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Holographic Principle and Large Scale Structure in the Universe

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Abstract

A reasonable representation of large scale structure, in a closed universe so large it's nearly flat, can be developed by extending the holographic principle and assuming the bits of information describing the distribution of matter density in the universe remain in thermal equilibrium with the cosmic microwave background radiation. The analysis identifies three levels of self-similar large scale structure, corresponding to superclusters, galaxies, and star clusters, between today's observable universe and stellar systems. The self-similarity arises because, according to the virial theorem, the average gravitational potential energy per unit volume in each structural level is the same and depends only on the gravitational constant. The analysis indicates stellar systems first formed at $z \approx 62$, consistent with the findings of Naoz *et al.*, and self-similar large scale structures began to appear at redshift $z \approx 4$. It outlines general features of development of self-similar large scale structures at redshift $z < 4$. The analysis is consistent with observations for angular momentum of large scale structures as a function of mass, and average speed of substructures within large scale structures. The analysis also indicates relaxation times for star clusters are generally less than the age of the universe and relaxation times for more massive structures are greater than the age of the universe.

Keywords: Holographic Principle, Large Scale Structure, Self-Similarity

1. Introduction

Formation of large scale structure in the universe is an important problem in cosmology [1], and the heuristic Press-Schechter excursion set model has been considered the only viable analytic approach to formation of large scale structure [2]. In contrast, this analysis extends the holographic principle [3] to consider formation of large scale structures, and stellar systems comprising those structures, in a closed Friedmann universe so large it's nearly flat. That may be a reasonable approximation to our universe.

In this analysis, $\rho_r(z)$ is the cosmic microwave background (CMB) radiation density at redshift z, where $\rho_r(z) = (1+z)^4 \rho_r(0)$ and the mass equivalent of today's radiation energy density $\rho_r(0) = 4.4 \times 10^{-34}$ g/cm^3 [4]. Correspondingly, $\rho_i(z)$ is the matter density within structural level i at redshift z and $\rho_0(0)$ is today's matter density in the universe as a whole. If the Hubble constant $H_0 = 71$ km/sec Mpc, the critical

density $\rho_{crit} = \dfrac{3H_0^2}{8\pi G} = 9.5 \times 10^{-30}$ g/cm^3 where $G = 6.67 \times 10^{-8}$ cm$^3 \cdot$g$^{-1} \cdot$sec^{-2}, and $c = 3.00 \times 10^{10}$ cm$\cdot$sec^{-1}. Assuming the universe is dominated by vacuum energy resulting from a cosmological constant Λ, matter accounts for about 26% [5] of the energy in today's universe. So, $\rho_0(0) = 0.26\rho_{crit} = 2.5 \times 10^{-30}$ g/cm^3 and the vacuum energy density $\rho_v = (1 - 0.26)\rho_{crit} = 7.0 \times 10^{-30}$ g/cm^3. The cosmological constant $\Lambda = \dfrac{8\pi G \rho_v}{c^2} = 1.3 \times 10^{-56}$ cm^2 and there is an event horizon in the universe at radius $R_H = \sqrt{\dfrac{3}{\Lambda}} = 1.5 \times 10^{28}$ cm. Therefore, the mass M_u of the observable universe is about

$$M_u = \frac{4}{3}\pi R_H^3 \rho_0(0) = 3.6 \times 10^{55} \text{ g}.$$

According to the holographic principle [3], the number of bits of information available on the light sheets of any

surface with area a is $\dfrac{a}{4\delta^2 ln(2)}$, where $\delta = \sqrt{\dfrac{\hbar G}{c^3}}$ is the Planck length and $\hbar$ is Planck's constant. So, only

$$N = \frac{\pi R_H^2}{\delta^2 ln(2)} = 4.0 \times 10^{122}$$ bits of information on the

event horizon will ever be available to describe all physics within the event horizon in our universe, The average mass per bit of information in the universe is $(3.6 \times 10^{55}\,g)/(4.0 \times 10^{122}) = 9.0 \times 10^{-68}\,g$ and the holographic principle indicates the total mass of the universe relates to the square of the event horizon radius by $M_u = f R_H^2$, where $f = 0.16$ g/cm^2.

In a closed universe, there is no source or sink for information outside the universe, so the total amount of information in the universe remains constant. Also, after the first few seconds of the life of the universe, energy exchange between matter and radiation is negligible compared to the total energy of matter and radiation separately [6]. So, in a closed universe, the total mass of the universe is conserved and the average mass per bit of information is constant. This suggests an extension of the holographic principle indicating the information describing the physics of an isolated gravitationally-bound astronomical system of total mass M is encoded on a spherical holographic screen with radius $R = \sqrt{\dfrac{M}{0.16}}$ cm around the center of mass of the system.

2. Assumptions

In a closed universe, a hierarchical self-similar description of the development of large scale structure in the universe can be obtained based on four assumptions:

1) Extend the holographic principle by assuming all information necessary to describe an isolated astronomical structure of mass M is available on the light sheets of a holographic spherical screen with radius $R = \sqrt{\dfrac{M}{0.16}}$ cm around the center of mass of the structure, so the average matter density within the spherical screen is $\rho_M = \dfrac{0.16 R^2}{\frac{4}{3}\pi R^3} = \dfrac{0.12}{\pi R}$ g/cm^3.

2) Assume the bits of information on the holographic spherical screens surrounding isolated astronomical structures are in thermal equilibrium with the CMB radiation.

3) Assume structures at any given self-similar structural level range in mass from the Jeans' mass at that level down to the Jeans' mass for the next finer level of structure.

4) Assume the number of structures of mass m in any structural level i is $\dfrac{K}{m}$, where K is constant, so the amount of information in any mass bin (proportional to $\dfrac{K}{m} m$) is the same in all mass bins. This is consistent with the $\dfrac{1}{m}$ behavior of the mass spectrum in the Press-Schechter formalism.

3. Analyses

Based on these assumptions, the following analysis identifies three levels of self-similar large scale structure (corresponding to superclusters, galaxies, and star clusters) between today's observable universe and stellar systems. Those self-similar large scale structures can be seen as gravitationally-bound systems of n widely separated units of the next lower structural level in a sea of cosmic microwave background photons. In this approach, today's speed of pressure waves affecting matter density at structural level i is

$$c_{si}(0) = \frac{2c}{3}\sqrt{\frac{\rho_r(0)}{\rho_i(0)}}$$ [7], and the corresponding Jeans'

length $L_{i+1}(0) = c_{si}(0)\sqrt{\dfrac{\pi}{G\rho_i(0)}}$ [7]. In today's universe, $c_{s0} = 2.7 \times 10^8$ cm/sec, and the first level (supercluster) Jeans' length $L_1(0) = 1.2 \times 10^{27}$ cm. The first level Jeans' mass, the mass of matter within a radius one quarter of the Jeans' wavelength $L_1(0)$, is

$$M_1(0) = \rho_0(0)\frac{4}{3}\pi\left(\frac{L_1(0)}{4}\right)^3 = 2.6 \times 10^{50}$$ g. All scales sma-

ller than the Jeans' wavelength are stable against gravitational collapse, and the radius of the spherical holographic screen for the first level Jeans' mass is $R_1 = 4.1 \times 10^{25}$ cm. The matter density within the spherical holographic screen for the first level Jeans' mass is $\rho_1(0) = \dfrac{0.16 R_1^2}{\frac{4}{3}\pi R_1^3} = \dfrac{0.12}{\pi R_1} = 9.1 \times 10^{-28}$ g/cm^3. Then, $c_{s1} = 1.4 \times 10^7$ cm/sec within the first level Jeans' mass, the second level (galaxy) Jeans' length is $L_2(0) = 3.2 \times 10^{24}$ cm, and the second level Jeans' mass is $M_2(0) = \rho_1(0)\dfrac{4}{3}\pi\left(\dfrac{L_2(0)}{4}\right)^3 = 1.9 \times 10^{45}$ g. Continuing in this way, the third level (star cluster) Jeans' mass $M_3(0) = 1.4 \times 10^{40}$ g, the fourth level (stellar system) Jeans' mass $M_4(0) = 1.0 \times 10^{35}$ g, and

$\dfrac{M_1(0)}{M_u} = \dfrac{M_2(0)}{M_1(0)} = \dfrac{M_3(0)}{M_2(0)} = \dfrac{M_4(0)}{M_3(0)} = 7.3 \times 10^{-6}$. The hierarchy of large scale structure stops with star clusters, because stellar systems cannot be treated as systems consisting of n widely separated subelements in a sea of cosmic microwave background photons.

Identify superclusters as structures with masses between the first and second level Jeans' masses, galaxies as structures with masses between the second and third level Jeans' masses, and star clusters as structures with mass between the third and fourth level Jeans' masses. Then, the universe can be seen successively as an aggregate of superclusters, an aggregate of galaxies, an aggregate of star clusters, or an aggregate of stellar systems. The Jeans' masses identify each structural level, but a mass distribution is needed to estimate the number of entities in each structural level and the average mass of structures at that level. Using the assumed $\dfrac{K}{m}$ behavior of the mass spectrum, the number of superclusters in the universe is $n = \int_{7.3 \times 10^{-6} M_1}^{M_1} \left(\dfrac{K}{m} \right) dm = 11.8\,K$ and the mass of the universe relates to the aggregate of supercluster masses by $M_u = \int_{7.3 \times 10^{-6} M_1}^{M_1} m \left(\dfrac{K}{m} \right) dm \approx KM_1$. So, $K = \dfrac{M_u}{M_1}$, the average mass of a supercluster

$\overline{M_1} = \dfrac{M_u}{n} = \dfrac{M_1}{11.8} = 2.2 \times 10^{49}$ g and the mass of the universe is the number of superclusters times the average supercluster mass. There are

$n = \int_{7.3 \times 10^{-6} M_2}^{M_2} \left(\dfrac{K}{m} \right) dm = 11.8\,K$ galaxies in a first level Jeans' mass, and the first level Jeans' mass is the aggregate of the galaxy masses within that Jeans' mass, so $M_1 = \int_{7.3 \times 10^{-6} M_2}^{M_2} m \left(\dfrac{K}{m} \right) dm \approx KM_2$. Then, $K = \dfrac{M_1}{M_2}$, and the average galaxy mass

$\overline{M_2} = \dfrac{M_1}{n} = \dfrac{M_2}{11.8} = 1.6 \times 10^{44}$ g. A similar analysis gives an average star cluster mass of 1.2×10^{39} g, and these results are consistent with observations [8-10].

Down to the third (star cluster) structural level, the total number $n = 11.8\,K = 1.6 \times 10^6$ of next lower level substructures inside the holographic screens for the Jeans' length at each structural level are the same as the total number of superclusters in the observable universe. Furthermore, considering the large scale structures within the universe, there are 1.4×10^5 average mass galaxies

in an average mass supercluster, 1.4×10^5 average mass star clusters in an average mass galaxy and (if the average stellar system mass is 4.3 times the solar mass) 1.4×10^5 average mass stellar systems in an average mass star cluster.

To understand the self-similarity (scale invariance) of large scale structures within the universe, consider gravitationally-bound systems of n entities with mass m and total mass $M = nm$. For structures with $n \approx 10^5$, the substructure mass m is much less than the mass M of the next highest level of structure. From the virial theorem, the gravitational potential energy of the systems is $V_G = -\dfrac{GM^2}{2R}$. The extended holographic principle indicates the information needed to describe gravitationally-bound astronomical systems of total mass M consisting of empty radiation-filled space and n smaller entities with mass $m \ll M$ is available on a spherical holographic screen of radius $R = \sqrt{\dfrac{M}{0.16}}$ surrounding the system. Then, the gravitational potential energy of the structure of mass M within the holographic screen is $V_G = -\dfrac{GM^2}{2R} = -\dfrac{G(0.16)^2 R^3}{2}$, so self-similarity (scale invariance) of large scale structures occurs because the average gravitational potential energy per unit volume at each structural level depends only on the gravitational constant and is identical for all levels of large scale structure.

Now consider development of large scale structure at $z > 0$. Stellar systems are the basic elements of self-similar large scale structures (star clusters, galaxies, superclusters, and the universe as a whole), and formation of the first stellar systems depended on thermonuclear reactions between (strongly interacting) protons in the baryon fraction of the matter density in the universe. This suggests the mass of the smallest gravitationally bound systems that become stellar systems at redshift z can be estimated by setting the escape velocity of protons on the holographic screen for the minimum mass stellar system, with radius R_{min}, equal to the average velocity of protons in equilibrium with CMB radiation outside the screen. For $R > R_{min}$, the escape velocity (escaping proton temperature) on the holographic screen is such that escaping protons are at higher temperature than the CMB and can transfer heat (and energy) to the CMB. Correspondingly, for $R < R_{min}$, the escape velocity (escaping proton temperature) on the holographic screen is such that escaping protons are at lower temperature than the CMB and cannot transfer heat (and energy) to the CMB. Any protons outside the holographic screen for the minimum mass stellar system

that are in equilibrium with the CMB (such as those escaping from structures larger than minimum size) can transfer heat (and energy) to structures less than minimum size until they grow to minimum size.

The escape velocity for a proton of mass m_p gravitationally bound at radius R from the centroid of a structure with mass M is calculated from $\frac{1}{2} m_p v^2 = \frac{GMm_p}{R}$. If the escape velocity of a proton on the holographic screen for the minimum mass stellar system at redshift z is the velocity of a proton in thermal equilibrium with the CMB, $\frac{3}{2} kT = \frac{GMm_p}{R}$, where the CMB temperature $T = (1+z)2.725°\text{K}$ and the Boltzmann constant $k = 1.38 \times 10^{-16}$ (g cm^2/sec^2)/$°$K. Since the radius R of the holographic screen for a structure of a mass M is $R = \sqrt{\frac{M}{0.16}}$, the minimum mass of a stellar system at redshift z is

$$M_{stellar} = \frac{1}{0.16}\left(\frac{1.5k(1+z)2.725}{Gm_p}\right)^2.$$ If outgoing protons near the holographic screen are in thermal equilibrium with the CMB and the outgoing photon flow from the minimum mass star, the outgoing photon flow from stellar systems with mass less than the minimum stellar system mass is at lower temperature than the CMB and cannot transfer energy to the CMB or appear as a star against the CMB background. Note that radii of holographic screens for stellar systems are considerably larger than radii of stars themselves. For example, the radius of the holographic screen for our sun is comparable to the radius of the entire solar system including the Oort cloud.

If the number of structures $n(m)$ in a mass bin m is $n(m) = \frac{K}{m}$, the smallest scale structures are most numerous. The mass of the largest known star is about 6.4×10^{35} g [11]. This holographic analysis suggests stellar systems with mass 6.4×10^{35} g would be the minimum mass stellar structures and the most numerous luminous structures in the universe at $z \approx 62$, consistent with indications that the first stars formed at $z \approx 65$ [12]. Today, at $z = 0$, this analysis indicates the smallest stellar systems have 0.08 times the solar mass, consistent with the mass of the smallest stars [13]. The fact that the mass of the smallest stars can be estimated from the extended holographic principle using only the Boltzmann constant, CMB temperature, gravitational constant and proton mass suggests a relation between organization of information and gravity, electromagnetism and strong interactions underlying that embodied in specific equations modeling details of thermonuclear reactions and stellar dynamics.

When matter dominates, the speed of pressure waves affecting matter density at redshift z within structural level i is $c_{si}(z) = c\sqrt{\frac{4(1+z)^4 \rho_r(0)}{9\rho_i(z)}}$ [7], and the Jeans' length at that level $L_{i+1}(z) = c_{si}(z)\sqrt{\frac{\pi}{G(1+z)^3 \rho_i(z)}}$ [7]. The first level of large scale structure within the universe is determined by the Jeans' mass $M_1(z) = \frac{4\pi}{3}\left(\frac{L_1(z)}{4}\right)^3 \rho_0(z)$, where

$$L_1(z) = \frac{(1+z)^2}{\rho_0(z)}\frac{2c}{3}\sqrt{\frac{\pi\rho_r(0)}{G}} = \frac{(1+z)^2 B}{\rho_0(z)}, \text{ and}$$

$B = \frac{2c}{3}\sqrt{\frac{\pi\rho_r(0)}{G}} = 2.89 \times 10^{-3} \frac{\text{g}}{\text{cm}^2}$, so the resulting Jeans' mass $M_1(z) = M_1 = \frac{\pi B^3}{48\rho_0^2(0)}$ is independent of z [7].

Evolution of large scale structure is characterized by $N(z)$, the number of structural levels between the Jeans' mass M_1 and stellar systems, and $n(z)$, the average number of next lower level structures within a structure at any given level, as structures in the $N(z)$ levels coalesce into the three levels present today. The Jeans' mass $M_i(z)$ of structures in level i is determined by the Jean's length $L_i(z)$ in the next highest structural level and the holographic density $\rho_{i-1}(z)$ inside the holographic screen for the Jeans' mass $M_{i-1}(z)$ of the next highest structural level. So, the ratio of the Jeans' mass $M_i(z)$ to the Jeans' mass $M_{i+1}(z)$ in the next subordinate level is

$$\frac{M_i(z)}{M_{i+1}(z)} = \frac{L_{i-1}^3(z)\rho_{i-1}(z)}{L_i^3(z)\rho_i(z)} = \frac{\rho_i^2(z)}{\rho_{i-1}^2(z)}.$$ The holographic density $\rho_i(z) = \frac{3A}{4\pi R_i(z)}$, where $A = 0.16 \frac{\text{g}}{\text{cm}^2}$ and the radius of the holographic screen for the Jeans' mass $M_i(z)$ is $R_i(z) = \sqrt{\frac{\pi B^3(1+z)^6}{48A\rho_i^2(z)}}$. So,

$$\frac{M_i(z)}{M_{i+1}(z)} = \frac{\rho_i^2(z)}{\rho_{i-1}^2(z)} = \left(\frac{3A}{\pi B}\right)^3 \frac{1}{(1+z)^6} = \frac{1.37 \times 10^5}{(1+z)^6}.$$

The average mass $\overline{M_i(z)}$ of structures in level i is the total mass of the next lowest level of structures

within level i divided by the total number of next lowest level of structures within level i. So,

$$\overline{M_i(z)} = \left(\int_{M_{i+1}(z)}^{M_i(z)} m\frac{K}{m}\,dm\right)\Big/\left(\int_{M_{i+1}(z)}^{M_i(z)} \frac{K}{m}\,dm\right)$$

$$= M_i(z)\left(1 - \frac{M_{i+1}(z)}{M_i(z)}\right)\Big/\left(\ln\left(\frac{M_i(z)}{M_{i+1}(z)}\right)\right).$$

Then, the number $n(z)$ of average mass structures of next lower level within the average mass at any structural level is

$$n(z) = \frac{\overline{M_i(z)}}{M_{i+1}(z)} = \frac{M_i(z)}{M_{i+1}(z)}$$

$$= \left(\frac{3A}{\pi B}\right)^3 \frac{1}{(1+z)^6} = \frac{1.37\times10^5}{(1+z)^6}.$$

The growth of $n(z)$ tracks development of self-similar large scale structure. Self-similar large scale structures began to emerge when $n \approx 10$ at $z = 3.9$, with 16 structural levels exceeding the minimum stellar system mass of $2\,M_\odot$. As time went on, $n = 100$ at $z = 2.3$ with eight structural levels exceeding the minimum stellar system mass of $0.9\,M_\odot$, $n = 1000$ at $z = 1.3$ with five structural levels exceeding the minimum stellar system mass of $0.4\,M_\odot$, and $n = 10,000$ at $z = 0.55$ with four structural levels exceeding the minimum stellar system mass of $0.2\,M_\odot$.

This analysis allows quick simulation of the formation of self-similar large scale structures, since the number $N(z)$ of self-similar structural levels exceeding the minimum stellar system mass $M_{\min stellar}(z)$ is the integer truncation of $\dfrac{1}{\log\left(\dfrac{M_i}{M_{i+1}}\right)}\log\left(\dfrac{M_1}{M_{\min stellar}(z)}\right)$, and

the number of average mass structures of next lower level within the average mass at any structural level, is

$$n(z) = \left(\frac{3A}{\pi B}\right)^3 \frac{1}{(1+z)^6} = \frac{1.37\times10^5}{(1+z)^6}.$$

Some other comparisons with observations are worth noting. First, combining the virial theorem with the holographic relation $M = 0.16R^2$, the average root mean square velocity of subelements in a self-similar large scale structure of mass M within the universe is

$$v_{rms} = \sqrt{\frac{G}{2}}(0.16M)^{\frac{1}{4}}.$$ For an average supercluster mass

of 2.2×10^{49} g, the r.m.s galaxy velocity is 2.5×10^8 cm/sec. This compares favorably with the estimated 4.8×10^8 cm/sec closing velocity of the colliding "bullet cluster" galaxies 1E0657-56 [14]. Second, the extended holographic principle can be used to derive a relation between angular momentum of large scale structures and

their mass, similar to that found by Wesson [15]. The angular momentum $J = I\omega$, where the moment of inertia I of a spherical system of mass M is $I = \frac{2}{5}MR^2$, and ω is the angular velocity of the system. Using the holographic relation $M = 0.16R^2$ yields $J = \left(\frac{2}{5}\right)0.16M^2\omega$. The angular velocity can be determined by considering a mass m fixed on the surface of the rotating structure just inside the holographic screen for the structure, with radius R_s. The radial acceleration of that particle $a_r = -\omega^2 R_s$ results from the gravitational force $F_r = -\dfrac{GmM}{R_s^2}$ attracting the particle to the centroid of the structure, so $\omega^2 = \dfrac{GM}{R_s^2} = \dfrac{G}{\sqrt{0.16M}}$. The result is $J = p(M)M^2 = \dfrac{2}{5}\dfrac{G^{0.5}}{(0.16M)^{0.25}}M^2$. Then, $p(M) = 1.5\times10^{-15}$ for an average galactic mass of 1.5×10^{44} g, about twice Wesson's empirical value $p = 8\times10^{-16}$ [15].

Finally, Forbes and Kroupa [16] suggest galaxies and star clusters can be distinguished by their relaxation times, with galaxies having relaxation times greater than the age of the universe and star clusters having relaxation times less than the age of the universe. Based on standard texts (Shu [17] and Binney & Tremaine [18]), Bhattacharya [19] considers a system of mass M and radius R composed of N stars with average mass m and number density $n = \dfrac{3N}{4\pi R^3}$. He then approximates the two body relaxation time for the system as $t_R \approx \dfrac{0.1N}{\ln N \sqrt{Gmn}}$. Using the holographic relation $R = \sqrt{\dfrac{M}{0.16}}$ between the mass and the radius of a system, its relaxation time is $t_R \approx \dfrac{0.1}{\ln N}\sqrt{\dfrac{4\pi N}{3Gm}}\left(\dfrac{M}{0.16}\right)^{\frac{3}{4}}$. This extended holographic analysis indicates the average star cluster today has mass 1.2×10^{39} g. If the (imprecisely known) mass of the average star is the solar mass 2×10^{33} g, the relaxation time for an average mass star cluster is 8.54×10^{17} sec. If the age of the universe is 13.6×10^9 yr $= 4.29\times10^{17}$ sec and the average stellar mass is about twice the solar mass, the relaxation time of the average mass star cluster equals the age of the universe. This indicates star clusters have relaxation times of the order of the age of the universe or less, and larger mass structures have longer relaxation times. So, a

direct consequence of the extended holographic principle and the fact that the average stellar mass is near the solar mass is that relaxation times for galaxies are greater than the age of the universe, consistent with Forbes and Kroupa [16].

4. Conclusions

The above analyses, based on four simple assumptions, produce numerical results in general agreement with astrophysical observations of large scale structures in our universe. It is unlikely that all of these results are mere coincidence, so the four assumptions probably provide a reasonable basis for studying development of large scale astrophysical structures if our universe turns out to be closed.

5. References

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