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Totally Anisotropic Cosmological Models with Bulk Viscosity for Variable G and Λ

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ABSTRACT

Einstein's field equations with variable gravitational and cosmological constants are considered in the presence of bulk viscous fluid for the totally anisotropic Bianchi type II space-time in such a way as to preserve the energy momentum tensor. We have presented solutions of field equations which represent expanding, shearing and non-rotating cosmological models of the universe. The physical behaviours of the models are discussed. We observe that the results obtained match with recent observations of SNIa.

1. Introduction

The simplest model of the observed universe is well represented by Friedmann-Robertson-Walker (FRW) models, which are both spatially homogeneous and isotropic. These models in some sense are good global approximation of the present-day universe. But on smaller scales, the universe is neither homogeneous and isotropic nor do we expect the universe in its early stages to have these properties. At very early times in the evolution of the universe, most of the radiations and matter currently observed are believed to have been created during the inflation. Modern cosmology is concerned with nothing less than a thorough understanding and explanation of the past history, the present state and the future evolution of the universe. In fact, these are theoretical arguments from the recent experimental data which support the existence of an anisotropic phase approaching to isotropic phase leading to consider the models of the universe with anisotropic background. Spatially homogeneous and anisotropic cosmological models play significant roles in the description of large-scale behaviours of the universe. Bianchi spaces I-IX play important roles in constructing models of spatially homogeneous and anisotropic cosmologies. Here we confine ourselves to totally anisotropic space-time of Bianchi type II space-time which have fundamental role in constructing cosmological models suitable for describing the early evolution of the universe. Much attention has been focused towards the study of locally rotationally symmetric (LRS) Bianchi type II space-times. Guzman [1] obtained the general vacuum solution of Brans-Dicke field equations for the totally anisotropic Bianchi type II space-time. Singh and Shri Ram [2] presented totally anisotropic Bianchi type II cosmological models in scalar tensor-theories of gravitation developed by Saez-Ballester [3], Lau and Prokhovnik [4]. Singh *et al.* [5] obtained exact solutions of Einstein's field equations in vacuum and in the presence of stiff matter for the totally anisotropic Bianchi type II space-time in normal gauge for Lyra's geometry when the gauge function is time-dependent. Recently, Yadav and Haque [6] obtained a spatially homogeneous and totally anisotropic Bianchi type II cosmological model representing massive string in normal gauge for Lyra's manifold.

At the early stages of the universe when neutrinos decoupling occurred, the matter behaved like a viscous fluid. The coefficient of viscosity decreases as the universe expands. Misner [7,8] studied the effect of viscosity on the evolution of the universe and suggested that the strong dissipation, due to the neutrino viscosity, may considerably reduce the anisotropy of the black body radiation. Murphy [9] developed a uniform cosmological model filled with fluid which possesses pressure and bulk viscosity exhibiting the interesting feature that the big-bang type singularity appears in the infinite past. Grøn [10], Dunn and Tupper [11], Coley and Tupper [12], Banerjee and Santos [13,14] etc. constructed and discussed cosmological models under the influence of both bulk and shear viscosities. Padmanabhan and Chitre [15] investigated the effect of bulk viscosity on the evolution of the universe at large.

The cosmological constant problem is one of the outstanding problems in cosmology. In recent years there has

been a lot of interests in the study of the role of cosmological constant Λ at very early and the later stages of the evolution of the universe. A wide range of observations suggest that the universe possesses a non-zero cosmological constant. The Λ term has been interpreted in terms of the Higgs scalar field by Bergmann [16]. Drietlein [17] suggested that the mass of Higgs boson is connected with Λ being a function of temperature and is related to the process of broken symmetries, and therefore it could be a function of time in a spatially homogeneous expanding universe. In quantum field theory, the cosmological constant is considered as the vacuum energy density. The general speculation is that the universe might have been created from an excited vacuum fluctuation (absence of inflationary scenario) followed by super cooling and reheating subsequently due to the vacuum energy.

Dirac [18] first introduced the idea of a variable G what he called Large Number Hypothesis and since then various works have been carried out for a modified general relativity theory with this variation in G. A number of authors such as Beesham [19,20], Berman [21], Kalligas *et al.* [22], Abdussattar and Vishwakarma [23] proposed the linking of variation of G and Λ within the frameworks of general relativity and studied several models with the Friedmann-Robertson-Walker (FRW) metric. This approach is appealing since it leaves the form of Einstein equations formally unchanged by allowing a variation of G to be accompanied by a change in Λ. Arbab [24,25] and Singh *et al.* [26] have considered cosmological models with viscous fluid considering variable cosmological and gravitational constants. Singh *et al.* [27] presented a number of classes of solutions of Einstein's field equations with variable G, Λ and bulk viscosity coefficient in the framework of non causal theory. Several authors investigated anisotropic bulk viscous fluid cosmological models of various Bianchi types time-dependent G and Λ (see Pradhan and Kumhar [28], Verma and Shri Ram [29,30] and references cited therein).

Bali and Tinker [31] investigated bulk viscous fluid flow for Bianchi type III space-time model with variable G and Λ, and obtained solutions of the field equations under certain physical and mathematical conditions. Motivated by this work, we present totally anisotropic Bianchi type-II bulk viscous barotropic cosmological models with variable G and Λ by making the following assumptions: 1) the conditions between the metric potentials A, B, C as $\dfrac{\dot{A}}{A} = \dfrac{m_1}{t^n}, \dfrac{\dot{B}}{B} = \dfrac{m_2}{t^n}, \dfrac{\dot{C}}{C} = \dfrac{m_3}{t^n}$; 2) the matter energy density and isotropic pressure satisfy the equation of state $p = \gamma\rho, 0 \le \gamma \le 1$; 3) the coefficient of bulk viscosity $\eta = \eta_0 \rho^\beta$ where η_0 and β are constants. We present the metric and field equations in Section 2. In Section 3, we deal with the solutions of the field equations and obtain two classes of solutions for $n \ne 1$ and $n = 1$.

We also discuss the physical features of the cosmological models. Some concluding remarks are given in Section 4.

2. Field Equations and General Expressions

We consider the totally anisotropic Bianchi type-II metric in the form

$$ds^2 = -dt^2 + A^2 \left(dx - zdy\right)^2 + B^2 dy^2 + C^2 dz^2 \qquad (1)$$

where the metric potentials A, B and C are functions of cosmic time t. Einstein's field equations with time-dependent cosmological and gravitational constants are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} + \Lambda g_{ij}. \qquad (2)$$

The energy-momentum tensor T_{ij} for a bulk viscous fluid distribution is given by

$$T_{ij} = \left(\rho + \overline{p}\right) v_i v_j + \overline{p} g_{ij} \qquad (3)$$

where $\overline{p} = p - \eta v^i {}_{;i}$ is the effective pressure, η is the coefficient of bulk viscosity, p is isotropic pressure, ρ is the energy density and v^i is fluid four-velocity vector satisfying $v_i v^i = -1$.

In commoving coordinates, Einstein's field Equation (2) for the metric (1) are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{3}{4} \frac{A^2}{B^2 C^2} = -8\pi G \overline{p} + \Lambda, \qquad (4)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{1}{4} \frac{A^2}{B^2 C^2} = -8\pi G \overline{p} + \Lambda, \qquad (5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4} \frac{A^2}{B^2 C^2} = -8\pi G \overline{p} + \Lambda, \qquad (6)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{4} \frac{A^2}{B^2 C^2} = 8\pi G \rho + \Lambda \qquad (7)$$

where the overdot denotes differentiation with respect to time t. Moreover, an additional equation for time changes of G and Λ is obtained by taking the divergence of Einstein tensor *i.e.*

$$\left(R_i^j - \frac{1}{2} R g_i^j\right); \; j = 0 \qquad (8)$$

which leads to

$$\left(8\pi G T_i^j - \Lambda g_i^j\right); \; j = 0. \qquad (9)$$

A semicolon denotes covariant differentiation. Equation (9) readily yields

$$\dot{\rho} + \left(\rho + \overline{p}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right). \qquad (10)$$

The conservation equation for energy-momentum $T_i^j; \; j = 0$ gives

$$\dot{\rho} + \left(\rho + \overline{p}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \qquad (11)$$

Using Equation (11), Equation (10) splits into the following equations

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \tag{12}$$

$$\dot{\Lambda} + 8\pi\dot{G}\rho = 8\pi G\eta\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)^2. \tag{13}$$

3. Solutions of Field Equations

Here we have four independent field equations containing eight unknowns viz. A, B, C, p, G, ρ, η, Λ. So we shall assume extra conditions to obtain unique solutions of the field equations.

In most of the investigations in cosmology, the bulk viscosity is assumed to be a simple power function of the energy density *i.e.*

$$\eta = \eta_0\rho^\beta \tag{14}$$

where η_0 and β are constants. Murphy [9] assumed $\beta = 1$ in the case of small density which corresponds to a radiative fluid. We also assume that the fluid obeys the barotropic equation of state

$$p = \gamma\rho, \ 0 \leq \gamma \leq 1. \tag{15}$$

3.1. Model I

We assume that solutions of the scale factors of the forms

$$\frac{\dot{A}}{A} = \frac{m_1}{t^n}, \frac{\dot{B}}{B} = \frac{m_2}{t^n}, \frac{\dot{C}}{C} = \frac{m_3}{t^n} \tag{16}$$

where *n* is a positive constant. On integration of Equation (16), we obtain

$$A = a\exp\left\{\frac{m_1 t^{1-n}}{1-n}\right\}, \tag{17}$$

$$B = b\exp\left\{\frac{m_2 t^{1-n}}{1-n}\right\}, \tag{18}$$

$$C = c\exp\left\{\frac{m_3 t^{1-n}}{1-n}\right\} \tag{19}$$

where *a*, *b*, *c* are constants of integration and $n \neq 1$.

Using Equations (15)-(16) into Equation (12), we obtain

$$\dot{\rho} + (1+\gamma)\left(\frac{m_1 + m_2 + m_3}{t^n}\right)\rho = 0. \tag{20}$$

Integration of Equation (20) yields

$$\rho = d\exp\left\{\frac{-(1+\gamma)(m_1 + m_2 + m_3)}{1-n}t^{1-n}\right\} \tag{21}$$

where *d* is a constant of integration. Differentiation of Equation (21) gives

$$\dot{\rho} = -d\frac{(1+\gamma)(m_1 + m_2 + m_3)}{t^n}$$
$$\times \exp\left\{\frac{-(1+\gamma)(m_1 + m_2 + m_3)}{1-n}t^{1-n}\right\} \tag{22}$$

Now using Equations (16)-(19) into Equation (7), we obtain

$$8\pi G\rho + \Lambda = \frac{(m_1 m_2 + m_2 m_3 + m_3 m_1)}{t^{2n}} - \frac{1}{4}\frac{a^2}{b^2 c^2}$$
$$\times \exp\left\{\frac{2(m_1 - m_2 - m_3)}{1-n}t^{1-n}\right\}. \tag{23}$$

Differentiation of (23) gives

$$8\pi\dot{G}\rho + 8\pi G\dot{\rho} + \dot{\Lambda}$$
$$= \frac{-2n(m_1 m_2 + m_2 m_3 + m_3 m_1)}{t^{2n+1}} - \frac{1}{4}\frac{a^2}{b^2 c^2}$$
$$\times \frac{2(m_1 - m_2 - m_3)}{t^n}\exp\left\{\frac{2(m_1 - m_2 - m_3)}{1-n}t^{1-n}\right\} \tag{24}$$

Substituting Equations (13) and (16) into Equation (24), we have

$$8\pi G\dot{\rho} + 8\pi G\eta\frac{(m_1 + m_2 + m_3)^2}{t^{2n}}$$
$$= \frac{-2n(m_1 m_2 + m_2 m_3 + m_3 m_1)}{t^{2n+1}}$$
$$- \frac{1}{4}\frac{a^2}{b^2 c^2}\frac{2(m_1 - m_2 - m_3)}{t^n}\exp\left\{\frac{2(m_1 - m_2 - m_3)}{1-n}t^{1-n}\right\} \tag{25}$$

Using Equations (14) and (22) into Equation (25), we find that

$$G = \left[\frac{n(m_1 m_2 + m_2 m_3 + m_3 m_1)}{t^{n+1}} + \frac{1}{4}\frac{a^2(m_1 - m_2 - m_3)}{b^2 c^2}\exp\left\{\frac{2(m_1 - m_2 - m_3)}{1-n}t^{1-n}\right\}\right]\exp\left\{\frac{(1+\gamma)(m_1 + m_2 + m_3)}{1-n}t^{1-n}\right\}$$

$$\left[4\pi d(1+\gamma)(m_1 + m_2 + m_3) - \frac{4\pi\eta_0 d^\beta}{t^n}(m_1 + m_2 + m_3)^2\exp\left\{\frac{(1-\beta)(1+\gamma)(m_1 + m_2 + m_3)}{1-n}t^{1-n}\right\}\right]^{-1}. \tag{26}$$

Again, from Equations (21), (23) and (26), we obtain the value of Λ as given in Equation (27).

The Gravitational constant G is zero at $t = 0$ and gradually increases and tends to infinity at late times. The cosmological term Λ is infinite at $t = 0$ and becomes zero as $t \to \infty$.

The scalar expansion (θ) and shear scalar (σ) are given by

$$\theta = \frac{m_1 + m_2 + m_3}{t^n}, \qquad (28)$$

$$\sigma^2 = \frac{\left(m_1^2 + m_2^2 + m_3^2\right) - \left(m_1 m_2 + m_2 m_3 + m_3 m_1\right)}{3t^{2n}}. \qquad (29)$$

The coefficient of bulk viscosity has the value given by

$$\eta = \eta_0 \, d^\beta \exp\left\{\frac{-\beta(1+\gamma)(m_1 + m_2 + m_3)}{1-n} t^{1-n}\right\} \qquad (30)$$

An important observational quantity is the deceleration parameter q which is defined as

$$q = -\frac{V\ddot{V}}{\dot{V}^2} \qquad (31)$$

where $V^3 = ABC$. The sign of q indicates whether the model inflates or not. The positive sign corresponds to standard decelerating model whereas negative sign indicates inflation. For the present solutions of A, B and C, the decelerating parameter has the value given by

$$q = -1 + \frac{3nt^{n-1}}{\left(m_1 + m_2 + m_3\right)}. \qquad (32)$$

Clearly q is positive for $t > \left(\dfrac{m_1 + m_2 + m_3}{3n}\right)^{1/(n-1)}$ and is negative for $t < \left(\dfrac{m_1 + m_2 + m_3}{3n}\right)^{1/(n-1)}$. The deceleration parameter indeed has a sign flip at $t = \left(\dfrac{m_1 + m_2 + m_3}{3n}\right)^{1/(n-1)}$. For $t < \left(\dfrac{m_1 + m_2 + m_3}{3n}\right)^{1/(n-1)}$, the solution gives an accelerating model of the universe. when

$t > \left(\dfrac{m_1 + m_2 + m_3}{3n}\right)^{1/(n-1)}$, our solution represents a decelerating model of the universe.

The spatial volume V of the model has the value given by

$$V = \left(abc\right)^{1/3} \exp\left\{\frac{(m_1 + m_2 + m_3)}{3(1-n)} t^{1-n}\right\}. \qquad (33)$$

We observe that the spatial volume is constant at $t = 0$. At this epoch the energy density ρ is finite and θ, σ are zero. For $0 < t < \infty$, the physical parameters ρ, p, θ, σ and η are well behaved and are decreasing functions of time. As $t \to \infty$, the spatial volume tends to infinity if $n < 1$ and the physical parameters tend to zero. Thus, for physical reality of the model, we must have $0 < n < 1$. The model essentially gives an empty space-time for large time. We also find that $\dfrac{\sigma}{\theta}$ tends to a constant limit as $t \to \infty$, which shows that the anisotropy in the universe is maintained throughout. Since $\eta = \eta_0 \rho^\beta$ and $\beta > 0$, the model leads to the inflationary phase of the universe [32].

3.2. Model II

We now obtain solution of the field Equations (4)-(7) for $n = 1$. For $n = 1$, the scale factors in Equation (16) are given by

$$\frac{\dot{A}}{A} = \frac{m_1}{t}, \quad \frac{\dot{B}}{B} = \frac{m_2}{t}, \quad \frac{\dot{C}}{C} = \frac{m_3}{t} \qquad (34)$$

which, on integration, gives

$$A = k_1 t^{m_1}, \quad B = k_2 t^{m_2}, \quad C = k_3 t^{m_3} \qquad (35)$$

where k_1, k_2, k_3 are constants of integration.

Substituting Equations (15) and (35) into Equation (12), we obtain

$$\dot{\rho} + (1+\gamma)\left(\frac{m_1 + m_2 + m_3}{t}\right)\rho = 0 \qquad (36)$$

which, on integration, leads to

$$\rho = Mt^{-(1+\gamma)(m_1 + m_2 + m_3)} \qquad (37)$$

$$\Lambda = \frac{(m_1 m_2 + m_2 m_3 + m_3 m_1)}{t^{2n}} - \frac{1}{4}\frac{a^2}{b^2 c^2}\exp\left\{\frac{2(m_1 - m_2 - m_3)}{1-n}t^{1-n}\right\} - \left[8\pi d\exp\left\{\frac{-(1+\gamma)(m_1 + m_2 + m_3)}{1-n}t^{1-n}\right\}\right]$$

$$\times \left[\frac{n(m_1 m_2 + m_2 m_3 + m_3 m_1)}{t^{n+1}} + \frac{1}{4}\frac{a^2(m_1 - m_2 - m_3)}{b^2 c^2}\exp\left\{\frac{2(m_1 - m_2 - m_3)}{1-n}t^{1-n}\right\}\right]\exp\left\{\frac{(1+\gamma)(m_1 + m_2 + m_3)}{1-n}t^{1-n}\right\}$$

$$\times \left[4\pi d(1+\gamma)(m_1 + m_2 + m_3) - \frac{4\pi\eta_0 d^\beta}{t^n}(m_1 + m_2 + m_3)^2\exp\left\{\frac{(\beta-1)(1+\gamma)(m_1 + m_2 + m_3)}{1-n}t^{1-n}\right\}\right]^{-1}. \qquad (27)$$

where M is a constant of integration.

The coefficient of bulk viscosity has the value given by

$$\eta = \eta_0 M^\beta t^{-\beta(1+\gamma)(m_1+m_2+m_3)}. \tag{38}$$

The effect of bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. The effect is clearly visible in isotropic pressure and energy density.

Using Equations (34) and (35) into Equation (7), we have

$$8\pi G\rho + \Lambda = \frac{(m_1 m_2 + m_2 m_3 + m_3 m_1)}{t^2} - \frac{1}{4}\frac{k_1^2}{k_2^2 k_3^2}t^{2(m_1-m_2-m_3)} \tag{39}$$

Equation (39), on differentiation, yields

$$8\pi\dot{G}\rho + 8\pi G\dot{\rho} + \dot{\Lambda} = \frac{-2(m_1 m_2 + m_2 m_3 + m_3 m_1)}{t^3}$$
$$-\frac{1}{2}\frac{k_1^2(m_1-m_2-m_3)}{k_2^2 k_3^2}t^{2(m_1-m_2-m_3)-1} \tag{40}$$

Combining Equations (13), (34) (37), (38) and (40), we obtain Equation (41).

Substituting for G and ρ in Equation (11), we obtain Equation (42).

The expansion (θ) and shear scalar (σ) have values given by

$$\theta = \frac{(m_1+m_2+m_3)}{t}, \tag{43}$$

$$\sigma^2 = \frac{(m_1^2+m_2^2+m_3^2)-(m_1 m_2+m_2 m_3+m_3 m_1)}{3t^2}. \tag{44}$$

We observe that the gravitational constant G is zero at

$t=0$ and gradually increases and tends to infinite as $t \to \infty$. We also see that the cosmological term Λ is infinite at $t=0$ and a decreasing function of time, and it approaches a small positive value at late time which is supported by recent results from the observations of the type Ia supernova explosion (SNIa). Naturally a cosmological model is required to explain acceleration in the present universe. Thus, this model is consistent with the results of recent observations.

The deceleration parameter q has the value given by

$$q = -1 + \frac{3}{(m_1+m_2+m_3)}. \tag{45}$$

From Equation (45), we observe that

$$q < 0 \quad \text{if} \quad m_1+m_2+m_3 > 3$$

and

$$q > 0 \quad \text{if} \quad m_1+m_2+m_3 < 3.$$

Thus, our solution represents an accelerating model of the universe if $(m_1+m_2+m_3) > 3$ and decelerating model if $(m_1+m_2+m_3) < 3$.

The spatial volume V of the model is given by

$$V^3 = (k_1 k_2 k_3)t^{(m_1+m_2+m_3)} \tag{46}$$

which is zero at $t=0$. At $t=0$ the energy density ρ, expansion θ and shear scalar σ all are infinite. Thus, the model starts with a big-bang singularity at $t=0$. The above parameters decrease with passage of time. The spatial volume increases as time increases and becomes infinite at late time. As $t \to \infty$ ρ, p, θ, σ and η tend to zero. Thus, the model represents an expanding shearing and non-rotating universe which essentially gives an empty space for large time. We also find that $\frac{\sigma}{\theta}$ does not

$$G = \left[\frac{(m_1 m_2+m_2 m_3+m_3 m_1)}{t^2} + \frac{1}{4}\frac{k_1^2(m_1-m_2-m_3)}{k_2^2 k_3^2}t^{(m_1-m_2-m_3)}\right] \times \left[t^{(1+\gamma)(m_1+m_2+m_3)}\right]$$
$$\left[\{4\pi M(m_1+m_2+m_3)\}\left\{(1+\gamma) - \frac{\eta_0 M^{\beta-1}(m_1+m_2+m_3)}{t}t^{-(\beta-1)(1+\gamma)(m_1+m_2+m_3)}\right\}\right]^{-1} \tag{41}$$

$$\Lambda = \left[\frac{(m_1 m_2+m_2 m_3+m_3 m_1)}{t^2} - \frac{1}{4}\frac{k_1^2}{k_2^2 k_3^2}t^{2(m_1-m_2-m_3)}\right]$$
$$-8\pi M t^{-(1+\gamma)(m_1-m_2-m_3)}\left[\frac{(m_1 m_2+m_2 m_3+m_3 m_1)}{t^2} + \frac{1}{4}\frac{k_1^2(m_1-m_2-m_3)}{k_2^2 k_3^2}t^{(m_1-m_2-m_3)}\right]$$
$$\times\left[t^{(1+\gamma)(m_1+m_2+m_3)}\right]\left[\{4\pi M(m_1+m_2+m_3)\}\left\{(1+\gamma) - \frac{\eta_0 M^{\beta-1}(m_1+m_2+m_3)}{t}t^{-(\beta-1)(1+\gamma)(m_1+m_2+m_3)}\right\}\right]^{-1}. \tag{42}$$

tend to zero as $t \to \infty$. Therefore, the anisotropy in the model is maintained throughout.

4. Conclusion

In this paper we have studied totally anisotropic Bianchi type-II bulk viscous fluid cosmological models with time-dependent gravitational and cosmological constants. We have presented two classes of physically viable cosmological models for $n \neq 1$ and $n = 1$. We have obtained expressions for physical parameter ρ, p, η, G and Λ as functions of time t. For $n \neq 1$, the model evolves with a finite volume at $t = 0$ and does not approach isotropy as $t \to \infty$. For large time, the energy density becomes zero.

The model is accelerating for $t < \left(\dfrac{m_1 + m_2 + m_3}{3n} \right)^{1/(n-1)}$

and is decelerating for $t > \left(\dfrac{m_1 + m_2 + m_3}{3n} \right)^{1/(n-1)}$. For

$n = 1$, the model starts evolving with a big-bang singularity at $t = 0$. This model represents an accelerating or decelerating universe according as $t = m_1 + m_2 + m_3$ is greater than 3 or less than 3. The anisotropy is maintained throughout in the model. The cosmological term is infinite initially and approaches to zero at late time. The gravitational constant G is zero initially and gradually increases and tends to infinity at late time. These are supported by recent results from the observations of the type Ia supernova explosion (SNIa).

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