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Semileptonic Decay of B_c Meson into *S* Wave Charmonium in a QCD Potential Model with Coulombic Part as Perturbation

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ABSTRACT

We present the semileptonic decay of B_c meson in a QCD potential model with the coulombic part of the Cornell potential $-\frac{4\alpha_S}{3r}+br+c$ as perturbation. Computing the slope and curvature of Isgur-Wise function in this approach, we study the pseudoscalar and vector form factors for the transition of B_c meson to its *S* wave charmonium $c\bar{c}$ states. Numerical estimates of widths for the transitions of $B_c \to J/\psi(\eta_c) l\nu_l$ are presented.

Keywords: Dalgarno Method; Isgur-Wise Function; Form Factors; Decay Width

1. Introduction

The investigation of weak decays of mesons composed of a heavy quark and antiquark gives a very important insight in the heavy quark dynamics. The exclusive semileptonic decay processes of heavy mesons generated a great excitement not only in extracting the most accurate values of Cabbibo-Kobayashi Maskawa (CKM) matrix elements but also in testing diverse theoretical approaches to describe the internal structure of hadrons. The great virtue of semileptonic decay processes is that the effects of the strong interaction can be separated from the effects of the weak interaction into a set of Lorentz-invariant form factors, *i.e.*, the essential informations of the strongly interacting quark/gluon structure inside hadrons. Thus, the theoretical problem associated with analyzing semileptonic decay processes is essentially that of calculating the weak form factors.

The decay properties of the B_c meson are of special interest, since it is the only heavy meson consisting of two heavy quarks with different flavor. This difference of quark flavors forbids annihilation into gluons. As a result, the excited B_c meson states lying below the B_D meson threshold undergo pionic or radiative transitions to the pseudoscalar ground state which is considerably more stable than corresponding charmonium or bottomonium states and decays only weakly. The CDF Collaboration reported the discovery of the B_c ground state in $p\bar{p}$ collisions already more than ten years ago [1]. However, up till recently its mass was known with a very large error. Now it is measured with a good precision in the decay channel $B_c \to J/\psi\pi$. More experimental data on masses and decays of the B_c mesons are expected to come in near future from the Tevatron at Fermilab and the Large Hadron Collider (LHC) at CERN. The estimates of the B_c decay rates indicate that the c quark transitions give the dominant contribution while the b quark transitions and weak annihilation contribute less. However, from the experimental point of view the B_c decays to charmonium are easier to identify. Indeed, CDF and D0 observed the B_c meson and measured its mass analyzing its semileptonic and nonleptonic decays $B_c \to J/\psi l\nu$.

There are many theoretical approaches to the calculation of exclusive B_c semileptonic decay modes. Some of them are: QCD sum rules [2-4], the relativistic quark model [5-7] based on an effective Lagrangian describing the coupling of hadrons to their constituent quarks, the quasipotential approach to the relativistic quark model [8-10], the instantaneous nonrelativistic approach to the Bethe-Salpeter (BS) equation [11], the relativistic quark model based on the BS equation [12,13], the QCD relativistic potential model [14], the relativistic quark-meson model [15], the nonrelativistic quark model [16], the covariant light-front quark model [17], and the constituent quark model [18-21] using BSW (Bauer, Stech, and Wirbel) model [22] and ISGW (Isgur, Scora, Grinstein, and Wise) model [23]. The purpose of this paper is to extend a QCD potential model [24] with coulombic part as perturbation to calculate the hadronic form factors and decay widths for the exclusive semileptonic decay of B_c meson.

Recently, we have reported the slope and curvature of I-W function for D and B mesons with the coulombic part of the potential as the perturbation in two particular renormalisation schemes $\overline{MS}$ and V scheme [24]. Instead of using a particular renormalisation schemes, here in this manuscript we use the strong coupling constant as a scale dependent parameter and compute the slope and curvature of I-W function for B_c meson with a different set of mass input parameters than that of reference [24]. We then use the I-W function to study the form factors and decay rates of B_c meson into its S wave charmonium $c\bar{c}$ states within the framework of the potential model.

The rest of the paper is organised as follows: Section 2 contains the formalism with its subsections containing the model wavefunction, Masses, hadronic form factors and decay widths. In Section 3 we place our results and conclusions.

2. Formalism

2.1. The Wavefunction

The wavefunction computed by Dalgarno method [25,26] with coulombic part $-\frac{4\alpha_s}{3r}+c$ of the potential as perturbation and linear part br as parent has been reported in reference [24] and the alternate approach of choosing the linear part as perturbation has been reported earlier [27,28]. For the calculations on the hadronic matrix elements, only the long-distance behaviours of the wavefunctions are important or dominant, so that taking the Coulomb piece as a perturbation becomes legitimate. The wavefunction with linear part as parent becomes an Airy function, which in fact is a diverging function. So in this manuscript we consider the wave function upto the order r^2. For completeness we summarise the main equations in this section.

The total wave function corrected upto first order with normalisation is [24]

$$\psi_{coul}(r) = \psi^{(0)}(r) + \psi^{(1)}(r) \tag{1}$$

$$= \frac{N_1}{2\sqrt{\pi}}\left[\frac{Ai\left((2\mu b)^{\frac{1}{3}}+\rho_{01}\right)}{r} - \frac{4\alpha_s}{3}\left(\frac{a_0}{r}+a_1+a_2 r\right)\right] \tag{2}$$

where N_1 is the normalisation constant for the total wave function $\psi_{coul}(r)$ with subscript "coul" means coulombic potential as perturbation.

Where ρ_{0n} are given as [29,30]:

$$\rho_{0n} = -\left[\frac{3\pi(4n-1)}{8}\right]^{\frac{2}{3}} \tag{3}$$

$$a_0 = \frac{0.8808(b\mu)^{\frac{1}{3}}}{(E-c)} - \frac{a_2}{\mu(E-c)} + \frac{4W^1\times 0.21005}{3\alpha_s(E-c)} \tag{4}$$

$$a_1 = \frac{ba_0}{(E-c)} + \frac{4\times W^1\times 0.8808\times(b\mu)^{\frac{1}{3}}}{3\alpha_s(E-c)} - \frac{0.6535\times(b\mu)^{\frac{2}{3}}}{(E-c)} \tag{5}$$

$$a_2 = \frac{4\mu W^1\times 0.1183}{3\alpha_s} \tag{6}$$

$$W^{(1)} = \int_0^{+\infty} r^2 H' \left|\psi^{(0)}(r)\right|^2 \mathrm{d}r \tag{7}$$

and

$$E = -\left(\frac{b^2}{2\mu}\right)^{\frac{1}{3}}\rho_{0n} \tag{8}$$

here b and c are the model input parameters as is used in our previous works [24,27,28]. "n" is the principal quantum no. ($n = 1$ for ground state), μ is the reduced mass of mesons and α_s is the strong running coupling constant.

2.2. Pseudoscalar and Vector Form Factors

In the case of the final $c\bar{c}$ states corresponds to the $J=0$, η_c states, as the matrix element of any axial current A^μ between the two pseudoscalar mesons vanishes, only vector current V^μ contributes. Unlike in the case of electromagnetic current of the charged pions, here the vector current $V^\mu = \bar{c}\gamma^\mu b$ is not conserved as $q_\mu V^\mu \propto (m_b - m_c) \neq 0$. So the matrix element of the hadronic current, V^μ between the two $J^P = 0^-$ mesons is expressed in terms of two form factors $f_\pm(q^2)$ as

$$\begin{aligned}&\left\langle \eta_c(p')\middle|V^\mu\middle|B_c(p)\right\rangle \\ &= f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu\end{aligned} \tag{9}$$

where $q = p - p' = k_1 + k_2$ is the four momentum transfer and $f_+(q^2)$ and $f_-(q^2)$ are the dimensionless weak transition form factors corresponds to $B_c \to \eta_c$, which are functions of the invariant q^2. Here q^2 varies within the range

$$m_\ell^2 \le q^2 \le \left(m_{B_c} - m_{\eta_c}\right)^2 = q^2_{\max}.$$

The transition between the pseudoscalar B_c and the vector $J/\psi(p',\varepsilon)$ mesons depends on four independant form factors as,

$$\left\langle J/\psi\left(p',\varepsilon\right)\right|\bar{c}\gamma^{\mu}b\left|B_c\left(p\right)\right\rangle = 2i\varepsilon^{\mu\nu\alpha\beta}\frac{\varepsilon_{\nu}p'_{\alpha}p_{\beta}}{M_{B_c}+M_{J/\psi}}V\left(q^2\right) \tag{10}$$

$$\begin{aligned}&\left\langle J/\psi\left(p',\varepsilon\right)\right|\bar{c}\gamma^{\mu}\gamma_5 b\left|B_c\left(p\right)\right\rangle\\ &=\left(M_{B_c}+M_{J/\psi}\right)\left[\varepsilon^{\mu}-\frac{\varepsilon\cdot qq^{\mu}}{q^2}\right]A_1\left(q^2\right)\\ &-\varepsilon\cdot q\left[\frac{\left(p+p'\right)^{\mu}}{M_{B_c}+M_{J/\psi}}-\frac{\left(M_{B_c}-M_{J/\psi}\right)q^{\mu}}{q^2}\right]A_2\left(q^2\right)\\ &2M_{J/\psi}\frac{\varepsilon\cdot qq^{\mu}}{q^2}A_0\left(q^2\right)\end{aligned} \tag{11}$$

In the present study we treat B_c system similar to $D\left(c\bar{d}\right)$ system as the ratio of the constituent quark masses in the B_c meson is very close to that of D meson. So we extend the HQET of the $q\bar{Q}$ system for the study of B_c meson also. And on the basis of HQET, the most general form of the transition discussed by Equations (9) and (10) can be expressed as [31],

$$\begin{aligned}&\frac{1}{\sqrt{M_{B_c}M_{\eta_c}}}\left\langle\eta_c\left(v'\right)\right|V^{\mu}\left|B_c\left(v\right)\right\rangle\\ &=\left(v+v'\right)^{\mu}\xi\left(\omega\right)\end{aligned} \tag{12}$$

$$\begin{aligned}&\frac{1}{\sqrt{M_{B_c}M_{J/\psi}}}\left\langle J/\psi\left(v',\varepsilon_3\right)\right|V^{\mu}\left|B_c\left(v\right)\right\rangle\\ &=i\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\nu}v'_{\alpha}v_{\beta}\xi\left(\omega\right)\end{aligned} \tag{13}$$

$$\begin{aligned}&\frac{1}{\sqrt{M_{B_c}M_{J/\psi}}}\left\langle J/\psi\left(v',\varepsilon_3\right)\right|A^{\mu}\left|B_c\left(v\right)\right\rangle\\ &=\left[\left(1+\omega\right)\varepsilon^{\mu}-\left(\varepsilon\cdot v\right)v'^{\mu}\right]\xi\left(\omega\right)\end{aligned} \tag{14}$$

where $\xi\left(\omega\right)$ is the Isgur-Wise function. For small, nonzero recoil, it is conventional to write the Isgur-Wise function as [32]:

$$\xi\left(v.v'\right)=\xi\left(\omega\right)=1-\rho^2\left(\omega-1\right)+C\left(\omega-1\right)^2+\cdots \tag{15}$$

where ω is given by,

$$w=v\cdot v'=\frac{\left[m_{B_c}^2+m_{c\bar{c}}^2-q^2\right]}{2m_{B_c}m_{c\bar{c}}} \tag{16}$$

The quantity ρ^2 is the slope of I-W function at $\omega=1$ and known as charge radius:

$$\rho^2=\left.\frac{\partial\xi}{\partial\omega}\right|_{\omega=1} \tag{17}$$

The second order derivative is the curvature of the I-W function known as convexity parameter:

$$C=\frac{1}{2}\left.\left(\frac{\partial^2\xi}{\partial\omega^2}\right)\right|_{\omega=1} \tag{18}$$

For the heavy-light flavor mesons the I-W function can also be written as [27,33]:

$$\xi\left(\omega\right)=\int_0^{+\infty}4\pi r^2\left|\psi\left(r\right)\right|^2\cos pr\,\mathrm{d}r \tag{19}$$

where

$$p^2=2\mu^2\left(\omega-1\right). \tag{20}$$

In Equation (1), the strong coupling constant connected to the potential is a function of the momentum as

$$\alpha_s\left(\mu_1^2\right)=\frac{4\pi}{\left(11-\frac{2n_f}{3}\right)\ln\left(\frac{\mu_1^2}{\Lambda^2}\right)} \tag{21}$$

where n_f is the number of flavour and μ_1 is the renormlistion scale related to the constituent quark mass and Λ is the QCD scale which is taken as 0.150 GeV by fixing $\alpha_s=0.118$ at the Z boson mass (91 GeV). We use the most common renormalisation scale as

$$\mu_1=4\frac{m_i m_j}{m_i+m_j}$$

with $n_f=4$ [34] and then evaluate α_s. For numerical calculation, we use the model input parameters as, $m_c=1.55$ GeV and $m_b=4.79$.

Using Equations (1), (15) and (19), we compute the slope and curvature of I-W function as

$$\rho_{B_c}^2=0.67 \quad \text{and} \quad C_{B_c}=0.06$$

Consequently, the form factors $f_{\pm}\left(q^2\right)$ correspond to the $c\bar{c}\left(\eta_c\right)$ final state are related to the Isgur-Wise function as [31]

$$f_{\pm}\left(q^2\right)=\xi\left(\omega\right)\frac{m_{B_c}\pm m_{\eta_c}}{2\sqrt{m_{B_c}m_{\eta_c}}} \tag{22}$$

and those related to the J/ψ as the final hadronic state are given by

$$\begin{aligned}V\left(q^2\right)&=A_2\left(q^2\right)=A_0\left(q^2\right)\\ &=\left[1-\frac{q^2}{\left(M_{B_c}+M_{J/\psi}\right)^2}\right]^{-1}A_1\left(q^2\right)\\ &=\frac{\left(M_{B_c}+M_{J/\psi}\right)^2}{4M_{B_c}M_{J\psi}}\xi\left(\omega\right)\end{aligned} \tag{23}$$

It is evident from Equation (23) that at $q^2\to0$

$$V\left(q^2\right)=A_2\left(q^2\right)=A_1\left(q^2\right)=A_0\left(q^2\right) \tag{24}$$

Thus knowing the masses and Isgur-Wise function of the transition $B_c \to c\bar{c}\left(\eta_c, J/\psi\right)\ell^+\nu_\ell$, we will be able to compute respective form factors correspond to $B_c \to c\bar{c}\left(\eta_c, J/\psi\right)\ell^+\nu_\ell$ transitions.

It should be noted that by virtue of transversality of the lepton current $l_\mu = l\gamma_\mu\left(1+\gamma_5\right)\nu l$ in the limit $m_l \to 0$, the probabilities of semileptonic decays into $e^+\nu_e$ and $\mu^+\nu_\mu$ are independent of f_-. Thus, in calculation of these particular decay modes of B_c meson this form factor can be consistently neglected [3].

The differential semileptonic decay rates can be expressed in terms of these form factors by

1) $B_c \to Pev$ decay ($P = \eta_c$)

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\left(B_c \to Pev\right) = \frac{G_F^2\Delta^3\left|V_{qb}\right|^2}{24\pi^3}\left|f_+\left(q^2\right)\right|^2. \tag{25}$$

2) $B_c \to Vev$ decay ($V = J/\psi$) The decays rate in transversally and longitudinally polarized vector mesons are defined by [35]

$$\frac{\mathrm{d}\Gamma_L}{\mathrm{d}q^2} = \frac{G_F^2\Delta\left|V_{qb}\right|^2}{96\pi^3}\frac{q^2}{M_B^2}\left|H_0\left(q^2\right)\right|^2, \tag{26}$$

$$\begin{aligned}\frac{\mathrm{d}\Gamma_T}{\mathrm{d}q^2} &= \frac{\mathrm{d}\Gamma_+}{\mathrm{d}q^2} + \frac{\mathrm{d}\Gamma_-}{\mathrm{d}q^2}\\ &= \frac{G_F^2\Delta\left|V_{qb}\right|^2}{96\pi^3}\frac{q^2}{M_B^2}\left(\left|H_+\left(q^2\right)\right|^2 + \left(H_-\left(q^2\right)\right)^2\right).\end{aligned} \tag{27}$$

where helicity amplitudes are given by the following expressions

$$H_\pm\left(q^2\right) = \frac{2M_{B_c}\Delta}{M_{B_c}+M_V}\left[V\left(q^2\right) \mp \frac{\left(M_{B_c}+M_V\right)^2}{2M_{B_c}\Delta}A_1\left(q^2\right)\right], \tag{28}$$

$$\begin{aligned}&H_0\left(q^2\right)\\ &= \frac{1}{2M_V\sqrt{q^2}}\Bigg[\left(M_{B_c}+M_V\right)\left(M_{B_c}^2 - M_V^2 - q^2\right)A_1\left(q^2\right)\\ &\qquad - \frac{4M_B^2\Delta^2}{M_{B_c}+M_V}A_2\left(q^2\right)\Bigg]\end{aligned} \tag{29}$$

Thus the total decay rate is given by

$$\begin{aligned}&\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\left(B_c \to Vev\right)\\ &= \frac{G_F^2\Delta\left|V_{qb}\right|^2}{96\pi^3}\frac{q^2}{M_{B_c}^2}\left(\left|H_+\left(q^2\right)\right|^2 + \left|H_-\left(q^2\right)\right|^2 + \left|H_0\left(q^2\right)\right|^2\right),\end{aligned} \tag{30}$$

where G_F is the Fermi constant, V_{qb} is CKM matrix element ($q = c$),

$$\Delta \equiv \left|\Delta\right| = \sqrt{\frac{\left(M_{B_c}^2 + M_{P,V}^2 - q^2\right)^2}{4M_{B_c}^2} - M_{P,V}^2}.$$

Integration over q^2 of these formulas gives the total rate of the corresponding semileptonic decay. The computed results for the form factors and the total decay rate is shown in **Tables 1-3** respectively and compared with the other theoretical values.

Table 1. Form factors for $B_c \to \eta_c\ell^+\nu_\ell$.

Work	$f_+(0)$	$f_-(0)$	$f_+\left(q^2_{\max}\right)$	$f_-\left(q^2_{\max}\right)$
Our Work	0.875	−0.311	1.070	−0.381
Faustov [35]	0.47		1.07	
Choi and Ji [36]	0.546		1.035	

Table 2. Form factors for $B_c \to J/\psi\ell^+\nu_\ell$.

Work	$V(0)$	$A_1\left(q^2_{\max}\right)$	$V\left(q^2_{\max}\right)$
Our Work	0.91	0.90	1.063
Faustov [35]	0.49	0.88	1.34

Table 3. Decay width for $B_c \to c\bar{c}\left(\ell^+\nu_\ell\right)$ in 10^{-15} GeV.

	$B_c \to \eta_c\left(\ell^+\nu_\ell\right)$	$B_c \to J/\psi\left(\ell^+\nu_\ell\right)$
	12.7	38.4
[6]	10.7	28.2
[9]	5.9	17.7
[36]	14.2	34.4
[13]	11.1	30.2
[37]	11 ± 1	28 ± 5
[38]	10	42

3. Results and Discussion

In this paper, we have computed the slope and curvature of Isgur-Wise function for B_c meson, considering the coulombic part of the cornell potential as perturbation and compute the different formfactors. We summarise and comment the result in the present work as follows.

1) We relate the form factors to the Isgur-Wise function in equations [22,23] which are based on the heavy flavour symmetry and is broken in the case of mesons containing two heavy quarks [39]. Spin symmetry breaking effects can occur when the c-quarks recoil momentum is larger than m_c. However we expects that the equations are applicable to other kinematic point since the recoil momentum $c\bar{c}$ state is small ($y_{\max} - 1 = 0.26$) due to its heavy mass [40]. We nave computed the hadronic form factors of B_c meson using HQET in analogy

to D meson system and are compared with other available results which are shown in **Tables 1** and **2**.

2) The values of ρ^2 and C are found to be smaller than the other theoretical values for D and B mesons. The reason is presumably due to the truncating of higher terms in the Airy function (which is an infinite series).

3) Numerical solution of Schrodinger equation with the specific potential also gives more accurate result than the present one. However, such approach appears to lack of physical insight into the problem unlike the relatively crude potential model approach pursued here.

4) The computed decay width for $B_c \to \eta_c\left(\ell^+\nu_\ell\right)$ in this approach is found to be in good agreement with other theoretical results but the corresponding one for $B_c \to J/\psi\left(\ell^+\nu_\ell\right)$ overshoots the other results and is shown in **Table 3**. The branching ratio obtained for this semileptonic transition becomes $\approx 2.3\times10^{-2}$. This value is larger than the inclusive branching ratio reported in the PDG 2011 [41] $\left[B_c \to J/\psi\left(\ell^+\nu_\ell + \text{anything}\right) = 5.2\times10^{-5}\right]$, which in principle includes the exclusive channel under consideration.

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