

CP Violation in Kaon Decay in the Scalar Strong Interaction Hadron Theory

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ABSTRACT

CP conservation and violation in neutral kaon decay are considered from a first principles' theory, recently published as "Scalar Strong Interaction Hadron Theory". The arbitrary phase angle relating K^0 and \bar{K}^0 in current phenomenology is identified to be related to the product of the relative energy to the relative time between the s and d quarks in these kaons. The argument of the CP violating parameter ε is predicted to be 45° without employing measured data. The K_S^0 decay rate is twice the $K_L^0 - K_S^0$ mass difference, in near agreement with data, and both are proportional to the square of the relative energy 29.44 eV. Any pion from K_L^0 decay will also have a mass shift of $\approx 1.28 \times 10^{-5}$ eV. The present first principles' theory is consistent with CP conservation. To achieve CP violation, the relative time cannot extend to both $+\infty$ and $-\infty$ but is bounded in at least one direction. The values of these bounds lie outside the present theory and it is unknown how they can be brought forth. $\bar{B}^0 - B^0$ mixing is also considered and the relative energy is 663.66 eV.

Keywords: CP Violation; Kaon Decay; Relative Energy; Relative Time; Scalar Strong Interaction

1. Introduction

CP violation in neutral kaon decay has been treated phenomenologically [1-3] but its origin remains a mystery ever since its discovery nearly 50 years ago. This problem is now treated employing the recently developed first principles' theory [4] in which this mysterious origin is shown to be connected to the relative energy and relative time between the s and d quarks in the kaons.

Section 2 reproduces some phenomenological results. In Section 3, the arbitrary phase angle relating \bar{K}^0 to K^0 in Section 2 is identified to be connected to the relative energy and time among the quarks in the kaon. When applied to $K_S^0 \rightarrow 2\pi$, the phase of the CP violating parameter ε in Section 2 is predicted by means of the degeneracies of SU(3) gauge fields to known SU(2) ones. The decay rate depends upon the relative energy. In Section 4, the CP violating $K_L^0 \rightarrow 2\pi$ is found to be forbidden unless the relative time is bounded between certain finite values. The mass shift of K_L^0 depends upon the relative energy 29.44 eV and is half the $K_S^0 \rightarrow 2\pi$ decay rate. Any pion from K_L^0 decay will also have a mass shift of $\approx 1.28 \times 10^{-5}$ eV. In Section 5, the semileptonic decays of kaons are treated and CP violation also requires that the relative time be bounded in one direction. Section 6 summarizes the roles of relative energy

and time and considers the possible origins of CP violation. $B^0 - \bar{B}^0$ mixing is similarly treated in Section 7.

2. Phenomenology [2,3]

The decays $K_S^0, K_L^0 \rightarrow 2\pi, 3\pi$ have been considered using the time-dependent Schrödinger equation without specifying the Hamiltonian in connection with CP non-conservation [2]. The starting point is the ansatz [2 (15.28)]

$$|\bar{K}^0\rangle = \exp(iS\Theta)CPT|K^0\rangle \quad (2.1)$$

where S denotes strangeness, CPT the conventional discrete operator and Θ an arbitrary phase angle. Under CPT invariance, [2] gives

$$\begin{aligned} |K_S^0\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1+\varepsilon \\ 1-\varepsilon \end{pmatrix}, \\ |K_L^0\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1+\varepsilon \\ -(1-\varepsilon) \end{pmatrix} \end{aligned} \quad (2.2)$$

where the upper row refers to K^0 and the low row to \bar{K}^0 and ε is a small, complex quantity [2]. Using the measured [1]

$$m_{KL} - m_{KS} = 3.483 \times 10^{-6} \text{ eV}$$

$$\Gamma(K_S^0 \rightarrow 2\pi) = 7.352 \times 10^{-6} \text{ eV} \approx 2(m_{KL} - m_{KS}) \quad (2.3)$$

the argument of $\varepsilon \approx 45^\circ$ or 225° has been deduced. Also, unitarity provides an upper bound for $|\varepsilon|$. Thus,

$$\varepsilon \approx \pm |\varepsilon| (1+i) / \sqrt{2},$$

$$|\varepsilon| < 4.3 \times 10^{-3} \quad (2.4)$$

Further, (2.1, 2) with (7.2.18) leads to

$$2\varepsilon \approx -i\Theta \quad (2.5)$$

so that the argument of $\Theta \approx -45^\circ$ or -225° .

[2 (15.105, 109, 113)] define the ratio of the semileptonic decay amplitudes

$$x = \frac{S_{fi}(\bar{K}^0 \rightarrow \pi^- L_s^+ \nu_L)}{S_{fi}(K^0 \rightarrow \pi^- L_s^+ \nu_L)} \quad (2.6a)$$

where L stands for the lepton species, as in (7.1.11-13), and s the lepton helicity. [2 (15.116, 117)] give the ratios of the decay rates to first order in ε ,

$$\frac{\Gamma(K_S^0 \rightarrow \pi^- L_s^+ \nu_L)}{\Gamma(K_S^0 \rightarrow \pi^+ L_s^- \bar{\nu}_L)} = \left| \frac{1 + \varepsilon + (1 - \varepsilon)x}{(1 + \varepsilon)x^* + (1 - \varepsilon)} \right|^2$$

$$\cong 1 + 4\text{Re}(\varepsilon) \quad (2.6b)$$

$$\cong \frac{\Gamma(K_L^0 \rightarrow \pi^- L_s^+ \nu_L)}{\Gamma(K_L^0 \rightarrow \pi^+ L_s^- \bar{\nu}_L)}$$

where x turns out to be small and has been dropped.

From the measured semileptonic decay ratios [1], (2.6b, 4) yields

$$\text{Re}(\varepsilon) = 1.612 \times 10^{-3}, \quad |\varepsilon| \approx 2.28 \times 10^{-3} \quad (2.7)$$

Furthermore, [1] gives the nonleptonic CP violating amplitude ratios

$$|\eta_{00}| = \sqrt{\frac{\Gamma(K_L^0 \rightarrow \pi^0 \pi^0)}{\Gamma(K_S^0 \rightarrow \pi^0 \pi^0)}} = 2.221 \times 10^{-3}$$

$$|\eta_{+-}| = \sqrt{\frac{\Gamma(K_L^0 \rightarrow \pi^+ \pi^-)}{\Gamma(K_S^0 \rightarrow \pi^+ \pi^-)}} = 2.232 \times 10^{-3} \quad (2.8)$$

$$\left(\delta^{ab} \partial_0 + i \frac{1}{4} g \sqrt{2} W_U^{ab}(X) \right) \left(\frac{1}{2} (-\delta^{be} \partial_{x_0} - \underline{\sigma}^{be} \partial_{\underline{x}}) - \underline{\sigma}^{be} \partial \right) \frac{\chi_0(r)}{\sqrt{2}} \exp(-iE_0 X^0 + i\omega_0 x^0)$$

$$\chi_0(r) = \chi_0(\underline{x}), \quad |\underline{x}| = r \quad (3.1)$$

With the degeneracy (A15), (3.1) can be written as

$$\left(\delta^{ab} i\omega_0 + i \frac{1}{4} g \frac{1+i}{\sqrt{2}} \left(-\frac{Z^{ab}(X)}{\cos \vartheta_w} \right) \right) \left(\frac{1}{2} \delta^{be} iE_0 - \underline{\sigma}^{be} \partial \right) \frac{\chi_0(r)}{\sqrt{2}} \exp(-iE_0 X^0 + i\omega_0 x^0) \quad (3.2)$$

The numbers in (2.7) and (2.8) show that

$$|\varepsilon| \approx |\eta_{00}| \approx |\eta_{+-}| \quad (2.9)$$

This numerical agreement suggests that CP nonconservation for the semileptonic and nonleptonic decays can be characterized by one parameter ε . However, the above results say nothing about the nature of this CP violation.

3. Relative Energy and Time and

$$K_S^0 \rightarrow 2\pi, 3\pi$$

The references in form of (x, y, z) or $\S x, y, z$ below refer to those in [4], mostly Chapter 7. This chapter has earlier appeared in a more general form as the first article in another volume [5].

In the ansatz (2.1), the nature of the phase angle Θ is not specified. Here, it is naturally identified to be related to the product of the relative energy ω_0 and the relative time x^0 in (A7), as will be specified in (6.1) ff below. In the absence of gauge field, (7.1.9) reverts to (A4) which led to (A8). With the association of Θ with $\omega_0 x^0$ above, however, $\omega_0 \neq 0$ and (A9) has to be *relaxed* for the present application.

In the absence of weak interaction, $g \rightarrow 0$ in (7.1.5) and K^0 and \bar{K}^0 are complex conjugate of each other in (7.2.19) and are stable, physical states with the same mass given by (A10). The phase angle Θ in (2.1), hence also the relative energy ω_0 , drops out. Turning on the weak interaction igW_U or $\bar{W}_U gi$, K^0 and \bar{K}^0 are no longer physical states and Θ and $\omega_0 \neq 0$ become small quantities of the same first order as igW_U . The physical states are now K_S^0 and K_L^0 shown in (2.2) or (7.2.18) incorporating the phase factor $\exp(i\omega_0 x^0)$ according to (A7). Therefore, ω_0 in (A8) cannot be dropped but is of the same order as the igW_U term.

Consider one of the two first order terms in (7.1.9a) for $\bar{K}^0 \rightarrow \pi^0 W_U$ of (7.3.27b). Let the first operator in (7.1.9a) be of first order g included in (7.1.4-5) and the second operator be of zeroth order given by the last of (3.1.4). By (A5), $\chi_{(qr)} = \chi_{(32)}$ in (7.1.9a) refers to \bar{K}^0 which by (7.2.18) becomes $K_S^0 / \sqrt{2}$. Using the wave function (A7), this first order term takes the form

Since ω_0 is generated by the weak interaction g term, it must have the same phase in (3.2);

$$\omega_0 = (1+i)\omega_{00}, \quad \omega_{00} = \text{real constant} \quad (3.3)$$

Turning to the first order term in the second operator in (7.1.9a) and repeat the above procedure. It is found that

$$\omega_0 = -(1+i)\omega_{00} \quad (3.4)$$

These two relations are related to the two phases in (2.4) but without using the empirical data (2.3); they are the consequences of the necessary degeneracy (A15). There is no conflict between (3.3) and (3.4) because they cancel out upon summing the two first order terms in (7.1.9a), as is reflected in (A8) which contains no linear ω_0 term.

Follow now Section 7.3.2-4 and obtain the first order decay amplitude $S_{fi}(K_S^0 \rightarrow 2\pi)$. In the source term (7.3.5a), the first term in the braces is replaced by (3.2) multiplied by a final state wave function $\chi_{0(22)}^+$:

$$\begin{aligned} & \frac{ig_l}{4} \chi_{0(rp)ea}^+(\lambda_l)_{ps} W_l^{ab} \left(\partial_{ll}^{fe} \chi_{0(sr)bf} \right) \\ & \rightarrow \chi_{0(22)ea}^+ \times \left(\delta^{ab} i\omega_0 + i \frac{1}{4} g \frac{1+i}{\sqrt{2}} \left(-\frac{Z^{ab}(X)}{\cos \mathcal{G}_w} \right) \right) \quad (3.5) \\ & \cdot \left(\frac{1}{2} \delta^{be} iE_0 - \underline{\sigma}^{be} \underline{\partial} \right) \frac{\chi_0(r)}{\sqrt{2}} \exp(-iE_0 X^0 + i\omega_0 x^0) \end{aligned}$$

The final state 2π comes from the decay of the Z boson via (7.7.2, 7) and the final state $\chi_{0(22)}^+$ represents a vacuum meson state π^0 given by (7.3.20) with $\psi \rightarrow \chi$ which does not contain any phase factor. Consider the integral over the relative time x^0 in the source (7.3.5a) with the replacement (3.5),

$$\begin{aligned} \int_{T_1}^{T_2} dx^0 \exp(i\omega_0 x^0) &= \int_{T_1}^{T_2} dx^0 \exp((i-1)\omega_{00} x^0) \quad (3.6) \\ T_2 = -T_1 = T_0 &\rightarrow \infty \end{aligned}$$

where (3.3) has been consulted. In the surface term (7.3.4), the exponential $\exp(i\omega_0 x^0)$ in (3.5) also enters but, in addition, also its complex conjugate $\exp(-i\omega_0^* x^0)$ contained in $\chi_{0(23)}^+$ in (7.3.4). The integral over the relative time x^0 in the surface integral (7.3.4) reads

$$\begin{aligned} & \int_{T_1}^{T_2} dx^0 \exp(i\omega_0 x^0 - i\omega_0^* x^0) \\ &= \int_{T_1}^{T_2} dx^0 \exp(-2\omega_{00} x^0) \quad (3.7) \end{aligned}$$

Equations (7.3.4) and (7.3.5) are sandwiched between the initial state $|i = K_S^0\rangle$ and final state $\langle f = 0, Z = 2\pi|$ as was mentioned beneath (7.3.13). The 0 that follows $\langle f =$ refers to the above-mentioned vacuum meson state π^0 . By (7.3.3 - 5, 13), $S_{fi}(K_S^0 \rightarrow 2\pi)$ is proportional to the ratio of (3.6) to (3.7) which vanishes for $T_0 \rightarrow \infty$ in (3.6);

$$\begin{aligned} & \frac{\int_{T_1}^{T_2} dx^0 \exp(i\omega_{00} x^0 - \omega_{00} x^0)}{\int_{T_1}^{T_2} dx^0 \exp(-2\omega_{00} x^0)} \quad (3.8) \\ &= -(1+i) \frac{\exp((i-1)\omega_{00} T_2) - \exp((i-1)\omega_{00} T_1)}{\exp(-2\omega_{00} T_2) - \exp(-2\omega_{00} T_1)} \rightarrow 0 \end{aligned}$$

This also holds if (3.3) is replaced by (3.4). Therefore, $K_S^0 \rightarrow 2\pi$ is forbidden to first order in igW_U . Even if this first order $S_{fi}(K_S^0 \rightarrow 2\pi)$ does not vanish, for instance by letting T_1, T_2 be finite such that (3.8) becomes of the magnitude of unity, it is expected to lead to a decay rate $\Gamma(K_S^0 \rightarrow 2\pi)$ of the same magnitude as $\Gamma(K^\pm \rightarrow 2p) \cong 1.1 \times 10^{-8} \text{ eV}$ from phase space considerations. This rate is 668 times smaller than the observed rate in (2.3). This is ‘‘one of the remaining unsolved problems of the weak interaction’’ [2].

However, there is now a second order term $-(\omega_0)^2 = -i2(\omega_{00})^2$ in (A8). Let E_{00} be the K^0 mass in (A10) and $E_0 = E_{00} + \Delta E_{0S}$ in (A8), the exponent in (3.5) will contain $-i\Delta E_{0S} X^0$ corresponding to the decay rate

$$\Gamma(K_S^0 \rightarrow 2\pi) = i\Delta E_{0S} = -4\omega_{00}^2/E_{00} \quad (3.9)$$

where the lower sign in (A10) is chosen. Note that the relative energy ω_{00} is a hidden variable but its square in (3.9) is visible. Although (3.9) comes from a second order term, its amplitude is of first order in ω_0 , hence of order g . In (3.2), the ratio of ω_0 to the g term that leads to the last mentioned $S_{fi}(K_S^0 \rightarrow 2\pi)$ is not fixed or known. This allows $\Gamma(K_S^0 \rightarrow 2\pi)$ to be $\gg \Gamma(K^\pm \rightarrow 2\pi)$; it may be regarded that the nonvanishing $S_{fi}(K_S^0 \rightarrow 2\pi)$ in the last paragraph is greatly amplified by the relative energy ω_{00} .

The factor $2 \approx \Gamma(K_S^0 \rightarrow \pi^-\pi^+)/\Gamma(K_S^0 \rightarrow 2\pi^0)$ [1] comes from the factor $1/\sqrt{2}$ in front of π^0 in (A5) that enters (7.7.2) via (7.7.7).

Instead of $\bar{K}^0 \rightarrow \pi^0 W_U$ of (7.3.27b) considered above, $K^0 \rightarrow \bar{W}_U \pi^0$ of (7.3.29) can be treated in the same way. The only differences are that $(1+i) \rightarrow (1-i)$ in (3.3 - 4) and the upper sign in (A10) is used when deriving the corresponding (3.9); the above results remain unchanged.

If the vacuum meson state π^0 assigned to $\chi_{0(22)}^+$ in (3.5) is replaced by a real final state π^0 , $K_S^0 \rightarrow 2\pi$ there goes over to $K_S^0 \rightarrow 3\pi$. Because π^\pm and π^0 form a triplet in the limit of SU(2) symmetry, they must have the same phase. Since π^\pm cannot contain a phase factor with complex argument, $\exp((i-1)\omega_{00} x^0)$, which would cause them to decay like $K_S^0 \rightarrow 2\pi$ in (3.9), this final state π^0 can at most contain a phase factor of the form $\exp(-i\omega_{00} x^0)$, similar to that for K_L^0 following (4.6) below. The $\exp(-\omega_{00} x^0)$ factor in (3.6) remains unchanged and the ratio (3.8) remains 0; $K_S^0 \rightarrow 3\pi$ is *forbidden*. However, if $T_1 \rightarrow -T_S$ is finite, (3.8) becomes

$2\exp(-\omega_0 T_S)$ and some $K_S^0 \rightarrow 3\pi$ will be seen. This form is the same as that for the semileptonic decay in (5.4) below. The difference is that T_K in (5.4) given by (5.8) is small. Here, T_S is much larger and is estimated to be $\sim 3 \times 10^{-15}$ sec if the small $\Gamma(K_S^0 \rightarrow 3\pi) \approx 2.57 \times 10^{-12}$ eV [1] is to be accounted for.

4. $K_L^0 \rightarrow 2\pi, 3\pi$ and $K_L^0 - K_S^0$ Mass Difference

The couplings $\bar{K}^0 W_U$ and $\bar{W}_U K^0$ of (7.3.27b) and the second of (7.3.29) are the same with respect to K_S^0 but not to K_L^0 according to (7.2.18). This leads to results mentioned in the next to last paragraph of Section 3 for K_S^0 . For K_L^0 , the W_U and \bar{W}_U contributions to $K_L^0 \rightarrow 2\pi$ needs be summed, as has been done in revised (7.3.27, 29) [unpublished]. The first order (3.1) for K_S^0 can be taken over here for K_L^0 if its sign is changed, as is seen in (7.2.18). Take the hermitian adjoint of (7.1.9a) and consider the first order term corresponding to (3.1) for K_L^0 ,

$$\left(\delta^{be} \partial_0 - i \frac{1}{4} g \sqrt{2} \bar{W}_U^{be}(X) \right) \left(\frac{1}{2} (-\sigma^{ab} \partial_{x_0} - \underline{\sigma}^{ab} \partial_{\underline{x}}) - \underline{\sigma}^{ab} \partial \right) \cdot \frac{\chi_0(r)}{\sqrt{2}} \exp(-iE_0 X^0 + i\omega_0 x^0) \quad (4.1)$$

Add this expression to the negative of (3.1) and work out the resulting expression using (A15-A16). The result shows that the imaginary part is of the form $ig(\text{real operator})$. Since $\bar{\omega}_0 \rightarrow i\omega_0$ must have the same phase, analogous to that mentioned above (3.3), ω_0 must be real here. The summation removes the imaginary parts in the above (real operator). The expressions (3.3, 4) are replaced by

$$\omega_0 = \omega_{00} \quad (4.2a)$$

$$\omega_0 = -\omega_{00} \quad (4.2b)$$

respectively, for $K_L^0 \rightarrow 2\pi$. The above (real operator) contains both W_6 and W_7 which are the same by (A15-A16).

The treatment of for $K_S^0 \rightarrow 2\pi$ in (3.3-5), using (4.2) instead, can be taken over and the ratio (3.8) is here replaced by

$$\frac{\int_{-T}^T dx^0 \exp(i\omega_0 x^0)}{\int_{-T}^T dx^0 \exp(\omega_0 x^0 (i-i))} = \frac{\sin(\omega_0 T)}{\omega_0 T} \rightarrow 0 \quad (4.3)$$

$T = \tau_0/2 \rightarrow \infty$

where (7.3.16) has been noted. Thus, $K_L^0 \rightarrow 2\pi$, just like $K_S^0 \rightarrow 2\pi$, is also *forbidden* to first order in igW_U . This result agrees with the requirement of CP invariance

[2]. Being a first principles' theory, consequences of conservation laws, including CP conservation, are included in the equations of motion (7.1.8-9).

Turning to the second order term $-(\omega_0)^2 = -(\omega_{00})^2$ in (A8) via (4.2). Let E_{00} be the K^0 mass in (A10) and $E_0 = E_{00} + \Delta E_{0L}$ in (A8), the equivalent of (3.9) becomes the mass shift

$$\Delta E_{0L} = 2\omega_{00}^2/E_{00} \quad (4.4)$$

when the upper sign in (A10) is chosen. The square of the relative energy among the s and d quarks in the hidden space x is visible in the laboratory space X in form of the mass shift (4.4), just like it is in form of the decay rate (3.9). Again, (4.4) comes from the second order $-(\omega_0)^2$ in (A8) but its amplitude is of first order in ω_0 , hence of order g . Comparison of (4.4) to (3.9) yields

$$\Gamma(K_S^0 \rightarrow 2\pi) = 2\Delta E_{0L} \quad (4.5)$$

which is 5.25% smaller than data (2.3). This discrepancy has been used to modify the argument 45° in (2.4) to about 43.5° [1-3]. Here, this 45° cannot be changed due to the constraint (3.3, 4). This discrepancy cannot be accounted for in the present theory. Similar to the CP violation cases mentioned above, finite T_2, T_1 in (3.8) will render the first order $S_{fi}(K_S^0 \rightarrow 2\pi)$ mentioned below it to contribute and to reduce this discrepancy.

Using (2.3) for ΔE_{0L} , (4.4) gives the relative energy between the s and d quarks in neutral kaons,

$$\omega_{00} = \pm 29.44 \text{ eV} \quad (4.6)$$

The upper sign is used. The lower sign can also be used if it is accompanied by $x^0 \rightarrow -x^0$, as is evident from the phase factor $\exp(i\omega_0 x^0)$ in (3.1), noting (3.3, 4). This leads to that the T 's in (3.6), at the end of Sec. 3 and in (5.8) and (6.1) below also change sign.

If the vacuum meson state π^0 in (3.5) adapted for K_L^0 is changed to a real π^0 , $K_L^0 \rightarrow 2\pi$, forbidden by (4.3) above, turns into $K_S^0 \rightarrow 3\pi$. This real final state π^0 must have a phase factor $\exp(i\omega_0 x^0)$ which is to be inserted into the upper integral in (4.3) to make this ratio to become unity, as is implicit in [4]. This implies that any π^0 from K_L^0 decay will also have a mass shift analogous to (4.4). According to the last paragraph of Section 3, π^\pm and π^0 have the same phase in the limit of SU(2) symmetry so that any π^\pm from K_L^0 decay will also have a mass shift. With (4.6) and the π masses [1], (4.4) yields the shifts

$$\begin{aligned} \Delta E_{\pi_0} &= 1.284 \times 10^{-5} \text{ eV} \\ \Delta E_{\pi^\pm} &= 1.277 \times 10^{-5} \text{ eV} \end{aligned} \quad (4.7)$$

Although these shifts are far less than the error margin 0.35 keV for the π masses and are not observable, it signifies that there exists *two different species of π triplets*

with slightly different masses.

$K_S^0 \rightarrow 3\pi$ has been treated in [4] employing the null relative energy condition (A9). A relaxation of this condition here however does not affect the results obtained there.

To achieve CP violation, let $T \rightarrow T_L \neq \infty$ in (4.3), like those mentioned in Section 3 for $\Gamma(K_L^0 \rightarrow 2\pi, 3\pi)$. For $T_L = 0$, (4.3) becomes unity and $\Gamma(K_L^0 \rightarrow 2\pi)$ like $\Gamma(K_S^0 \rightarrow 2\pi)$ mentioned beneath (3.8), $\approx \Gamma(K^\pm \rightarrow 2\pi)$ from phase space considerations. This rate is much greater than data. To account for the measured $\Gamma(K_L^0 \rightarrow 2\pi)$, T_L is chosen such that (4.3) becomes of the magnitude $\text{Re}(\eta_{00}, \eta_{+-})\sqrt{668} \approx 0.0407$, where $\text{Re}(\eta_{00}, \eta_{+-})$ are given by (2.7-9) and 668 has been mentioned beneath (3.8). Thus, CP is correctly violated if

$$\begin{aligned} \sin(\omega_0 T_L) / \omega_0 T_L &\approx 0.0407 \\ T_L &\approx \pm 0.1025, \pm 0.226, \pm 0.712 \text{ eV}^{-1} \end{aligned} \tag{4.8}$$

where (4.6) has been used. The angles $\omega_0 T_L$ are close to multiples of π . These relative times are very short, of the order of 10^{-16} sec. These values, like the finite T s mentioned in Section 3, cannot be predicted from the present theory and have to come outside of it.

5. Semileptonic Decay of K_L^0 and K_S^0

The amplitude for $\bar{K}^0 \rightarrow \pi^+ W^-$ is given by (7.3.27a) and for $K^0 \rightarrow \pi^- W^+$ by the first of (7.3.29). Subsequently, $W^- \rightarrow \pi^+ L \nu_L$ and $W^+ \rightarrow \pi^- L \nu_L$, where $L = \mu, e$. But the amplitudes for $\bar{K}^0 \rightarrow \pi^- W^+$ and

$K^0 \rightarrow \pi^+ W^-$ do not follow from the internal index combinations in (7.3.18-19) and hence do not appear in (7.3.26-29). These decays are therefore forbidden, in agreement with the selection rule $\Delta S = \Delta Q$ [2]. This rule is a consequence of the present theory and has been verified by that x of (2.6a) is consistent with 0 [1].

The amplitudes (7.3.27a, 29) assume the null relative energy condition (A9). The rates of these decays have not been evaluated because the pions are relativistic and their wave functions unknown; the coarse nonrelativistic (3.5.23), (7.7.13) used for $K_L^0 \rightarrow 3\pi$ is insufficient here and in other $K \rightarrow 2\pi$ decays. Nevertheless, the forms of (7.3.27a) and the first of (7.3.29) are the same so that the decay rates $\Gamma(K^0 \rightarrow \pi^- L \nu_L)$ and $\Gamma(\bar{K}^0 \rightarrow \pi^+ L \nu_L)$ are the same. By (7.2.18), K_S^0 and K_L^0 contain equal amount of \bar{K}^0 and K^0 , they contribute about equally to these semileptonic decays. This is approximately verified by data [1] and corresponds to the CP conserving part of these decays.

As was mentioned in the beginning of Section 3, (A9) has been relaxed here. Since the amplitudes for $\bar{K}^0 \rightarrow \pi^+ W^-$ and $\bar{K}^0 \rightarrow \pi^0 W_0$ have the same form according to (7.3.27), the developments (3.1-4) can with

some modifications be taken over. The expression corresponding to (3.1) reads

$$\begin{aligned} &\left(\delta^{ab} \partial_0 + i \frac{1}{4} g \sqrt{2} (W_1^{ab}(X) \mp i W_2^{ab}(X)) \right) \\ &\cdot \left(\frac{1}{2} (-\delta^{be} \partial_{x^0} - \underline{\sigma}^{be} \underline{\partial}_x) - \underline{\sigma}^{be} \underline{\partial} \right) \\ &\cdot \frac{\chi_0(r)}{\sqrt{2}} \exp(-iE_0 X^0 + i\omega_0 x^0) \end{aligned} \tag{5.1}$$

where the degeneration (A14) for W_V^\pm together with (A13) have been used. Analogous to $W_6(X) = W_7(X)$ in (A15-A16), $W_1(X) = W_2(X)$ here inasmuch as the both $W_I^\pm(X)$ have the same X dependence, so that the lower sign form of (5.1) turns to a form nearly the same as (3.2). Thus, (3.3) holds for $\bar{K}^0 \rightarrow \pi^+ W^-$. For $K^0 \rightarrow \pi^- W^+$, the upper sign of (5.1) lead to

$$\omega_0 = (1-i)\omega_{00} \tag{5.2}$$

The phase factor considerations for $K_S^0 \rightarrow 3\pi$ in the last paragraph of Sec. 3 can be taken over here. The phase factor $\exp(-i\omega_0 x^0)$ for the final state π^0 there can also be used for the final state π^+ in $\bar{K}^0 \rightarrow \pi^+ W^-$ here. Such a factor is required for in $K_L^0 \rightarrow 3\pi$ mentioned beneath (4.6). Inserting this factor into (3.6) turns (3.8) into

$$\begin{aligned} &\frac{\int_{T_1}^{T_2} dx^0 \exp(-\omega_0 x^0)}{\int_{T_1}^{T_2} dx^0 \exp(-2\omega_0 x^0)} \\ &= 2 \frac{\exp(-\omega_0 T_2) - \exp(-\omega_0 T_1)}{\exp(-2\omega_0 T_2) - \exp(-2\omega_0 T_1)} \rightarrow 0 \\ &T_2 = -T_1 = T_0 \rightarrow \infty \end{aligned} \tag{5.3}$$

Analogous to that $K_S^0 \rightarrow 2\pi$ is forbidden by (3.8), the semileptonic decay $\bar{K}^0 \rightarrow \pi^+ W^- \rightarrow \pi^+ L \nu_L$ is also forbidden when the relative energy $\omega_{00} \neq 0$, contrary to observation. If $\omega_{00} = 0$ or the null relative energy condition (A9) holds, (5.3) turns into $\infty/\infty \rightarrow 1$ and these semileptonic decays can take place, as was implied in the second paragraph of this section. These decays however conserve CP.

CP violation can be obtained in a way similar to that achieved in the last paragraph of Section 4, in which the limits of the relative time x^0 were not allowed to extend to $\pm\infty$. Let $T_2 = T_0 = \infty$ as in (5.3) and (3.6) but $T_1 = -T_K$ finite, (5.3) turns into

$$\frac{\int_{-T_K}^{\infty} dx^0 \exp(-\omega_0 x^0)}{\int_{-T_K}^{\infty} dx^0 \exp(-2\omega_0 x^0)} = 2 \exp(-\omega_0 T_K) \tag{5.4}$$

For $K^0 \rightarrow \pi^- W^+ \rightarrow \pi^- L \nu_L$, (5.2) replaces (3.3). Let $-T_1 = T_0 = \infty$ as in (5.3) and (3.6) but $T_2 = -T_K$, the

equivalent of (5.4) reads

$$\frac{\int_{-\infty}^{-T_K} dx^0 \exp(\omega_{00}x^0)}{\int_{-\infty}^{-T_K} dx^0 \exp(2\omega_{00}x^0)} = 2 \exp(\omega_{00}T_K) \quad (5.5)$$

As was mentioned in the second paragraph of this section, K_S^0 and K_L^0 contain equal amount of K^0 and \bar{K}^0 and $\Gamma(K^0 \rightarrow \pi^- L^+ \nu_L) = \Gamma(\bar{K}^0 \rightarrow \pi^+ L^- \nu_L)$ for the dominating CP conserving part. This together with (5.4-5) give the expression corresponding to (2.6b)

$$\frac{\Gamma(K_S^0 \rightarrow \pi^- L_s^+ \nu_L)}{\Gamma(K_S^0 \rightarrow \pi^+ L_s^- \bar{\nu}_L)} = \frac{\Gamma(K_L^0 \rightarrow \pi^- L_s^+ \nu_L)}{\Gamma(K_L^0 \rightarrow \pi^+ L_s^- \bar{\nu}_L)} \quad (5.6)$$

$$\approx \left| \frac{\exp(\omega_{00}T_K)}{\exp(-\omega_{00}T_K)} \right|^2 \cong 1 + 4\omega_{00}T_K$$

Comparison with (2.6b) gives the identification

$$\omega_{00}T_K = \text{Re}(\varepsilon) \quad (5.7)$$

This together with (2.7) and the lower of (4.6) yields

$$T_K \approx 5.5 \times 10^{-5} \text{ eV}^{-1} = 1.2 \times 10^{-11} \text{ s} \quad (5.8)$$

which is more than 10 times shorter than the K_S^0 decay time. Thus, by suitable choices of the limits of the relative times, the present theory reproduces the rather successful phenomenological (2.6b).

6. On the Origin of CP Violation in Neutral Kaon Decay

From the semileptonic decays in Section 5, the phenomenological phase angle Θ in (2.1) by (2.4-5) and (5.7) becomes

$$\Theta = i2\varepsilon \approx \pm i\sqrt{2}|\varepsilon|(1+i) \quad (6.1)$$

$$= \pm i2\omega_{00}T_K \mp 2\omega_{00}T_K$$

Here, Θ is identified with the product of a relative energy and a relative time, both finite. For the CP violating $K_L^0 \rightarrow 2\pi$ in Section 4, the phase angle $\omega_{00}T_L$ in (4.8) via (4.3) associated with K_L^0 is real. This corresponds to that the angle Θ in (2.1) is also real for this case and is related to $\omega_{00}T_L$, also a product of a finite relative energy and a finite relative time.

Equations (5.4-5) show that it is the imaginary part of the relative energy ω_0 in (3.3) and (5.2) that causes (5.6) to deviate from unity and thereby causes CP violation in the semileptonic decays, which takes place when the relative time x^0 between the s and d quarks does not extend to both $+\infty$ and $-\infty$ but is finite at one end.

This does not conflict with that $-\infty \leq x^0 \leq \infty$ in (7.4.6b) via (7.3.16) to generate the M_W mass. There, the actions in (7.4.3-4) involve stationary K^0 and \bar{K}^0 and not their decay so that (A9) can and has been applied in

that stage. The phase factor $\exp(i\omega_0x^0)$ here reduces to 1 there which allows for $-\infty \leq x^0 \leq \infty$, similar to that $\omega_{00} = 0$ in (5.3) renders it to be $\infty/\infty \neq 0$.

For the CP violating $K_L^0 \rightarrow 2\pi$, the relative time x^0 also does not extend to both $+\infty$ and $-\infty$ but is finite at both ends as is shown in (4.3) with $T \rightarrow T_L$ given in (4.8).

Summarizingly, CP conservation is related to the phase angle ω_0x^0 . The relative energy ω_0 gives rise to the mass shifts for K_L^0 in (4.4) and pions in (4.7) and to that the large $K_S^0 \rightarrow 2\pi$ decay rate in (3.9) is twice the K_L^0 mass shift in (4.5). These results are derived within the frame work of the present theory.

The relative energy ω_0 is necessary but not sufficient to account for the CP violating $K_S^0 \rightarrow 2\pi$ in Section 4 and the semileptonic decays in Section 5. To achieve these CP violations, bounds need be put on the values of the relative time x^0 so that it cannot run from $-\infty$ to ∞ . The values of these bounds, given in (4.8) and (5.8), respectively, lie outside the present theory.

It not known how such relative time bounds can be brought forth. One observation is that the relative time $|x^0| = |x_{II}^0 - x_I^0|$ must be shorter than twice the laboratory time $2X^0 = |x_{II}^0 + x_I^0|$ in (3.1.3a). Now this X^0 is limited by the finite decay time so that the relative time is also limited. Another one is that the relative time $|y^0|$ has been set to the finite $2/d_m$ in the last line of §7.7.2. Further, how the kaons are produced may also enter here. In any case, the CP violations here are not related to the CP violating phase in neutrino oscillation phenomenology [1 Neutrino Mixing].

In this connection, it may be pointed out that the boundary condition of the wave functions χ and ψ in (A1), hence also in the action (7.1.8), at $|x^0| \rightarrow \infty$, like T_0 and T in (3.6-7), (4.3) and (5.3), has been assumed to be the same as the corresponding ones (fixed) in the laboratory time $|x^0|$, as was pointed out in Section 6.1.1. The validity of this assumption has not been fully investigated and it is not clear whether this may impact upon the above relative time considerations.

7. $\bar{B}^0 - B^0$ Mixing

The $\bar{B}^0 - B^0$ mixing differs fundamentally from the $\bar{K}^0 - K^0$ mixing considered above. In **Table 1**, the u , d and s quarks have about the same mass. This leads to that M_{mpr}^2 in (2.4.1) differ by $\leq 12\%$, as was mentioned above (2.4.5). Therefore, the kaons and pions are members of an approximate SU(3) octet. In the limit of SU(3) symmetry, the u , d and s quark masses coalesce. \bar{K}^0 and K^0 belong to the same octet providing basis vectors of the *regular* representation of SU(3). They can be transformed into each other by suitable choice of the

Table 1. Quark masses and d_{m_0} obtained from (5.1.1-4) using the masses π^+ , K^+ , K^0 , D^0 , D_s^+ and B^0 , and quark contents of [P1].

m_1 (GeV)	$m_2 - m_1$	m_3	m_4	m_5	d_{m_0} (GeV ²)
0.6592	0.00215	0.7431	1.6215	4.7786	0.24455

transformation $U_{3qs}(X)$ of (7.1.7). Their linear combinations K_S^0 and K_L^0 in (7.2.18) are also vectors in this octet space on par with the pion vectors and represent physical mesons suitable for describing some weak decays.

The c and b quarks are much heavier and the D and B mesons cannot meaningfully be accommodated in SU(4, 5) multiplets together with the kaons and pions, as was indicated above (2.4.5). In the limit of SU(3) symmetry, B^0 belongs to the triplet (B^+ , B^0 , B_S^0) providing basis vectors of the *first fundamental* representation of SU(3). But \bar{B}^0 belongs to the antitriplet (B^- , \bar{B}^0 , \bar{B}_S^0) providing basis vectors of the *second fundamental* representation of SU(3) transforming differently [6]. Thus, B^0 and \bar{B}^0 cannot be transformed into each other and linear combinations of them of the type (7.2.18) do not have definite transformation properties and hence are not members of any SU(n) multiplet. They remain as physical mesons in two different triplets. The D meson triplets behave analogously.

As in Section 3, the null relative energy condition (A9) is also relaxed for B^0 and \bar{B}^0 . The complex ω_0 in (3.3) is due to the degeneration (A15) involving the complex neutral gauge boson W_U . Data [1] however show that the semileptonic decays of B^0 and \bar{B}^0 proceed dominantly via charged gauge bosons $W^\pm \rightarrow$ charged leptons; the branching ratio of $B^0 \rightarrow \bar{D}^0 Z \rightarrow \bar{D}^0 (2K, 2\pi)$ is very small. Therefore, $\omega_0 = \omega_{00} =$ real as in (4.2) for K_L .

Therefore, a mass shift ΔE_{B^0} equivalent to ΔE_{0L} in (4.4) also holds for B^0 . For \bar{B}^0 , the lower sign in (A10) is chosen. The laboratory time X^0 dependent part of the B^0 and \bar{B}^0 wave functions is

$$\begin{aligned} \Psi_{B^0}(X^0) &= \exp\left[\left(-\frac{\Gamma_{B^0}}{2} - iE_{00} - i\Delta E_{B^0}\right)X^0\right] \\ \Psi_{\bar{B}^0}(X^0) &= \exp\left[\left(-\frac{\Gamma_{B^0}}{2} - iE_{00} + i\Delta E_{B^0}\right)X^0\right] \end{aligned} \tag{7.1}$$

where E_{00} is the mass and Γ_{B^0} the decay rate of B^0 and \bar{B}^0 . The X^0 dependent part of the probability of mixed B^0 and \bar{B}^0 is

$$\begin{aligned} & \left| \Psi_{B^0}(X^0) \pm \Psi_{\bar{B}^0}(X^0) \right|^2 \\ &= \pm 2 \left(\exp(-\Gamma_{B^0} X^0) \right) \left(1 + \cos(2\Delta E_{B^0} X^0) \right) \tag{7.2} \\ & \Delta E_{B^0} = 2\omega_{00}^2 / E_{00} \end{aligned}$$

It is proportional to the phenomenological probability for remaining in the original bottom state [3]

$$\begin{aligned} P(B^0 \rightarrow B^0) &= P(\bar{B}^0 \rightarrow \bar{B}^0) \\ &= \frac{1}{2} \exp(-\Gamma t) (1 + \cos(\Delta m_d t)) \end{aligned} \tag{7.3}$$

Here, $\Gamma = \Gamma_{B^0}$, $t = X^0$ and the oscillation frequency $\Delta m_d = 2\Delta E_{B^0} = 3.337 \times 10^{-4}$ eV [1]. With the B^0 mass $E_{00} = 5.2796$ GeV, the second of (7.2) gives the relative energy $\omega_{00} = 663.66$ eV between the b and d quarks. This value is 22.5 times greater than 29.44 eV in (4.6) for that between the s and d quarks in K^0 . Note that the ratio between the B^0 and K^0 masses is 10.6, nearly half the above ratio.

The mass difference 3.337×10^{-4} eV is somewhat less than the B^0 decay rate $\Gamma_{B^0} = 4.43 \times 10^{-4}$ eV [1], similar to that the $K_L^0 - K_S^0$ mass difference is about half of the K_S^0 decay rate in (2.3). These mass differences, apart from their values, are outcomes of the present theory, irrespective CP violation. Analogously, CP violation in the $\bar{B}^0 - B^0$ system is also attributed to relative energy and time between the b and d quarks. Such a treatment however requires the knowledge of the amplitudes of 5 specific decays [3] and is beyond the scope of this paper.

In passing, it may be noted that while mixings of d , s and b quarks take place, the c and u quarks with charge $2e/3$ do not seem to mix, as $\bar{D}^0 - D^0$ mixing is absent in [1].

8. Conclusions

In the standard model, quarks in meson decay are treated [3] largely as leptons are in QED. The effects of quark confinement in the relative space and of the relative time between the quarks are practically lost.

In the paper, the relative energy between the quarks gives rise to the mass shift between K_S^0 and K_L^0 , equalling half the K_S^0 decay rate, and $\bar{B}^0 - B^0$ mixing. Hadron spectra stem from the relative space [4] and the relative time generates the W and Z boson masses [4] without Higgs. Here, CP violation is achieved by limiting the relative time to certain regions.

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Appendix Relevant Equations from the Book [4]

Parts of the book [4] often referred to are reproduced here. The references in form of (x, y, z) or $\xi x, y, z$ below refer to those in [4].

The starting point is the equations of motion for meson including internal coordinates ξ given by (2.3.19, 21, 23, 27),

$$\partial_1^{ab} \partial_{11ef} \chi_b^f(x_1, x_{11}) \xi_r^p(z_1, z_{11}) + (M_m^2 - \Phi_m(x_1, x_{11})) \psi_e^a(x_1, x_{11}) \xi_r^p(z_1, z_{11}) = 0 \tag{A1a}$$

$$\partial_{1bc} \partial_{11de} \psi_e^c(x_1, x_{11}) \xi_r^p(z_1, z_{11}) + (M_m^2 - \Phi_m(x_1, x_{11})) \chi_b^d(x_1, x_{11}) \xi_r^p(z_1, z_{11}) = 0 \tag{A1b}$$

$$\overline{\square}_1 \overline{\square}_{11} \Phi_m(x_1, x_{11}) = -\frac{1}{2} g_s^4 \text{Re} \psi^{ba}(x_1, x_{11}) \chi_{ab}^*(x_1, x_{11}) \tag{A2}$$

$$M_m = (m_p + m_r) / 2 \tag{A3}$$

$$\psi_{(pr)} \rightarrow \begin{pmatrix} \psi_{(11)} & \psi_{(12)} & \psi_{(13)} \\ \psi_{(21)} & \psi_{(22)} & \psi_{(23)} \\ \psi_{(31)} & \psi_{(32)} & \psi_{(33)} \end{pmatrix} - \frac{1}{3} (\psi_{(11)} + \psi_{(22)} + \psi_{(33)}) = \begin{pmatrix} \frac{1}{\sqrt{2}} \psi_{(\pi^0)} + \frac{1}{\sqrt{6}} \psi_{(\eta)} & \psi_{(\pi^+)} & \psi_{(K^+)} \\ \psi_{(\pi^-)} & -\frac{1}{\sqrt{2}} \psi_{(\pi^0)} + \frac{1}{\sqrt{6}} \psi_{(\eta)} & \psi_{(K^0)} \\ \psi_{(K^-)} & \psi_{(\bar{K}^0)} & -\frac{2}{\sqrt{6}} \psi_{(\eta)} \end{pmatrix}, \tag{A5}$$

$$\psi \rightarrow \chi_{ab}$$

Follow (3.1.3a) and (3.5.7) and introduce the relative and laboratory coordinates,

$$x^\mu = x_{11}^\mu - x_1^\mu, X^\mu = (1 - a_m) x_1^\mu + a_m x_{11}^\mu, a_m = 1/2 \tag{A6}$$

where the relative space time $x^\mu = (\underline{x}, x^0)$ are hidden variables [4]. The wave functions of a pseudosclar meson at rest reads (3.1.1, 5, 7, 9)

$$\psi^{be}(x_1, x_{11}) = \delta^{be} \psi_0(x_1, x_{11}) = \delta^{be} \psi_0(\underline{x}) \exp(-iE_0 X^0 + i\omega_0 x^0), \psi \rightarrow \chi \tag{A7}$$

where E_0 is the mass of the meson and ω_0 the relative energy among the quarks. Inserting (A7) into (A4a) for a given (pr) and taking the trace, one obtains from the first line of (3.1.8),

$$\partial_1^{ab} = \frac{1}{2} \partial_X^{ab} - \partial^{ab} \rightarrow D_{1ps}^{ab} = \frac{1}{2} \left(\partial_X^{ab} \delta_{ps} + i \frac{1}{2} g(\lambda_l)_{ps} W_l^{ab}(X) \right) - \partial^{ab} \delta_{ps} = \partial_1^{ab} \delta_{ps} + i \frac{1}{4} g(\lambda_l)_{ps} W_l^{ab}(X) \tag{A11}$$

$$\partial_{11}^{fe} = \frac{1}{2} \partial_X^{fe} + \partial^{fe} \rightarrow D_{11ps}^{fe} = \partial_{11}^{fe} \delta_{ps} + i \frac{1}{4} g(\lambda_l)_{ps} W_l^{fe}(X)$$

Here, x_1 and x_{11} are the quark and antiquark coordinates, respectively, $\partial_1 = \partial/\partial x_1, \partial_{11} = \partial/\partial x_{11}, \chi$ and ψ are the meson wave functions with the spinor indices a, b, \dots running from 1 to 2, Φ_m the scalar interquark potential, p the quark flavor, q the antiquark flavor, and z_1 and z_{11} respectively the internal coordinates for the quark and the antiquark in an abstract complex three dimensional space. $p, r = 1, 2$ and 3 refer to the u, d and s quarks, respectively. Because the quark masses m_p and m_r are different for different mesons, (A1) can, after cancelling out the ξ functions be written in the form (2.4.3)

$$\partial_1^{ab} \partial_{11}^{fe} \chi_{(pr)bf}(x_1, x_{11}) - (M_{m(pr)}^2 - \Phi_m(x_1, x_{11})) \psi_{(pr)}^{ae}(x_1, x_{11}) = 0 \tag{A4a}$$

$$\partial_{1bc} \partial_{11ed} \psi_{(pr)}^{ce}(x_1, x_{11}) - (M_{m(pr)}^2 - \Phi_m(x_1, x_{11})) \chi_{(pr)bd}(x_1, x_{11}) = 0 \tag{A4b}$$

where (pr) indicates the dependence upon the quark flavors. The matrix form of ψ is shown in (2.4.18),

$$\left[E_0^2/4 - \omega_0^2 + \Delta \right] \chi_0(\underline{x}) = (\Phi_m - M_m^2) \psi_0(\underline{x}) \tag{A8}$$

$$\Delta = \partial^2 / \partial \underline{x}^2$$

The null relative energy condition (3.5.6)

$$\omega_0 = 0 \tag{A9}$$

With this ansatz, (A8) together with Φ_m obtained from (3.2.8, 20) and (3.4.1), (4.3.2), (4.4.1) leads to the steady state meson mass (5.1.1)

$$E_{00} = \pm \sqrt{(m_p + m_r)^2 - 4d_{m0} - d_m^2} \tag{A10}$$

where $d_m = 0.864$ GeV is given by (5.2.3) and d_{m0} and the quarks masses m_p are given in **Table 1**.

In the presence of SU(3) gauge fields, the operators in (A4) are generalized according to (7.1.4, 5) and (7.2.14),

$$\begin{aligned}
 i \frac{g}{2} (\lambda_l)_{ps} W_{lab}(X) &\rightarrow i \frac{g_l}{2} (\lambda_l)_{ps} W_{lab}(X) \\
 &= i \frac{g}{2} \left(\begin{array}{ccc} 2A \sin \vartheta_W + Z \cos 2\vartheta_W / \cos \vartheta_W & \sqrt{2} W_I^- & (g_{4-7}/g) \sqrt{2} W_V^- \\ \sqrt{2} W_I^+ & -Z / \cos \vartheta_W & (g_{4-7}/g) \sqrt{2} W_U \\ (g_{4-7}/g) \sqrt{2} W_V^+ & (g_{4-7}/g) \sqrt{2} \bar{W}_U & -2A \sin \vartheta_W + 2Z \sin^2 \vartheta_W / \cos \vartheta_W \end{array} \right) \quad (A12)
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{2} W_I^\pm &= W_1 \pm i W_2, \quad \sqrt{2} W_V^\pm = W_4 \pm i W_5 \\
 \sqrt{2} W_U &= W_6 + i W_7, \quad \sqrt{2} \bar{W}_U = W_6 - i W_7
 \end{aligned} \quad (A13)$$

where A and Z are gauge fields (7.2.1), ϑ_W the Weinberg angle (7.2.12) and g and g_{4-7} weak coupling constants (7.2.7).

There are four known gauge fields W_1^\pm , Z and A in (7.2.2) but eight W_1^\pm , W_3 , W_8 , W_V^\pm , W_U , and \bar{W}_U in (7.1.5). The last four gauge fields are new, have not been observed and are converted into the four known ones via the following degeneracy scheme Section 7.2.3. Thus, (7.2.20) reads

$$\frac{g_{4-7}}{g} \sqrt{2} W_{V^{ab}}^\pm(X) \rightarrow \sqrt{2} W_{1^{ab}}^\pm(X) \quad (A14)$$

Equations (7.2.21, 22) have been written down heuristically and are now replaced by the formal analog of (A14) as is evident from (A12),

$$\frac{g_{4-7}}{g} \sqrt{2} W_U = \frac{g_{4-7}}{g} (W_6 + i W_7) \rightarrow \frac{1+i}{\sqrt{2}} \left(-\frac{Z}{\cos \vartheta_W} \right) \quad (A15)$$

$$\frac{g_{4-7}}{g} \sqrt{2} \bar{W}_U = \frac{g_{4-7}}{g} (W_6 - i W_7) \rightarrow \frac{1-i}{\sqrt{2}} \left(-\frac{Z}{\cos \vartheta_W} \right) \quad (A16)$$

where the magnitudes on both sides are equal.