

Prediction of Cosmological Constant Λ in Veneziano Ghost Theory of QCD*

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ABSTRACT

Based on the Veneziano ghost theory of QCD, we estimate the cosmological constant Λ , which is related to the vacuum energy density, ρ_Λ , by $\Lambda = 8\pi G\rho_\Lambda$. In the recent Veneziano ghost theory ρ_Λ is given by the absolute value of the product of the local quark condensate and quark current mass: $\rho_\Lambda = \frac{2N_f H}{m_q} c |m_q < 0| : \bar{q}q : |0 >|$. By solving Dyson-

Schwinger Equations for a dressed quark propagator, we found the local quark condensate $\langle 0| : \bar{q}q : |0 \rangle \approx -(235 \text{ MeV})^3$, the generally accepted value. The quark current mass is $m_q \approx 4.0 \text{ MeV}$. This gives the same result for ρ_Λ as found by previous authors, which is somewhat larger than the observed value. However, when we make use of the nonlocal quark condensate, $\langle 0| : \bar{q}(x)q(0) : |0 \rangle = g(x)\langle 0| : \bar{q}q : |0 \rangle$, with $g(x)$ estimated from our previous work, we find Λ is in a good agreement with the observations.

Keywords: Cosmological Constant Λ ; Veneziano Ghost Theory of QCD; Local Quark Vacuum Condensate; Nonlocal Quark Condensate; Quantum Chromodynamics-QCD

1. The Cosmological Constant Λ and the QCD Veneziano Ghost Theory

The starting point of most cosmological study is Albert Einstein's Equations, which is a set of ten equations in Einstein's theory of general relativity. The original Einstein field equations can be written as the form [1]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} \quad (1)$$

in units of $\hbar = c = 1$, where G is the gravitational constant ($G = 6.7087(10) \times 10^{-39} \text{ GeV}^{-2}$, sometime called Newton's constant), $R_{\mu\nu} (\mu, \nu = 0, \dots, 3)$ is the Ricci tensor, R is the trace of Ricci tensor (it is like the radius of curvature of space-time), $g_{\mu\nu}(x)$ represents the metric tensor, which is a function of position x in spacetime.

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$T^{\mu\nu}$ is the energy-momentum tensor, which describes the distribution of matter and energy. Equation (1) describes a non-static universe. However, Einstein believed, at that time, that our universe should be static. In order to get a static universe, in 1917 Einstein introduced a new term, $\Lambda g_{\mu\nu}$, in Equation (1) to balance the attractive force of gravity, giving his modified equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu}. \quad (2)$$

The Λ in Equation (2) is the so-called cosmological constant, which is a dimensional parameter with units of $(\text{length})^{-2}$. Indeed, Equation (2) allows a static universe [2], called Einstein's universe, which is one of the solution [3] of Friedmann's simplified form of Einstein's equation with a Λ term. However, almost one hundred years ago the observations of redshifts of galaxies led to Hubbles Law [4] and the interpretation that the universe is expanding. This led Einstein to declare his static cosmological model, and especially the introduction of the Λ term to his original field equation theory, his "biggest blunder".

Note that the term $\Lambda g_{\mu\nu}$ in Equation (2) corresponds

to adding a vacuum term to $T_{\mu\nu}$,

$$T_{\mu\nu}(vac) = \rho_\Lambda g_{\mu\nu}. \quad (3)$$

Therefore, the cosmological constant Λ is related to the vacuum energy density, ρ_Λ by [3]

$$\Lambda = 8\pi G\rho_\Lambda. \quad (4)$$

The vacuum energy density, called dark energy density, and a model with Λ representing dark energy were reintroduced about three decades ago. See Ref. [5] for a review of the physics and cosmology of Λ , with references to the many models that have been published. To explain our uniform and flat universe via inflation a cosmological constant was added to the Friedmann equation [6]. From studies of radiation from the early universe, the Cosmic Microwave Background Radiation (CMBR), by a number of projects, including WMAP [7], the inflation scenerio was verified, and it was shown that about 73% of the total energy in the universe is dark energy. As clearly shown by Friedmann's equation with a cosmological constant, dark energy corresponds to negative pressure, or anti-gravity. This was confirmed by studies of distant type Ia supernovae [8,9], which showed an acceleration of the expansion of the universe, and was consistent with dark energy being 73% of the energy in the universe. Also, dark energy causes distant galaxies to accelerate away from us, in contrast to the tendency of ordinary forms of energy to slow down the recession of distant objects. See Ref. [5] for other of the many references to CMBR, supernovae, galaxy and other studies of dark energy.

The existence of a non-zero vacuum energy would, in principle, have an effect on gravitational physics on all scales. The value of Λ in our present universe is not well known, and it is an empirical issue which will ultimately be settled by observation. A precise determination of this number (Λ) or ρ_Λ will be one of the primary goals of observational cosmology in the near future. Recently the possiblity of determining the cosmological constant by observations has been discussed [10].

A major outstanding problem is that most quantum field theories predict a huge cosmological constant Λ from the energy of the quantum vacuum. This conclusion also follows from dimensional analysis and effective field theory down to the Planck scale, by which we would expect a cosmological constant of the order of M_{pl}^4 (M_{pl} is the Planck mass with $M_{pl} = G^{-1/2} = 1.22 \times 10^{19}$ GeV. The Planck energy is thought to be the energy where conventional physical theories break down and a new theory of quantum gravity is required). We know that the measured value is on the order of 10^{-35} s^{-2} , or 10^{-47} GeV^4 , or 10^{-29} g/cm^3 , or about 10^{-120} in reduced planck units (M_{pl}). That is, there is a large difference between the magnitude of the vacuum

energy expected from zero-point fluctuations and scalar potential, $\rho_\Lambda^{theory} = 2 \times 10^{110} \text{ erg/cm}^3$, and the observed value, $\rho_\Lambda^{observe} = 2 \times 10^{-10} \text{ erg/cm}^3$, a discrepancy of a factor of 10^{120} . This is the largest discrepancy—the worst theoretical prediction in the history of physics. At the same time, some supersymmetric theories require a cosmological constant that is exactly zero. Therefore, we face a big difficulty in understanding the observational $\rho_\Lambda^{observe}$. This problem has been referred to as the long-standing cosmological constant problem.

Vacuum energy is predicted to be created in cosmological phase transitions. In the standard model of particle physics with the temperature (T) of the universe as a function of time (t), there are two important phase transitions. At $t \approx 10^{-11}$ seconds, with $T \approx 140 \text{ GeV}$ the universe undergoes the electroweak phase transition (EWPT), with the vacuum expectation value of the Higgs field, $\langle 0 | \Phi^{Higgs} | 0 \rangle$, going from zero to a finite value corresponding to a Higgs mass $\approx 140 \text{ GeV}$. At $t \approx 10^{-5}$ seconds, with $T \approx 150 \text{ MeV}$, the universe undergoes the QCD phase transition (QCDPT), when a universe consisting of a dense quark-gluon plasma becomes our current universe with hadrons. The latent heat for this phase transition is the quark condensate, $\langle 0 | \bar{q}q | 0 \rangle$, also a vacuum energy, which is an essential part of the present work.

First we review the work of F. R. Urban, A. R. Zhitnitsky [11,12], which is based on the QCD Veneziano ghost theory [13-16]. In this model the cosmological vacuum energy density ρ_Λ can be expressed in terms of QCD parameters for $N_f = 2$ light flavors as follows [10,11]

$$\rho_\Lambda = c \frac{2HN_f}{m_{\eta'}} \left| m_q \langle 0 | \bar{q}(0)q(0) | 0 \rangle \right|, \quad (5)$$

where m_q is the current quark mass and $c = c_{QCD} \times c_{grav}$. The first factor c_{QCD} is a dimensionless coefficient with value of $c_{QCD} \approx 1$ [10,11], which is entirely of QCD origin and is related to the definition of QCD on a specific finite compact manifold such as a torus,

$$\rho_\Lambda \approx c_{QCD} \frac{2N_f |m_q \langle \bar{q}q \rangle|}{Lm_{\eta'}} \quad \text{with } L \text{ being the size of the}$$

manifold and $m_{\eta'}$ the mass of η' meson. A precise computation of c_{QCD} has been calculated in a conventional lattice QCD approach by studying corrections of order $1/s$ to the vacuum energy [10,11]. Note that c_{QCD} depends on the manifold where the theory is defined. The second factor c_{grav} has a purely gravitational origin and is defined as the relation between the size L of the manifold we live in, and the Hubble constant H , $L = (c_{grav} H_0)^{-1}$. One can define this size of the manifold as $L \approx 17H_0^{-1}$ where $H_0 = 2.1 \times 10^{-42} \times h \text{ GeV}$ and $h = 0.71$ (H_0 , Hubble

constant today). Therefore, one can explicitly obtain an estimate for the linear length L of the torus, and then obtain the value of c_{grav} , with $c_{grav} = 0.0588$.

In Section 2 we briefly review our previous calculation of the quark condensate [17] using Dyson-Schwinger equations (DSEs) [18,19], and discuss the quark current mass m_q , which are needed to calculate ρ_Λ , as shown in Equation (5). Since our values for the local quark condensate $\langle 0 | \bar{q}(0)q(0) | 0 \rangle$ and the current quark mass are approximately the same as in Ref. [10,11] we find the same value for ρ_Λ as in that work, with a factor 6 discrepancy when compared to the observed vacuum energy density. In Section 3 we use a nonlocal quark condensate, based on earlier research, and find good agreement between $\rho_\Lambda^{nonlocal\ theory}$ and $\rho_\Lambda^{observed}$. Finally, we give our Summary and concluding remarks in Section 4.

2. Local Quark Condensate, Current Quark Mass, ρ_Λ

In this section we review our previous work on the quark condensate, the current quark mass, and the resulting value for the cosmological constant/vacuum energy density.

2.1. The Local Quark Condensate

The quark propagator is defined by

$$S_q^{ab}(x) = \langle 0 | T [q^a(x) \bar{q}^b(0)] | 0 \rangle, \quad (6)$$

where $q^a(x)$ ($\bar{q}^b(x)$) is a quark field with color a (b), and T is the time-ordering operator. The nonperturbative part of the quark propagator is given by

$$S_q^{NP}(x) = -\frac{1}{12} \left[\langle 0 | \bar{q}(x)q(0) | 0 \rangle + x_\mu \langle 0 | \bar{q}(x)\gamma^\mu q(0) | 0 \rangle \right]. \quad (7)$$

For short distances, the Taylor expansion of the scalar part, $\langle 0 | \bar{q}(x)q(0) | 0 \rangle$, of $S_q^{NP}(x)$ can be written as (see, e.g., Refs.[17,20])

$$\begin{aligned} \langle 0 | \bar{q}(x)q(0) | 0 \rangle &= \langle 0 | \bar{q}(0)q(0) | 0 \rangle \\ &- \frac{x^2}{4} \langle 0 | \bar{q}(0) [ig_s \sigma G(0)] q(0) | 0 \rangle + \dots \end{aligned} \quad (8)$$

In Equation (8) the vacuum expectation values in the expansion are the local quark condensate, the quark-gluon mixed condensate, and so forth.

The Dyson-Schwinger Equations [18,19] were used to derive the local quark condensate in Ref. [17]. See this reference for details and a discussion of approximations. Note that as shown in Equation (8), the quark-gluon mixed condensate provides the small- x dependence of the nonlocal $\langle 0 | \bar{q}(x)q(0) | 0 \rangle$ quark condensate. How-

ever, for the present work this small- x expansion is not useful, and we shall use a known expression for the nonlocality, described below. Therefore we only give the results for the local quark condensate. Also note that the vacuum condensates can act as a medium [21,22], which influences the properties of particles propagating through it.

Using the solutions of DSEs with three different sets of the quark-quark interaction parameters (see Ref.[17]) leads to our theoretical predictions for the local quark vacuum condensate listed in **Table 1**.

Set 1 results are consistent with many other calculations, such as QCD sum rules [23-25], Lattice QCD [26-28] and Instanton model predictions [29-31]. These numerical results will be used to calculate Λ/ρ_Λ in the Subsection 2.3 below.

2.2. The Current Mass of Light Quarks

As we have seen from Equation (5) to predict Λ we need to know the basic quark current mass m_q . Since one cannot produce a beam of quarks, it is difficult to determine the quark masses. Using various models the effective quark masses have been estimated, but we need the current quark masses of the light u and d quark. Estimates of these masses and references can be found in the Particle Data Physics booklet [32]. They are

$$\begin{aligned} 1.7 < m_u < 3.3 \text{ MeV} \\ 4.1 < m_d < 5.8 \text{ MeV}e. \end{aligned} \quad (9)$$

From this we estimate that the current quark mass is

$$m_q \simeq 4.0 \text{ MeV} \quad (10)$$

2.3. Cosmological Constant Λ with $\langle 0 | \bar{q}(0)q(0) | 0 \rangle$ and m_q

From Equation (2), $\Lambda = 8\pi G\rho_\Lambda$, the vacuum energy density, while ρ_Λ , is given in Equation (5) as

$$\rho_\Lambda = c \frac{2HN_f}{m_\eta} m_q \langle 0 | \bar{q}(0)q(0) | 0 \rangle \quad (11)$$

Since our values for m_q and $\langle 0 | \bar{q}(0)q(0) | 0 \rangle$ are the standard ones, we find the same value for ρ_Λ as in

Table 1. Predictions of local quark condensate in QCD vacuum, $\langle 0 | \bar{q}q | 0 \rangle_\mu^f$ with f standing for quark flavor and μ denotes renormalization point, $\mu^2 = 10 \text{ GeV}^2$.

Set no. of quark interactions	$\langle 0 \bar{q}q 0 \rangle_{u,d}$ for u and d quarks
Set 1	$-0.0130(\text{GeV})^3 \sim -(235 \text{ MeV})^3$
Set 2	$-0.0078(\text{GeV})^3 \sim -(198 \text{ MeV})^3$
Set 3	$-0.0027(\text{GeV})^3 \sim -(139 \text{ MeV})^3$

Ref.[11]

$$\rho_{\Lambda}^{theory} \approx (3.6 \times 10^{-3} \text{ eV})^4, \quad (12)$$

while the value observed [33] is

$$\rho_{\Lambda}^{observed} \approx (2.3 \times 10^{-3} \text{ eV})^4. \quad (13)$$

Although the theoretical and observed values are similar, they still differ by

$$\rho_{\Lambda}^{theory} / \rho_{\Lambda}^{observed} \approx 6.0$$

3. Cosmological Constant Λ with Nonlocal Quark Condensate

As mentioned above, the expression

$$\begin{aligned} \langle 0 | \bar{q}(x)q(0) | 0 \rangle &= \langle 0 | \bar{q}(0)q(0) | 0 \rangle \\ &- \frac{x^2}{4} \langle 0 | \bar{q}(0) [ig_s \sigma G(0)] q(0) | 0 \rangle + \dots \end{aligned}$$

does not work except for very small x . Therefore we shall use the nonlocal quark condensate derived from the quark distribution function (see Refs.[34,35]). Using the form in Ref.[35],

$$\langle 0 | \bar{q}(x)q(0) | 0 \rangle = g(x^2) \langle 0 | \bar{q}(0)q(0) | 0 \rangle, \quad (14)$$

with

$$g(x) = \frac{1}{(1 + \lambda^2 x^2 / 8)^2}. \quad (15)$$

The value of λ^2 estimated in Ref.[36] is $\lambda^2 \approx 0.8 \text{ GeV}^2$. Using $1/\Lambda_{QCD}$ as the length scale, or $x^2 = (1/0.2 \text{ GeV})^2$, one obtains

$$g(1/\Lambda_{QCD}) = \frac{1}{2.25^2} = \frac{1}{6.25}. \quad (16)$$

From this we obtain

$$\langle 0 | \bar{q}(x)q(0) | 0 \rangle = \frac{1}{6.25} \langle 0 | \bar{q}(0)q(0) | 0 \rangle, \quad (17)$$

and

$$\begin{aligned} \rho_{\Lambda}^{nonlocal\ theory} &\approx \frac{1}{6} (3.6 \times 10^{-3} \text{ eV})^4 \\ &= (2.3 \times 10^{-3} \text{ eV})^4 \approx \rho_{\Lambda}^{observed} \end{aligned} \quad (18)$$

Therefore, using the modification of the quark condensate via the nonlocal condensate, one obtains excellent agreement between the theoretical and observed cosmological constants.

4. Summary and Concluding Remarks

The cosmological constant Λ is an important physical quantity, which was introduced by A. Einstein who mo-

dified the field equations of his general theory of relativity to obtain a stationary universe. The constant has recently been used to explain the observed accelerated expansion of the universe, but its observational value is about 120 orders of magnitude smaller than the one theoretically computed in the framework of the currently accepted quantum field theories. Namely, quantum field theory predicted that vacuum energy density, ρ_{Λ} , is of the order of M_{pl}^4 , with $M_{pl} = 1.22 \times 10^{19} \text{ GeV}$, which is about 120 orders of magnitude larger than the observed value of $\rho_{\Lambda}^{observed} = (2.3 \times 10^{-3} \text{ eV})^4$. This difference is the so called cosmological constant problem, the worst problem of fine-tuning in physics.

Based on the Veneziano ghost theory of QCD, using a local quark condensate, we obtained the same result for ρ_{Λ} as in Refs[11,12], about a factor of 6 larger than $\rho_{\Lambda}^{observed}$. However, $\langle 0 | \bar{q}(0)q(0) | 0 \rangle$ is just an approximation to $\langle 0 | \bar{q}(x)q(0) | 0 \rangle$. Using the nonlocal quark condensate $\langle 0 | \bar{q}(x)q(0) | 0 \rangle = g(x) \langle 0 | \bar{q}(0)q(0) | 0 \rangle$ we find that the theoretical and observed values of ρ_{Λ} are approximately equal.

The cosmological constant Λ is a potentially important contributor to the dynamical history of the universe. Unlike ordinary matter, which can clump together or disperse as it evolves, the vacuum energy is a property of spacetime itself, and is expected to be the same everywhere. If the cosmological constant is the valid model of dark energy, a sufficiently large cosmological constant will cause galaxies and supernovae to accelerate away from us, as has been observed, in contrast to the tendency of ordinary forms of energy to slow down the recession of distant objects. The value of Λ in our present universe is not well known. A precise determination of this constant will be one of the primary goals of both theoretical cosmology and observational cosmology in the near future.

One might doubt the correctness of the Veneziano QCD ghost theory that we used in this work, since it is an analogue of two-dimensional theory based on the Schwinger model [18,19], replacing the vector gauge field by two scalar fields. These scalar fields have positive and negative norms and cancel with each other, leaving no trace in the physical subspace. They have small contribution to the vacuum energy in the curved space. It is known that the QCD ghost must be an intrinsically vector field in order for the $U(1)$ problem to be consistently resolved within the framework of QCD. It seems to be necessary to examine if the Veneziano mechanism works in terms of the vector ghost fields instead of the scalar fields used here. However, Ohta and others in Refs. [36-38] have discussed the same problem in more realistic four dimensional models, and show that the QCD

ghost produces vacuum energy density ρ_Λ proportional to the Hubble parameter which has approximately the right magnitude $\sim (3 \times 10^{-3} \text{ eV})^4$.

There is now considerable evidence that the universe began as fireball in the cosmological vacuum, the so-called “Big Bang”, with extremely high temperature and high energy density. One knows that the quark condensate is vastly changed by the QCD phase transition, and this implies that there is a temperature (T) dependence of $\langle 0 | \bar{q}(x)q(0) | 0 \rangle$ and Λ . Λ is probably dependent on temperature T and momentum p of virtual particles which produce vacuum condensates, as mentioned above. We can predict the Λ dependence on temperature T and momentum p by solving the temperature dependent Dyson-Schwinger Equations. In this case, Λ is a function of T and p . Such a new study could show the behavior of the Λ during the evolution of the universe. This work is under its way and should be complete soon.

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