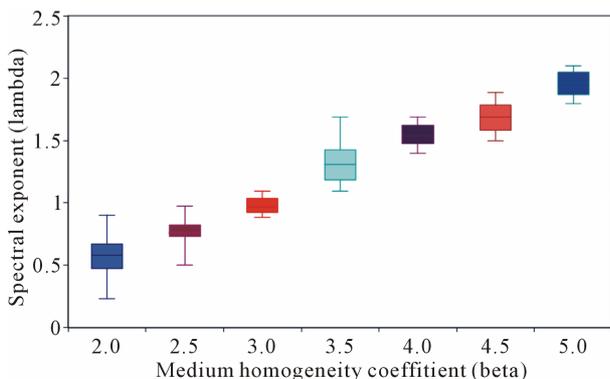


Figure 2. For each realization the power spectrum was constructed and the spectral exponent calculated. Then averages were taken for 50 independent realization.

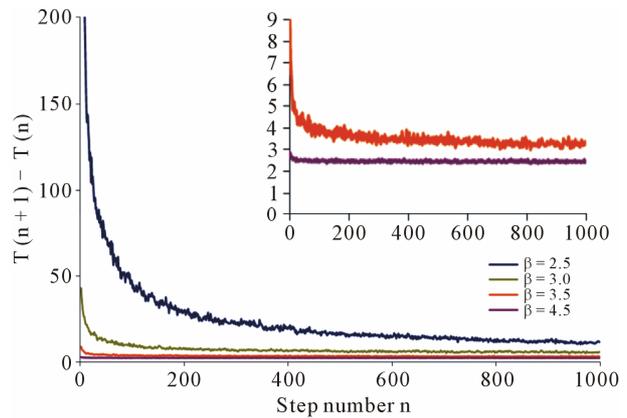


Figure 3. Discrete first derivative of time in function of the step number.

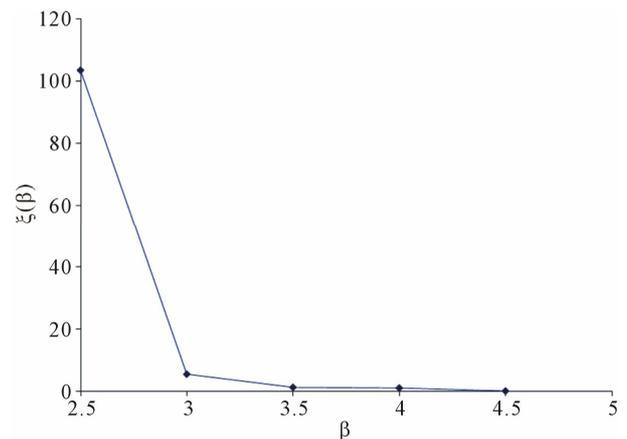


Figure 4. Discrete second derivative of time as a function of step number n , calculated in the third time step of the walk for capture the transitory part of the process.

noise for a traveling agent in a heterogeneous medium depends on the degree of heterogeneity of the medium. If the medium homogeneity coefficient is distributed as a power law $P(K) = cK^{-\beta}$, then a $1/f$ noise will be obtained when $\beta = 3$. Thus, this dynamical behavior may naturally arise from the motion of agents in a complex medium.

4. Conclusions

Our results suggest that $1/f$ noise may be a fingerprint of a statistical phase transition from randomness (low correlation associated with white noise), to predictability (high correlation associated to brown noise) an idea suggested in [37]. The authors proved that a transitional state in two different regimes implies the occurrence of $1/f$ time series and that this property is generic in both classical and quantum systems. They showed this by studying a classical system, the one-dimensional module-1 logistic map and a quantum one, the nuclear excitation spectra obtained with a schematic shell-model Hamilto-

nian.

Moreover, Relano *et al.* [38] proposed to interpret fluctuations in quantum excitation spectra as generalized time series and to study the corresponding power spectral density. While quantum excitation spectra (which is known to be chaotic) result in $1/f$ noise, the integrable excitation spectra follow instead a Brownian noise signal $1/f^2$. In the same line of thought, autonomous (involuntary) physiological time series are found to correspond with $S(f) \sim 1/f^\lambda$ power spectral density. Time series from healthy subjects are found to approach $1/f$ ($\lambda = 1$), and results have been reported for the fluctuations associated to heartbeat, gait, temperature, respiration [32,39-41]. On the other hand, time series from aging subjects tend toward $1/f^2$ ($\lambda = 2$) [42], while particular phenomena, such as heart fibrillation, tend to flatten out the power spectral density towards ($\lambda = 0$) [43,44]. It has been suggested that the $1/f$ in healthy physiological systems indicates a critical state, and that deviations indicate aging or disease.

The main result of our work is the relationship between the media's homogeneity of a traveling agent motion and the noise type observed. Of special interest is that $1/f$ noise is found when medium is complex. Additionally, we conjecture that $1/f$ noise is a fingerprint of a statistical phase (perhaps of second order) transition from randomness (low correlation associated with white noise), to predictability (high correlation associated to brown noise). Nevertheless a more detailed analysis should be carried out for prove this last point.

Finally, most interesting, but at this point somehow speculative, is the possible relation between $1/f$ and statistical contextuality. Meanwhile Eliazar and Klafter [33, 34] have proven that both Lévy walks and $1/f$ are the result of systems which superimpose the transmissions of infinite independent stochastic signals, it has been proposed that this would require a non-classical probabilistic theory, *i.e.* statistical contextuality [45-47]. Then we propose that $1/f$ ubiquity is a consequence of contextuality ubiquity in Nature.

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