

Charge Radius and Effective Couplings via the Process $\nu_\mu e^- \rightarrow \nu_\mu e^-$

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Received July 8, 2012; revised October 19, 2012; accepted October 26, 2012

ABSTRACT

In this work the neutral-current scattering cross-section for neutrinos on electrons is calculated assuming that a massive Dirac neutrino is characterized by a phenomenological parameters, a charge radius $\langle r^2 \rangle$ and the right-handed currents are present in the framework of a Left-Right symmetric model (LR). Using the CHARM II result for the charge radius of the muon-neutrino $|\langle r^2 \rangle| < 6.0 \times 10^{-33} \text{ cm}^2$, we place a bound on $-7.9 \times 10^{-33} \text{ cm}^2 \leq \langle r^2 \rangle_{LR} \leq 7.9 \times 10^{-33} \text{ cm}^2$. We discuss the relationship between the electron neutral couplings g_V^{ve} and g_A^{ve} and the LR model parameters. We also estimate a bound on the heavy massive neutral vector boson mass $M_{Z_R} > 650 \text{ GeV}$ of the LR model. These results have never been reported in the literature before and could have practical or theoretical interest, such as in the case of the neutrinos produced in core-collapse supernova explosions, that is to say, right-handed Dirac neutrinos emission from supernova core $\nu_L e^- \rightarrow \nu_R e^-$.

Keywords: Weak and Electromagnetic Interactions; Non-Standard-Model Neutrinos; Neutral Currents; Electromagnetic Form Factors

1. Introduction

Although in the framework of the Standard Model (SM) [1-3] neutrinos are assumed to be electrically neutral, the electromagnetic properties of the neutrino are discussed in many gauge theories beyond the SM. Electromagnetic properties of the neutrino [4-8] may manifest themselves in a non vanishing charge radius, thus the neutrino is subject to the electromagnetic interaction.

The charge radius of ν_e has been bounded by the LAMPF [9] experiment

$$\langle r^2 \rangle = 0.9 \pm 2.7 \times 10^{-27} \text{ cm}^2$$

obtaining, more recently LSND [10] limit from measurement of electron-neutrino electron elastic scattering is

$$-2.97 \times 10^{-32} \text{ cm}^2 < \langle r_e^2 \rangle < 4.14 \times 10^{-32} \text{ cm}^2,$$

while for the muon-neutrino the bound from the scattering experiment of CHARM II [11] is

$$|\langle r^2 \rangle| < 0.6 \times 10^{-32} \text{ cm}^2.$$

To our knowledge, there are no bounds on the charge radius of τ -neutrinos from scattering experiments. How-

ever, the corresponding bounds from Super-K and SNO observations [12] for the neutrino charge radius are $|\langle r_{\nu_\tau}^2 \rangle|$,

$$|\langle r_{\nu_\mu}^2 \rangle| < 2.08(1.53) \times 10^{-31} \text{ cm}^2$$

and

$$|\langle r_{\nu_e}^2 \rangle| < 6.86(5.26) \times 10^{-32} \text{ cm}^2.$$

Experimental evidence for neutrino flavor transformation [13] implies that neutrinos are the first elementary particles whose properties cannot be fully described within the SM. This hints to the possibility that other properties of these intriguing particles might substantially deviate from the predictions of the SM. As a result, this has motivated vigorous efforts, both on the theoretical and experimental sides, to understand the detailed properties of neutrinos and of their interactions more in depth. In particular, electromagnetic properties of the neutrinos can play important roles in a wide variety of domains such as cosmology [14] and astrophysics [15, 16], and can play a role in the deficit of electron neutrinos from the sun [17-19].

In general, a photon may couple to charged leptons

through its electric charge, magnetic dipole moment (MM), electric dipole moment (EDM) and the anapole moment (AM). This coupling may be parameterized using a matrix element in which the usual γ^α is replaced by a more general Lorentz-invariant form [4]:

$$\Gamma^\mu = F_Q(q^2)\gamma^\mu + F_M(q^2)i\sigma^{\mu\nu}q_\nu + F_E(q^2)\sigma^{\mu\nu}q_\nu\gamma_5 + F_A(q^2)(q^2\gamma^\mu - q^\mu\not{q})\gamma_5 \quad (1)$$

where $F_{Q,M,E,A}(q^2)$ are the electromagnetic form factors of the neutrino, corresponding to the charge radius, MM, EDM and AM, respectively.

Electromagnetic properties of neutrinos are of fundamental importance and serve as a probe of physics beyond the SM. Several authors have shown that the charge radius of the neutrino is not a physical quantity [6,20], as demonstrated by the fact that it is gauge-dependent [21]. However, other authors claim that they can extract a gauge-independent neutrino charge radius, which is, therefore, a physical observable [22]. A definition of the neutrino charge radius that satisfies physical requirements, *i.e.* it is a physical observable, has recently been provided [22] in the framework of the Pinch Technique formalism [23].

In this paper, we start from a Left-Right symmetric model (LR) [24] and assuming that a massive Dirac neutrino is characterized by a phenomenological parameter, a charge radius $\langle r_\nu^2 \rangle$ we calculate the cross-section of the process $\nu_\mu e^- \rightarrow \nu_\mu e^-$. We also estimate bounds on the charge radius of the muon-neutrino in the framework of the LR mode

$$-7.9 \times 10^{-33} \text{ cm}^2 \leq \langle r^2 \rangle_{LR} \leq 7.9 \times 10^{-33} \text{ cm}^2,$$

using the limit of CHARM II for the charge radius of the muon-neutrino $|\langle r^2 \rangle| < 0.6 \times 10^{-33} \text{ cm}^2$.

In a previous papers [25], possible corrections at the couplings of the fermion with the gauge boson were calculated, in particular the lepton couplings g_V and g_A with the neutral boson Z_L , which were measured with great precision in LEP and CHARM II [11]. In the present work, we calculate the simultaneous contribution of the neutrino charge radius, the additional neutral vector boson Z_R , the mixing angle ϕ of the LR model on the electron couplings constant $(g_V^{ve})_{LR}$ and $(g_A^{ve})_{LR}$. We also obtain bounds on the heavy massive neutral vector boson mass M_{Z_R} of the LR model. The neutrino charge radius in the LR model is simply treated as a new parameter. One is thus dealing with a purely phenomenological analysis. For an analysis of the electromagnetic form factors of the neutrino from a theoretical point of view in Left-Right models, see [26].

This paper is organized as follows: in Section 2 we carry out the calculus of the process $\nu_\mu e^- \rightarrow \nu_\mu e^-$. In

Section 3 we achieve the numerical computations and, finally, we summarize our results in Section 4.

2. Muon-Neutrino Electron Cross-Section

In this section we obtain the corresponding amplitude for the process

$$\nu_\mu(k_1) + e^-(p_1) \rightarrow \nu_\mu(k_2) + e^-(p_2), \quad (2)$$

mediated by the photon γ and the neutral gauge bosons Z_L and Z_R . We assume that a massive Dirac neutrino is characterized by a phenomenological parameter, a charge radius $\langle r_\nu^2 \rangle$. Therefore, the expression for the cross-section of the process $\nu_\mu e^- \rightarrow \nu_\mu e^-$ is given by

$$\sigma_T^{LR} = \frac{G_F^2 m_e E_\nu}{2\pi} \left\{ 2\delta^2 + 2\delta(P+S) + \frac{(P+S)^2 + (Q+R)^2}{2} \right. \\ \left. + \frac{1}{3} \left[2\delta^2 + 2\delta(P+S) + \frac{(P-S)^2 + (Q-R)^2}{2} \right] \right\}, \quad (3)$$

where the neutrino charge radius, the heavy massive neutral vector boson and the mixing angle ϕ contribute to the total cross-section. P, Q, R, S are given by

$$\begin{aligned} P &= (A + 2B + C)g_V, \\ Q &= (-A + C)g_A, \\ R &= (-A + C)g_V, \\ S &= (A - 2B + C)g_A. \end{aligned} \quad (4)$$

The constants A, B and C [27] depend only on the parameters of the LR model

$$\begin{aligned} A &= \left(c_\phi - \frac{s_W^2}{r_W} s_\phi \right)^2 + \gamma \left(\frac{s_W^2}{r_W} c_\phi + s_\phi \right)^2, \\ B &= \left(c_\phi - \frac{s_W^2}{r_W} s_\phi \right) \left(-\frac{c_W^2}{r_W} s_\phi \right) + \gamma \left(\frac{s_W^2}{r_W} c_\phi + s_\phi \right) \left(\frac{c_W^2}{r_W} c_\phi \right), \\ C &= \left(\frac{c_W^2}{r_W} s_\phi \right)^2 + \gamma \left(\frac{c_W^2}{r_W} c_\phi \right)^2, \\ \gamma &= \left(\frac{M_{Z_L}}{M_{Z_R}} \right)^2, \\ \delta &= \frac{\sqrt{2}\pi\alpha}{3G_F} \langle r^2 \rangle, \end{aligned} \quad (5)$$

where $c_\phi = \cos \phi$, $s_\phi = \sin \phi$, $c_W = \cos \theta_W$, $s_W = \sin \theta_W$, $r_W = \sqrt{\cos 2\theta_W}$ and M_{Z_L} , M_{Z_R} are the masses of the light and heavy massive neutral vector bosons that participate in the interaction. γ together with ϕ are the two new parameters that are introduced in the LR model, while $\langle r^2 \rangle$ is the charge radius of the muon-neutrino. Bounds on this quantity are reported in the literature

[11,28,29]. We can in addition take the limit $\phi = 0$ and $M_{Z_R} \rightarrow \infty$ and get the standard model cross-section. Notice that it is not possible to factorize the neutrino current in Equation (4), since P , Q , R and S are different, and therefore we cannot parameterize the electron weak coupling in terms of the effective couplings g_V^{eff} and g_A^{eff} .

2.1. Charge Radius of the Muon-Neutrino

From the cross-section expression (3), we obtain the interference cross-section which is given by

$$\sigma_I^{LR} = \frac{2G_F^2 m_e E_\nu \delta}{3\pi} (2P + S), \quad (6)$$

where P , S and δ are defined in Equations (4) and (5), respectively.

We rewrite the interference cross-section as the follow

$$\sigma_I^{LR} = K \langle r^2 \rangle (2g_V + g_A) \cdot \left[1 + \left(\frac{2g_V a_W + g_A b_W}{2g_V + g_A} \right) \gamma + (\gamma - 1) \left(\frac{2g_V + g_A b'_W}{2g_V + g_A} \right) s_\phi c_\phi \right], \quad (7)$$

where

$$K = \frac{4G_F^2 m_e \alpha E_\nu}{9\sqrt{2}}, \quad a_W = \frac{1}{r_W^2}, \quad b_W = r_W^2, \quad b'_W = -2r_W.$$

Evaluating the limit when the mixing angle is $\phi = 0$ and $M_{Z_R} \rightarrow \infty$, $\gamma \rightarrow 0$, the second and third terms in (7) is zero and Equation (7) is reduced to the expressions (A.5, 17) given in Refs. [30,31]:

$$\sigma_I^0 = K \langle r^2 \rangle (2g_V + g_A). \quad (8)$$

In order to identify the neutrino charge radius in the LR model, of Equations (7) and (8), we define

$$\langle r^2 \rangle_{LR} = \langle r^2 \rangle \left[1 + \left(\frac{2g_V a_W + g_A b_W}{2g_V + g_A} \right) \gamma + (\gamma - 1) \left(\frac{2g_V a_W + g_A b'_W}{2g_V + g_A} \right) s_\phi c_\phi \right], \quad (9)$$

where $\langle r^2 \rangle_{LR}$ is the charge radius of the muon-neutrino in the LR model and $\langle r^2 \rangle$ is the charge radius in the minimal extension of the standard model.

2.2. Electron Coupling Constants $(g_V^{ve})_{LR}$ and $(g_A^{ve})_{LR}$

In this subsection, the total cross-section Equation (3) is written in such a way that we can express the theoretical predictions for the electron couplings constant $(g_V^{ve})_{LR}$ and $(g_A^{ve})_{LR}$ such that give the SM couplings in the limit $\phi = 0$ and $M_{Z_R} \rightarrow \infty$:

$$\sigma_T^{LR} = \frac{G_F^2 m_e E_\nu}{2\pi} \left\{ \delta^2 + 2\delta (f_1 g_V^{ve} + f_3 g_A^{ve}) + \frac{1}{2} (f_1 g_V^{ve} + f_3 g_A^{ve})^2 + \frac{1}{2} f_2^2 (g_V^{ve} + g_A^{ve})^2 + \frac{1}{3} \left[\delta^2 + 2\delta (f_1 g_V^{ve} - f_3 g_A^{ve}) + \frac{1}{2} (f_1 g_V^{ve} - f_3 g_A^{ve})^2 + f_2^2 (g_V^{ve} - g_A^{ve})^2 \right] \right\}, \quad (10)$$

where

$$f_1 = u^2 + \frac{v^2}{r_W^2} \gamma, \quad f_2 = uv(1 - \gamma), \quad f_3 = v^2 + u^2 r_W^2 \gamma, \quad (11)$$

and

$$u = \cos \phi - \frac{\sin \phi}{r_W}, \quad v = \cos \phi + r_W \sin \phi,$$

with γ and δ as defined in Equation (5). We get the SM [28] formula after taking $\delta = 0$, $\phi = 0$, and $M_{Z_R} \rightarrow \infty$ in Equation (10):

$$\sigma^{SM} = \frac{G_F^2 m_e E_\nu}{2\pi} \left[(g_V^{ve} + g_A^{ve})^2 + \frac{1}{3} (g_V^{ve} - g_A^{ve})^2 \right], \quad (12)$$

and by analogy with the standard model, likewise we have

$$\sigma_T^{LR} = \frac{G_F^2 m_e E_\nu}{2\pi} \cdot \left\{ \left[(g_V^{ve})_{LR} + (g_A^{ve})_{LR} \right]^2 + \frac{1}{3} \left[(g_V^{ve})_{LR} - (g_A^{ve})_{LR} \right]^2 \right\}. \quad (13)$$

Using Equation (10) which includes the right-handed current of the neutrino, we would obtain new limits on the LR model parameters. We already mentioned in this section that it was not possible at the amplitude level to define g_V^{eff} and g_A^{eff} for the electron couplings. However, looking at Equations (10) and (13) we see that this is not the case for the total cross-section. As a matter of fact we can identify the LR model couplings of the electron in the following way:

$$(g_A^{ve})_{LR} [M_{Z_L}, M_{Z_R}, \phi, \delta, \sin \theta_W, g_A^{ve}] = \left[\delta f_3 + \frac{1}{2} (f_1 f_3 + f_2^2) \right] (g_A^{ve})_{SM} \quad (14)$$

$$(g_V^{ve})_{LR} [M_{Z_L}, M_{Z_R}, \phi, \delta, \sin \theta_W, g_V^{ve}] = \delta f_3 + \frac{1}{2} (f_1 f_3 + f_2^2) (g_V^{ve})_{SM} \quad (15)$$

In these expressions, with the limits $\phi = 0$ and $M_{Z_R} \rightarrow \infty$, $\gamma \rightarrow 0$ the SM couplings are recovered.

3. Results

In this section, we present the numerical results obtained

for the charge radius of the muon-neutrino in the framework of a Left-Right symmetric model $\langle r^2 \rangle_{LR}$, the electron couplings constants $(g_V^{ve})_{LR}$ and $(g_A^{ve})_{LR}$ and of the mass of the heavy massive neutral vector boson M_{Z_R} . For the SM parameters, we adopted the following:

$$M_{Z_L} = 91.187 \pm 00.007 \text{ GeV}$$

and $\sin^2 \theta_W = 0.2312$ [28]. At the present time, the most precise direct measurements of $g_{V,A}^{ve}$ come from the LEP and CHARM II experiments [11,28]

$$g_V^{ve} = -0.035 \pm 0.017$$

and

$$g_A^{ve} = -0.503 \pm 0.017$$

at 1σ in agreement with the SM and the world average values. For the mixing angle ϕ between Z_L and Z_R , we use the reported data in [32-34]:

$$-1.66 \times 10^{-3} \leq \phi \leq 1.22 \times 10^{-3}, \quad (16)$$

with 90% C.L. Other limits on the mixing angle ϕ reported in the literature are given in [35,36].

In order to estimate a limit on the charge radius of the muon-neutrino $\langle r^2 \rangle_{LR}$ in the framework of the Left-Right symmetric model, we plot the expression (9) to analyze the general behavior of the $\langle r^2 \rangle_{LR}$ function (Figure 1). In this figure, we observe that the mixing angle ϕ , around -0.75 rad, $\langle r^2 \rangle_{LR}$ can be as high as 10.4, and for values of ϕ around 0.8 rad, $\langle r^2 \rangle_{LR}$ is as low as 5.5. This shows a strong dependence on ϕ ; therefore, if $\langle r^2 \rangle_{LR}$ is the charge radius, the restriction on the charge radius can be "softened" if we consider a LR model. In Figure 2, we show the allowed region for $\langle r^2 \rangle_{LR}$ as a function of ϕ with 90% C.L. The allowed region is the rectangle band that is a result of both factors in Equation (9). In this figure, the second factor gives the rectangle band while $\langle r^2 \rangle$ gives the bandwidth. This

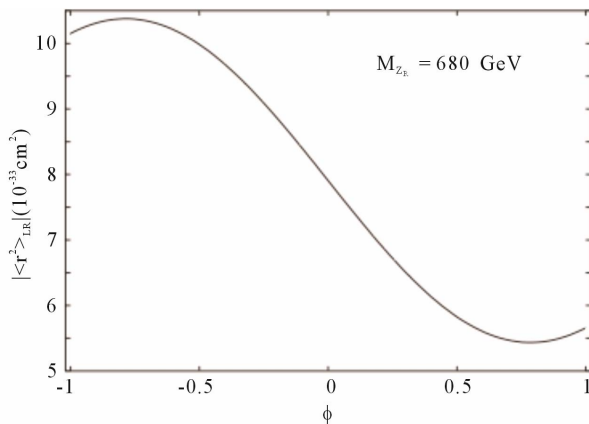


Figure 1. $\langle r^2 \rangle_{LR}$ as a function of the mixing angle ϕ .

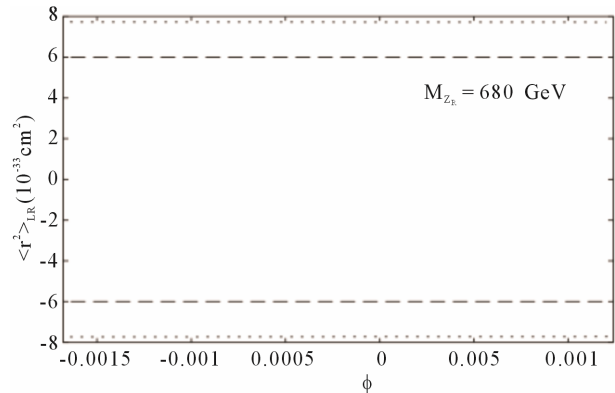


Figure 2. Allowed region for $\langle r^2 \rangle_{LR}$ as a function of the mixing angle ϕ with the value $\langle r^2 \rangle$. The dashed line shows the allowed region for $\langle r^2 \rangle$, while the dotted line shows the same result for the LR model.

analysis was done using the experimental value for $\langle r^2 \rangle$ reported by CHARM II [11] with a 90% C.L. In the same figure, we show the $\langle r^2 \rangle$ ($\phi = 0$) result at 90% C.L. with the dashed horizontal lines. The allowed region in the LR model (dotted line) for $\langle r^2 \rangle_{LR}$ is wider than the one for the $\langle r^2 \rangle$, and is given by:

$$\begin{aligned} -7.9 \times 10^{-33} \text{ cm}^2 &\leq \langle r^2 \rangle_{LR} \\ &\leq 7.9 \times 10^{-33} \text{ cm}^2, 90\% \text{ C.L.} \end{aligned} \quad (17)$$

whose value is quite close to that reported by other authors [5,6,8,11,12,28,29].

Figure 3 shows the charge radius $\langle r^2 \rangle_{LR}$ as a function of the LR parameters ϕ and M_{Z_R} . This figure shows a strong dependence of the charge radius with respect to the model parameters. In Figure 4 we plot χ^2 as a function of ϕ and M_{Z_R} . The minimum is obtained for $\phi = 1.22 \times 10^{-3}$ and $M_{Z_R} = 800 \text{ GeV}$ with $\chi^2 = 1.78 \times 10^{-3}$. In Figure 5 we have plotted $(g_A^{ve})_{LR}$ from Equation (14) as a function of the LR parameters ϕ and M_{Z_R} . We have chosen the range

$$-1.66 \times 10^{-3} \leq \phi \leq 1.22 \times 10^{-3} \text{ and } 300 \leq M_{Z_L} \leq 800 \text{ GeV}$$

where ϕ is measured in radians.

In Figure 6 we have plotted $(g_V^{ve})_{LR}$ from Equation (15) as a function of ϕ and M_{Z_R} . The range of variation for the LR parameters is the same as in Figure 5. In this case the experimental value, $(g_V^{ve})_{LR}^{\text{exp}} = -0.035$ is reached for small values of M_{Z_R} and ϕ . The effect of these two variables on $(g_A^{ve})_{LR}$ and $(g_V^{ve})_{LR}$ is similar.

In Figure 7 we plot χ^2 as a function of the LR parameters taking the same range of variation. The minimum is obtained for $\phi = 1.22 \times 10^{-3}$ and $M_{Z_R} = 800 \text{ GeV}$ with $\chi^2 = 3.85 \times 10^{-5}$. In Figure 8 we plot χ^2 , the minimum is obtained for

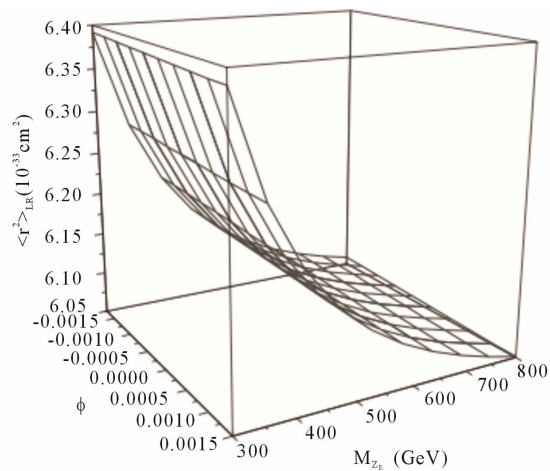


Figure 3. Plot of $\langle r^2 \rangle_{LR}$ as a function of the LR parameters ϕ and M_{Z_R} .

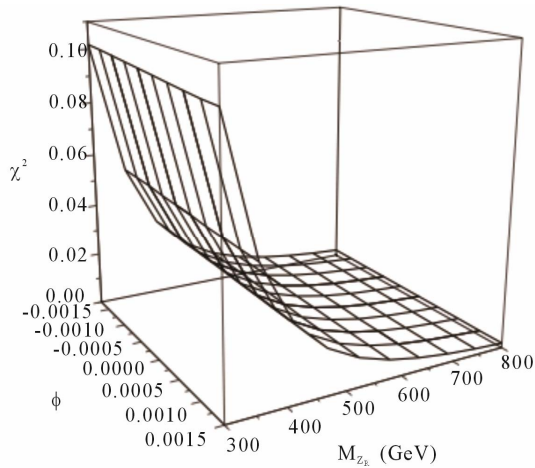


Figure 4. Plot of χ^2 as a function of the LR parameters ϕ and M_{Z_R} .

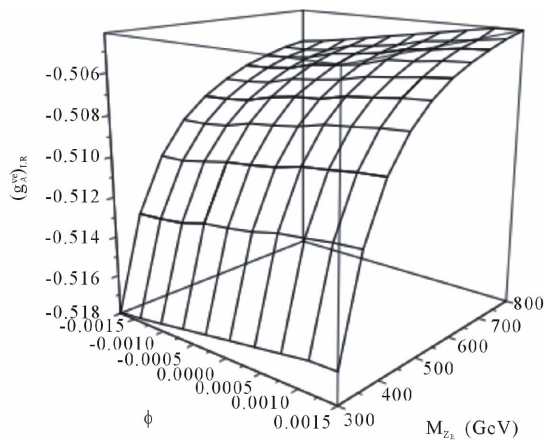


Figure 5. Plot of $(g_A^{ve})_{LR}$ as a function of the LR parameters ϕ and M_{Z_R} .

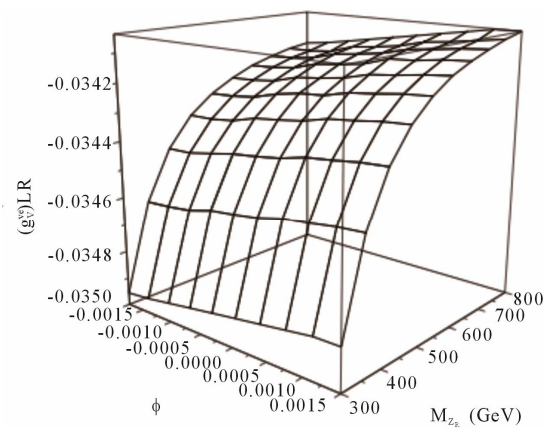


Figure 6. Plot of $(g_V^{ve})_{LR}$ as a function of the LR parameters ϕ and M_{Z_R} .

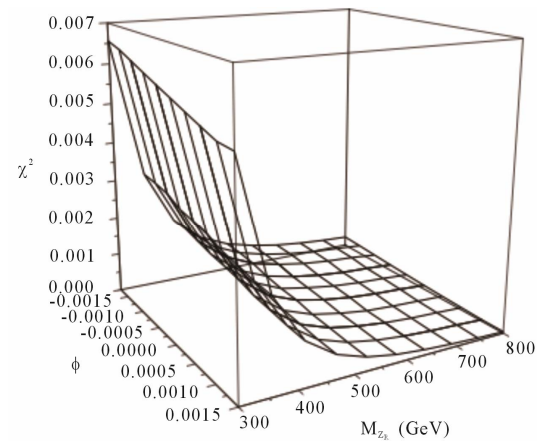


Figure 7. Plot of χ^2 as a function of the LR parameters ϕ and M_{Z_R} . The minimum of χ^2 is obtained at $\phi = 1.22 \times 10^{-3}$ and $M_{Z_R} = 800$ GeV.

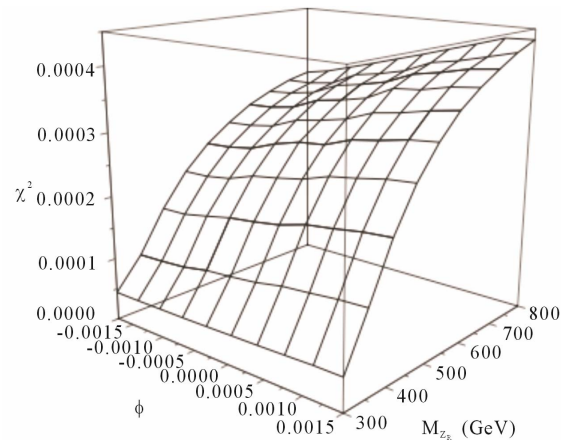


Figure 8. Plot of χ^2 as a function of the LR parameters ϕ and M_{Z_R} . The minimum of χ^2 is obtained at $\phi = -1.66 \times 10^{-3}$ and $M_{Z_R} = 300$ GeV.

$$\phi = -1.66 \times 10^{-3}$$

and

$$M_{Z_R} = 300 \text{ GeV} \quad \text{with} \quad \chi^2 = 4.41 \times 10^{-5}.$$

In these figures the effect of ϕ and M_{Z_R} on $(g_A^{ve})_{LR}$ and $(g_V^{ve})_{LR}$ are opposite.

Finally, in **Figure 9** we show the allowed region for the mass of the heavy massive neutral vector boson M_{Z_R} as a function of ϕ . The allowed region is obtained from Equation (14) and the analysis was done for $\delta = 0$ and $\delta = 0.01$ [11]. We obtain the bound:

$$M_{Z_R} > 650 \text{ GeV, 90\% C.L.}, \quad (18)$$

which is consistent with the bounds obtained in the literature [11,25,28,37] for M_{Z_R} .

4. Conclusions

The intrinsic properties of the neutrino are a matter of constant interest. Therefore, we have derived formulas for the total cross-section, the interference cross-section,

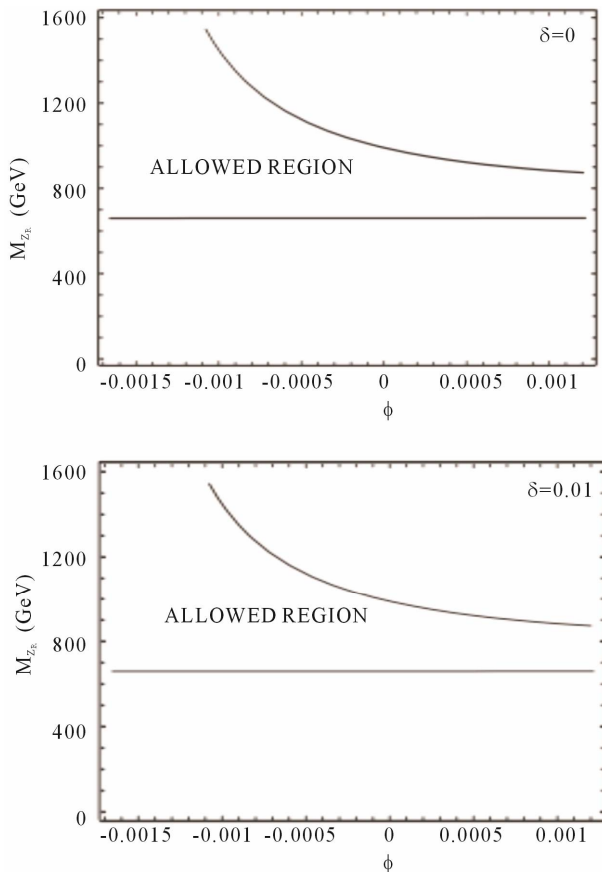


Figure 9. Contour in the $\phi - M_{Z_R}$ plane with $\delta = 0, 0.01$. Values outside of the allowed region are excluded.

the neutrino charge radius $\langle r^2 \rangle_{LR}$, and the electron couplings constants $(g_A^{ve})_{LR}$ and $(g_V^{ve})_{LR}$ via muon-neutrino electron scattering in the framework of the Left-Right symmetric model. We found that the contribution of the mixing angle ϕ , the heavy massive neutral vector boson mass M_{Z_R} of the LR model, and the charge radius $\langle r^2 \rangle$ is evident in the total cross-section and in the interference cross-section which are given in Equations (3) and (7), respectively. The SM prediction is obtained when we take the limits $\phi = 0$ and $M_{Z_R} \rightarrow \infty$, $\gamma \rightarrow 0$, resulting in Equation (8), which agrees with the term of interference reported in the literature in References [30,31,38]. Our bound obtained for the neutrino charge radius in the LR model is competitive with those reported in the literature [5,6,8,11,12,28,29]. In the case of non-standard couplings constants $(g_A^{ve})_{LR}$ and $(g_V^{ve})_{LR}$ (**Figures 5 and 6**), the bounds are dependent on the LR model parameters, and the bound on the mass of the heavy gauge boson (**Figure 9**) is consistent with that obtained in the literature [11,25,28,37].

In summary, we have estimated bounds that can be derived from the muon-neutrino electron scattering. Our bounds on the neutrino charge radius $\langle r^2 \rangle_{LR}$, the electron couplings constants $(g_A^{ve})_{LR}$ and $(g_V^{ve})_{LR}$ and the heavy massive neutral vector boson mass M_{Z_R} are consistent with those reported in the literature and in some cases, improve the existing bounds. However, new experiments dedicated to the detailed study of electron (anti) neutrino interactions with matter, for example the reactor MUNU [39], as well as radioactive sources of neutrinos such as the BOREXINO detector [40], should be able to improve existing limits on the neutrino charge radius, magnetic moment and other parameters. In addition, these results have never been reported in the literature before and could have practical or theoretical interest, such as in the case of the neutrinos produced in core-collapse supernova explosions, that is to say, right-handed Dirac neutrinos emission from supernova core $\nu_L e^- \rightarrow \nu_R e^-$ [41-43].

5. Acknowledgements

We acknowledge support from CONACyT, SNI and PROMEP (México).

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