

Radial Electric Field in Tokamak Plasmas as a Physical Consequence of Ehrenfest's Paradox

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ABSTRACT

A simplified form and some possible theoretical resolutions of the so-called Ehrenfest's Paradox are described. A relation between physical consequences of this relativistic paradox and charge density ρ of tokamak plasma is shown. Plasma experiments which could resolve the Ehrenfest's Paradox are presented.

Keywords: Tokamak; Ehrenfest's Paradox

1. Introduction

Ehrenfest's Paradox was presented in [1] for the first time in 1909. Detailed historical, physical, and geometrical descriptions of Ehrenfest's Paradox can be found in [2] and references therein. For our experimental purposes, let's present Ehrenfest's Paradox in the following simplified form. Consider two thin rings with radii R_1 and R_2 (and $R_1 = R_2$). The second ring is accelerated by an external force so that any point on the ring has linear velocity V . The observers in the laboratory frame measure circumferences of these rings (L_1 and L_2) and radii. The question is: what are the results of these measurements taking into account relativistic effects?

The analysis of relativistic contraction and of related effects in non-inertial rotating frame (including geometrical arguments, purely kinematical and dynamical grounds) is very complex. But physical consequences of these complex analyses are sufficiently simple. We can basically present three possible theoretical hypotheses for the results of circumference measurements.

The first hypothesis, which is not widely accepted, will only be mentioned here. Pursuant to this hypothesis, both the radius of the rotating ring R_2 and its circumference L_2 contract in the laboratory frame by relativistic effects, so that their ratio remains equal

$$2\pi \Rightarrow \frac{L_2}{R_2} = 2\pi = \frac{L_1}{R_1} \quad [3,4].$$

From the contemporary point of view, this resolution of the Ehrenfest's Paradox is highly unlikely see, for example [2].

For most researchers, a more acceptable resolution to Ehrenfest's Paradox in the laboratory frame is as follows:

$$R_1 = R_2 \text{ and } L_1 = 2\pi R_1 = 2\pi R_2 = L_2 \quad (1)$$

We shall refer to it as "hypothesis 1". Though the nuances of "hypothesis 1" are not so important for our discussion, we will consider this hypothesis in more detail. There are two possible ways to obtain this result. Let's introduce the circumference L' and radius R'_2 of a rotating ring in the rotating reference frame (with the linear velocity V at the radius R_2).

The first and less accepted approach is based on the assumption of "no Lorentz contraction for rotating reference frame" [5-8]. This would mean that $L_1 = L_2 = L'_2$ and $R_1 = R_2 = R'_2$.

The second and more widespread approach is based, in a simplified form, on the following ideas. According to

$$[9], \quad L' = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \cdot L_2 = \gamma \cdot L_2, \text{ and } R'_2 = R_2.$$

We shall refer to it as "condition (1)". The authors (see, for example, [10-16]), through the analysis of a metric tensor for a rotating reference frame, have come to the following conclusion. The condition (1) is fulfilled but the rotating observers would see the increase of the circumference in the form of $L' = \gamma \cdot L_1$. The laboratory observers would see a relativistic contraction of a rotating ring in the form of $L_2 = \gamma^{-1} \cdot L' = \gamma^{-1} \gamma \cdot L_1 = L_1$, and $R_1 = R'_2 = R_2$. It is important to emphasize that the majority of the advocates of the standard resolution of the Ehrenfest's Paradox *a priori* believe that the geometry of the rotated ring in the laboratory inertial frame is Euclidean, $L_2 = 2\pi R_2$, and they analyze mainly the geometry of the rotating ring in the rotating frame.

Let's follow the foregoing logic of the more widespread approach, but assume that the circumference measured by the observers in the rotating reference frame

does not change due to the rotation and is equal to the initial length of the non-rotating ring L_1 (see, for example, [17-19]). Then, the laboratory observers would conclude that:

$$L_2 = \gamma^{-1} \cdot L_1 \text{ and } R_1 = R'_2 = R_2 \quad (2)$$

We shall use the Equation (2) refer to it as “the hypothesis 2”. It is necessary to emphasize, that in the case of “hypothesis 2” the geometry of the rotating ring in the laboratory frame is Non-Euclidean.

Unfortunately, it is practically impossible to resolve the Ehrenfest’s Paradox by observing the real rotating disks or rings because of centrifugal forces that lead to significant deformation of the rings. Thus, it is difficult to measure very small relativistic effects against the background of the centrifugal deformation at accessible rotation velocities and size of rings. Note that the first experimental studies of the measurement of physical consequences of the Ehrenfest’s Paradox were presented only in [19], 2011.

2. Physical Consequences of the Ehrenfest’s Paradox in Tokamak Plasma

Recently, in [19,20] the effect of relativistic contraction of an “electron ring” circumference in steady state tokamak plasma rotating in toroidal direction with current velocity $V_e(r)$ has been analyzed. Let r be the minor radius of a tokamak magnetic surface, see **Figure 1**. The minor tokamak radius a was assumed to be much less than the major radius R , where $\frac{a}{R} \ll 1$, and electron

toroidal rotation velocity was assumed to be moderate, so that it would be possible to exclude centrifugal forces in the momentum balance of plasma [21]. The toroidal rotation velocity of “the ion ring” $V_i(r)$, as a general rule, is much less than the toroidal rotation velocity of the “electron ring”, a fact known from experiment [21]. It is assumed that at the moment of plasma creation (with no current) from neutral gas (hydrogen or deuterium), the electron density $n_e^0(r)$ and the ion density $n_i^0(r)$ are

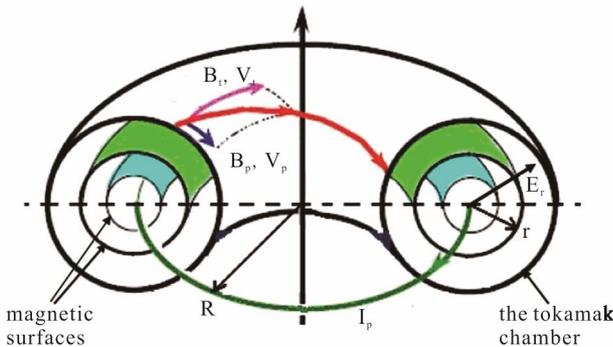


Figure 1. A sketch of tokamak.

equal, and the difference between the total number of electrons and total number of ions does not vary during a discharge and is equal to 0 in the tokamak chamber. We can use the last assumption because neutral gas is injected into the tokamak chamber, and neutral gas is pumped from the tokamak. Electrons and ions can move and can be redistributed in the minor radius direction of tokamak plasma after the occurrence of a current.

One can note that the peak value of the experimentally measured radial electric field $E_r(r)$ in tokamak coincides with the occurrence in plasma a small difference between electron density $n_e(r)$ and ion density $n_i(r)$ in the laboratory frame [19,20], of the order of

$$\frac{|n_i(r) - n_e(r)|^{\max}}{n_e(r)} \approx \left(\frac{V_e(r)}{c} \right)^2$$

and for the ohmic modes: $\frac{|n_i(r) - n_e(r)|}{n_e(r)} \approx \left(\frac{V_i(r)}{c} \right)^2$. We shall refer to it as “condition (2)”.

Let us assume at the beginning, that the total number of electrons and ions does not vary during a discharge, electron density and ion density, ion toroidal rotation velocity and electron rotation velocity values are constant and do not depend on the tokamak minor radius r . In this case, initial electron density (before plasma current) is n_e^0 and ion density is n_i^0 , where $n_e^0 = n_i^0$.

Therefore, this can be considered as two thin rings (the electron ring and the ion ring), originally having the same circumference $L^{\text{electron}} = 2\pi R = L^{\text{ion}}$, which are brought to different toroidal rotation velocities, V_e and V_i , where $V_i \ll V_e$. The situation is similar to the one considered above in the context of Ehrenfest’s Paradox. Following the ideas summarized by E. L. Feinberg in [22], it is not necessary to investigate in detail the process of electron ring and ion ring acceleration to given velocities. One can compare only an initial and a final states. In this case, measurement of the part of the electric field which can arise, for example, within the frame of “the hypothesis 2”, is much easier than the investigation of the deformations of a rotating rigid ring. The reason is as follows: on one hand, the current electron velocity can reach hundreds km/s, on the other hand, possible deformations of the “electron ring” due to centrifugal force will lead only to the occurrence of dipole components in the electric field associated with minor change of radius of the rotating ring. Relativistic contraction of the ring circumference without change of radius and conservation of total electron number (under the “the hypothesis 2”) can lead to the occurrence of a monopole component in electric field which is relatively easy to measure, as it will be shown below.

Following [19], let us consider the change in density of charges ρ in tokamak plasma under “hypothesis 1”

and “hypothesis 2”.

“Hypothesis 1”

It is obvious from Equation (1) that tokamak charge density ρ is not changed after the appearance of the toroidal rotating electron (current) ring and ion ring in plasma, that is:

$$\rho = 0 \tag{3}$$

“Hypothesis 2”

Following [19], it is possible to show that in this case rotation creates a finite charge density ρ in a tokamak plasma.. If we ignore higher-order terms in $\frac{V^2}{c^2}$ expansion, we can write:

$$\begin{aligned} \rho &= \frac{|e|n'_i}{\sqrt{1-\frac{V_i^2}{c^2}}} - \frac{|e|n'_e}{\sqrt{1-\frac{V_e^2}{c^2}}} \\ &\cong |e|n'_i \cdot \left(1 + \frac{1}{2} \frac{V_i^2}{c^2}\right) - |e|n'_e \cdot \left(1 + \frac{1}{2} \frac{V_e^2}{c^2}\right) \\ &\cong -\frac{(|e| \cdot n_e \cdot (V_i - V_e))^2}{2 \cdot c^2 \cdot |e|n_e} + \frac{(|e| \cdot n_e \cdot (V_i - V_e)) \cdot V_i}{c^2} \\ &\quad + |e|(n'_i - n'_e) \\ &= -\frac{j^2}{2 \cdot c^2 \cdot |e|n_e} + \frac{j \cdot V_i}{c^2} \end{aligned} \tag{4}$$

where n'_e and n'_i are: the electron density in the rotating frame with velocity V_e and the ion density in the rotating frame with the velocity V_i , respectively. We have taken into account the framework of “hypothesis 2”:

$$L_{rot}^{electron} = L^{electron} \cdot \sqrt{1-\frac{V_e^2}{c^2}} \neq L_{rot}^{ion} = L^{ion} \cdot \sqrt{1-\frac{V_i^2}{c^2}} \quad \text{and}$$

$$n'_e = n_i^0 = n_e^0 = n'_e.$$

Change of the charge density in this case is only associated with relativistic change of the denominator in the expression for the density. Let us note that ρ depends on parameters measured in the laboratory frame: the current density j , the electron density n_e and the ion toroidal rotation velocity V_i .

So, we have calculated the charge density ρ in each point inside a stationary tokamak chamber in the laboratory frame. One can “forget” about the particular nature of the charge density ρ relating to the Non-Euclidean geometry of a rotating electron (ion) ring of the tokamak plasma in the laboratory frame, and can instead use the Poisson equation (being the relativity theory equation in the laboratory frame) with ρ taken from Equation (4) to calculate the electrostatic radial electric field in tokamak plasma. Hence, $E_r(r)$ in a simple tokamak plasma is created by two relativistic terms in the density of charges ρ , Equation (4), which appeared in the labora-

tory frame.

In case of a real tokamak, plasma parameters depend on the minor radius of magnetic surfaces. Consideration of such dependence for the purpose of calculation of tokamak plasma charge density is shown in [19,20] in detail for $\frac{a}{R} \ll 1$. The principal point here is the consideration of each nested magnetic surface with plasma distributed in a thin, hollow ring.

Following [19], we can rewrite Equation (3) in the case of “hypothesis 1” so as to take into account the processes of redistribution of electrons and ions on the minor radius by plasma diffusion (convection):

$$\rho(r) \cong |e|(n_i^d(r) - n_e^d(r)) \tag{5}$$

Due to electron and ion diffusion (change of the numerator in the expression for the density), additional volume charge densities in plasma can arise, and this can be expressed by the term $|e|(n_i^d(r) - n_e^d(r))$ in (5). Hence, only the diffusion (convection) of ions and electrons could create $E_r(r)$ in tokamak plasma.

In the case of “hypothesis 2”, we can rewrite the Equation (4) in the following form:

$$\begin{aligned} \rho(r) \cong &-\frac{j^2(r)}{2 \cdot c^2 \cdot |e|n_e(r)} + \frac{j(r) \cdot V_i(r)}{c^2} \\ &+ |e|(n_i^d(r) - n_e^d(r)) \end{aligned} \tag{6}$$

where $j = |e| \cdot n_e(r) \cdot (V_i(r) - V_e(r))$.

Equation (6) has two relativistic terms, and “condition (2)” is not merely casual coincidence in this case. Let us emphasize again that $\rho(r)$ depends on the plasma parameters measured in the laboratory frame: the current density $j(r)$, the electron density $n_e(r)$, the ion toroidal rotation velocity $V_i(r)$ and the diffusion (convection) term. So, we have calculated the charge density ρ in each point inside a stationary tokamak chamber in the laboratory frame, Equation (6). As emphasized above, one can “forget” about the particular nature of the charge density ρ relating partially to the Non-Euclidean geometry of a rotating electron (ion) ring of the tokamak plasma in the laboratory frame, and can use the Poisson equation with ρ taken from Equation (6) to calculate the electrostatic radial electric field in the tokamak plasma. In our consideration, the diffusion (convection) term is not determined. We can mention one integral property of the diffusion (convection) term, which is a consequence of the physical assumption that the difference between the total number of electrons and total number of ions does not vary during a discharge and is equal to 0 in a tokamak chamber. It is:

$$\int_{V_{ch}} |e|(n_i^d(r) - n_e^d(r)) dV = 0 \tag{7}$$

where V_{ch} is the volume of the toroidal tokamak chamber. The diffusion (convection) term can be determined within the frame of different approaches, see [20].

3. T-11M Tokamak Experiment

Having accepted “hypothesis 2”, we have seen that plasma current creates relativistic volume charge density

equal to $-\frac{j^2}{2 \cdot c^2 \cdot |e| n_e}$. The second relativistic term on

the right-hand side of Equation (6) for plasma usually is

more than five times smaller than $-\frac{j^2}{2 \cdot c^2 \cdot |e| n_e}$. The

third term is the symmetrical redistribution of charges by the diffusion (convection) along the minor radius in a

plasma chamber. Thus $-\frac{j^2}{2 \cdot c^2 \cdot |e| n_e}$ can be crucial in

the creation of $E_r(r)$, especially at the beginning of a discharge, and if plasma has modulated current. For tokamak plasma contained in a metallic chamber, $E_r(r)$ can modify the electric potential of the chamber

$$\begin{aligned} \Delta\varphi &\cong \frac{\int_{V_{ch}} \rho(r) dV}{C} \\ &\approx \frac{\int_{V_{ch}} \left(-\frac{j^2(r)}{2 \cdot c^2 \cdot |e| n_e(r)} + |e| (n_i^d(r) - n_e^d(r)) \right) dV}{C} \\ &= \frac{\int_{V_{ch}} \left(-\frac{j^2(r)}{2 \cdot c^2 \cdot |e| n_e(r)} \right) dV}{C} \end{aligned}$$

with respect to the ground; see the Equation (7). The electric potential of the chamber is proportional in this case to the volume of plasma, the averaged value of

$-\frac{j^2}{2 \cdot c^2 \cdot |e| n_e}$, the electric capacitance of a closed metal-

lic tokamak chamber C , and relates to chamber RC time.

If one wants to measure the potential of the tokamak chamber $\Delta\varphi(t)$ during the discharge, one can expect two options. In the case of “hypothesis 1”, there would be no change of $\Delta\varphi(t)$ due to the plasma current, *i.e.* $\Delta\varphi(t) = 0$, see the Equation (7); in the case of “hypothesis 2”, the potential of the chamber will change proportional to

$$\frac{\int_{V_{ch}} \left(-\frac{j^2(r)}{2 \cdot c^2 \cdot |e| n_e(r)} \right) dV}{C}.$$

Thus, measurement of tokamak chamber potential $\Delta\varphi(t)$ during discharges could resolve Ehrenfest’s Paradox in principle.

The first series of special experiments for electric potential measurements at several points in a tokamak chamber were carried out at T-11M tokamak (main plasma parameters in presented shots were: deuterium plasma, the average steady-state electron density $\langle n_e \rangle \sim 10^{13} \text{ cm}^{-3}$, the plasma current $I_p \sim 50 \text{ kA}$, $r = 20 \text{ cm}$, $R = 70 \text{ cm}$) with modulated current [19,23]. The example of the typical measurement is shown on **Figure 2**. For the purpose of calculation of the theoretical dependence (triangles and dashed curve in **Figure 2**), we have used: 1) experimental data for plasma current and electron density; 2) experimental chamber resistivity $R = \sim 4 \text{ MOhm}$; 3) experimental chamber RC time $\sim 2.5 \text{ ms}$. Electron density diagnostics did not give us adequate information for several milliseconds at the beginning of discharge. We have extrapolated the electron density growth during the first $\sim 8 \text{ ms}$ by a linear function.

One can see satisfactory coincidence of theoretical calculation results based on “hypothesis 2” with the experimental results.

4. Physical Consequences of Equation (6) and Some Tokamak Experiments

Radial electric field $E_r(r)$ plays an important role in various modes of improved plasma confinement in a tokamak [24]. Some of those modes will be used in the thermonuclear reactor ITER [25], which is currently under construction.

Because “condition 2” exists in tokamak plasma, we usually cannot use the Poisson equation for the calculation of $E_r(r)$ —it is not possible to measure or even to calculate independently $n_e(r)$ and $n_i(r)$ in the laboratory frame with required accuracy.

Unfortunately, another approach—the ambipolarity equation for radial flows—cannot be used as well, since the ambipolarity emerges automatically from the toroidal symmetry of the considered configuration [26,27]. For those reasons, more complex approaches to the estimation of $E_r(r)$ are used. There are many successful methods of calculation of $E_r(r)$, see, for example, [21, 28,29] and references in [21,28]. Historically, the first calculation of $E_r(r)$ was presented in [30]. The value of $E_r(r)$ was calculated by taking into account the higher orders in the expansion in small parameter of the plasma kinetic theory. One use common approach for $E_r(r)$ that takes into account the influence of a small fraction of the locally trapped ions in the ripples of the toroidal magnetic field on the formation of $E_r(r)$ [31, 32]. It is necessary to emphasize that ion radial diffusion is often considered the most important determinant of $E_r(r)$, see, for example [21]. Effects of viscosity between main ions and neutrals in tokamak plasma [33] can sometimes self-consistently estimate $E_r(r)$ on the pe-

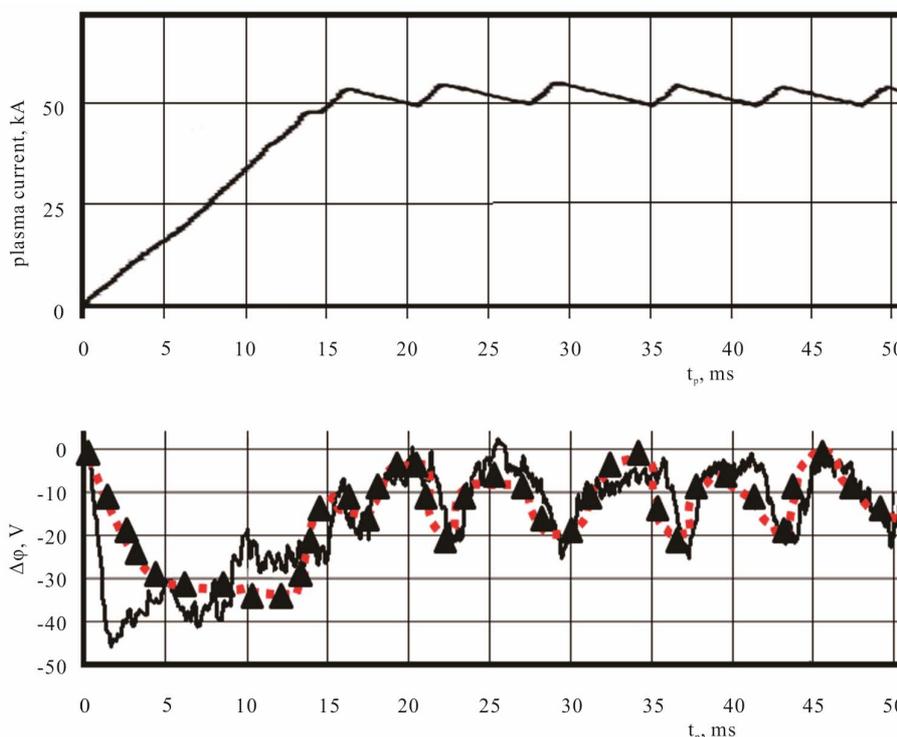


Figure 2. Time dependence of plasma current $I_p(t)$ and the tokamak chamber electric potential $\Delta\phi(t)$ during the discharge, T-11M tokamak [23]. Solid curve is the experimental dependence of $\Delta\phi(t)$; triangles and dashed curve are the theoretical dependence.

riphery of the plasma. The influence of large gradients of the radial electric field on the shape of ion trajectories [34] sometimes allows qualitative explanation of some of the issues of the formation of $E_r(r)$ in modes with internal thermal barriers. Some authors introduce anomalous viscosities in the standard neo-classical theory [35], etc.

It is necessary to note that it is very difficult to measure accurately the radial electric field profile in a tokamak.

This complexity gives rise to situations where experimental results concerning $E_r(r)$ are difficult to explain using simple approaches, see for example [36-40]. Therefore, it is often necessary to use complex contemporary theories. It will be shown below that same experimental results can be explained by enough simple approach based on Equation (6), too.

Let us assume that the charge density, Equation (6), creates $E_r(r)$ in tokamak plasma. To compare the results of this approach to $E_r(r)$ with the results of some actual experiments, we take into account the additional plasma equation, see, for example, [21] (which is derived from radial equilibrium of forces on a magnetic surface):

$$E_r(r) \cong \frac{V_t(r) \cdot B_p(r)}{c} + \frac{1}{|e| \cdot n_i(r)} \cdot \frac{dP_i(r)}{dr} - \frac{V_p(r) \cdot B_t}{c} \quad (8)$$

where $V_p(r)$ and $V_t(r)$ ($V_t(r) \equiv V_i(r)$ in the article are the velocities of the poloidal and toroidal rotation of plasma ions, respectively, and hence, of the plasma as a whole (the velocities are low enough so the centrifugal effect may be omitted); c is the speed of light, e is the electron charge; $n_i(r)$ and $P_i(r)$ are the density and pressure of plasma ions; $B_p(r)$ and $B_t(r)$ are poloidal and toroidal magnetic fields. To establish the main features of relations between $E_r(r)$ and $V_t(r)$, as well as other plasma parameters, we may ignore the weak poloidal dependence of parameters in Equation (8)

$(1 \pm \frac{r}{R} \cdot \cos \vartheta$, where R is the major radius of the to-

kamak, ϑ is the poloidal angle; $\frac{r}{R} \ll 1$). A relation of

type (8) is always true when the plasma is in the steady state (only these states are considered below). For the sake of simplicity, let's assume that magnetic surfaces are nested cylinders with small radii r , and the plasma consists of electrons and, for example, deuterium ions. Let's also assume that the velocity of poloidal rotation may be taken from experiments or derived from neo-classical theory.

We can express $E_r(r)$ and $V_t(r)$ independently using Equation (6), the Poisson equation and Equation (8):

$$E_r(r) \cong -\frac{B_p(r)}{c} \cdot \int_r^a \left[V_p(\xi) \cdot \frac{B_t}{B_p(\xi)} - \frac{c}{|e| \cdot n_e(\xi) \cdot B_p(\xi)} \cdot \frac{dP_i(\xi)}{d\xi} - \frac{j(\xi)}{2 \cdot |e| n_e(\xi)} + |e|(n_i^d(\xi) - n_e^d(\xi)) \cdot \frac{c^2}{j(\xi)} \right] \times \left(\frac{1}{\xi} + \frac{1}{B_p(\xi)} \frac{dB_p(\xi)}{d\xi} \right) d\xi + \left[\frac{V_p(r) \cdot B_t}{c} - \frac{1}{|e| \cdot n_e(r)} \cdot \frac{dP_i(r)}{dr} \right]_{r=a} \tag{9}$$

$$V_i(r) \cong V_p(r) \frac{B_t}{B_p(r)} - \frac{c}{|e| \cdot n_e(r) \cdot B_p(r)} \frac{dP_i(r)}{dr} - \left[V_p(r) \frac{B_t}{B_p(r)} - \frac{c}{|e| \cdot n_e(r) \cdot B_p(r)} \cdot \frac{dP_i(r)}{dr} \right]_{r=a} - \int_r^a \left[V_p(\xi) \cdot \frac{B_t}{B_p(\xi)} - \frac{c}{|e| \cdot n_e(\xi) \cdot B_p(\xi)} \cdot \frac{dP_i(\xi)}{d\xi} - \frac{j(\xi)}{2 \cdot |e| n_e(\xi)} + |e|(n_i^d(\xi) - n_e^d(\xi)) \cdot \frac{c^2}{j(\xi)} \right] \times \left(\frac{1}{\xi} + \frac{1}{B_p(\xi)} \frac{dB_p(\xi)}{d\xi} \right) d\xi \tag{10}$$

We take into account the assumption that the toroidal rotation velocity at the plasma boundary with $r = a$ is close to zero [41]. One can find an example of the derivation procedure and interpretation of equations similar to Equations (9) and (10) in [41].

Below we present important quantitative and qualitative correlations of theoretical results (using Equations (6), (9) and (10)) with tokamak experimental data [20,36-42].

The first important conclusion from Equation (10): if $-\frac{j^2(r)}{2 \cdot c^2 \cdot |e| n_e(r)} + |e|(n_i^d(\xi) - n_e^d(\xi))$ is not close to zero

in the plasma core then a toroidal ion beam is created in that region (its velocity logarithmically tending to infinity [20,23]). So, we have to suppose that

$-\frac{j^2(r)}{2 \cdot c^2 \cdot |e| n_e(r)} + |e|(n_i^d(\xi) - n_e^d(\xi)) \cong 0$ at the core of the plasma.

Consequently, one can write the Poisson equation for this case:

$$\frac{1}{r^2} \frac{d}{dr} (r^2 E_r(r)) \cong \frac{j(r) \cdot V_i(r)}{c^2}$$

and calculate the radial electric field profile:

$$E_r(r) \cong \frac{B_p(r) \cdot V_i(r)}{c} - \frac{1}{r \cdot c} \int_0^r B_p(\xi) \cdot \frac{dV_i(\xi)}{d\xi} \cdot \xi \cdot d\xi \tag{11}$$

where $B_p(r) \cong \frac{4\pi}{r \cdot c} \int_0^r j(r) \cdot \xi \cdot d\xi$.

It is possible to derive Equation (11) from Equation (9) directly, see [19].

As very good know from experiments, see, for example, [36], the radial electric field for tokamak Ohmic modes is equal zero in the plasma center; is close to zero in the plasma periphery; and has the minimum (negative

value in several kV/m) in the middle region of the plasma.

$E_r(r)$ as calculated using Equation (11) is presented in **Figure 3** for typical ohmic discharge in TCV tokamak [36]. One can see satisfactory agreement of theoretical calculation based on Equation (11) with experimental results.

Investigation of so-called “locked” mode is another quantitative example of this approach. In this mode, plasma stops rotating in the toroidal direction on all magnetic surfaces, and experimental $E_r(r)$ becomes close to zero

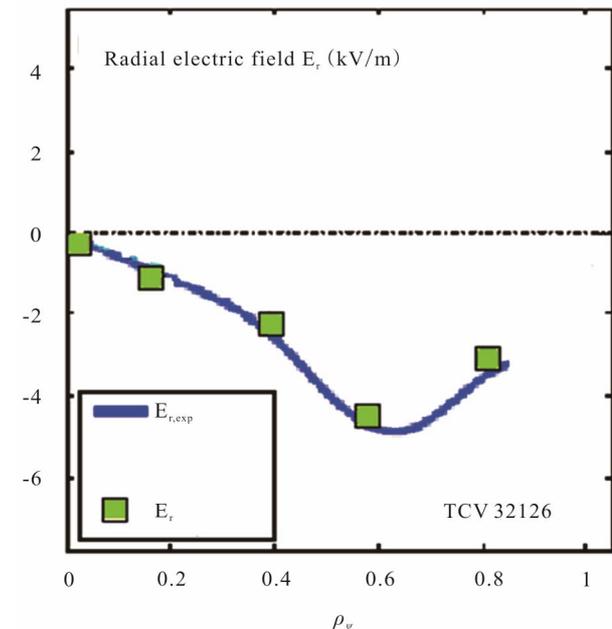


Figure 3. Example of radial electric field $E_r(r)$ profile calculation in a typical ohmic discharge in TCV tokamak [36]. ρ_w is the effective radius of the tokamak magnetic surface. Solid curve is the experimental $E_r(r)$ profile; squares are the $E_r(r)$ values calculated with Equation (11) using the experimental toroidal rotation plasma velocity profile.

everywhere [37]. For example, obtaining the result $E_r(r) \cong 0$ with the condition that $V_i(r) \cong 0$ is difficult to explain with approaches based only on Equation (8), but it is the trivial consequence of Equation (11).

Polarity and typical values of toroidal rotation velocity in plasma cores in most ohmic modes and some modes with ICRH (if ion toroidal rotation velocity is opposite to the plasma current and is equal to $10 \div 100$ km/s [21,39]) are correctly described by Equation (10) with

$$-\frac{j^2(r)}{2 \cdot c^2 \cdot |e|n_e(r)} + |e|(n_i^d(\xi) - n_e^d(\xi)) \cong 0$$

for real ion pressure profiles [20,23].

$$\text{If the sum of } -\frac{j^2(r)}{2 \cdot c^2 \cdot |e|n_e(r)} + |e|(n_i^d(\xi) - n_e^d(\xi))$$

in Equation (10) becomes negative a plasma periphery (this often indicates the suppression of electron losses at plasma periphery) then the plasma core begins rotating in the current direction (which correlates with experimental data [20,38]).

An important consequence of Equations (9) and (10) is the fact that plasma confinement is better in the mode with co-current plasma rotation and positive $E_r(r)$ than in the mode with the counter-current plasma rotation and negative $E_r(r)$ [20] (if other plasma parameters are similar and two modes differ from each other only in the direction of the rotation and $E_r(r)$ polarity). This is confirmed by experimental data [40].

The integral relation between plasma parameters, Equations (9) and (10), leads us to the following fact: a local variation of plasma parameters in a given region, for example on a region of peripheral magnetic surface, leads to the “instantaneous” total change of toroidal rotation plasma velocity and $E_r(r)$ on the whole magnetic surfaces inside the perturbed magnetic surface. Such non-diffusive penetration of perturbations was observed in experiments; see, for example, [42].

5. Conclusions

1) The presented relativistic theory of radial electric field formation, based on Equation (6), can sometimes explain quantitatively and more often qualitatively, many experimental tokamak results for $E_r(r)$ and $V_i(r)$. Specific examples are presented in the article.

2) Tokamak plasma can be a useful tool for the research of possible physical consequences of Ehrenfest's Paradox. Measurement of the tokamak chamber potential $\Delta\phi(t)$ with respect to ground during discharges could resolve that Paradox in principle. The plasma discharge created from initially neutral gas inside a metallic tokamak chamber can affect $\Delta\phi(t)$ in two ways:

a) The most expected effect is $\Delta\phi(t) = 0$. In this case

Ehrenfest's Paradox should be resolved within the framework of “hypothesis 1”.

b) Another possibility is

$$\Delta\phi(t) \approx \frac{\int_{V_{ch}} \left(-\frac{j^2(r)}{2 \cdot c^2 \cdot |e|n_e(r)} \right) dV}{C}.$$

In this case Ehrenfest's Paradox should be resolved within the framework of “hypothesis 2”.

Available experimental measurements of $\Delta\phi(t)$, which were described above, and comparison with theoretical results, based on Equation (6), see the first item of Conclusion, show, that the Ehrenfest's Paradox potentially should be resolved within the frame of “hypothesis 2”, as result of Non-Euclidean geometry of the rotating electron (ion) rings in the laboratory should be.

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