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Dark Energy as a Property of Dark Matter

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Abstract

A novel model of dark matter (DM), elastically compressible, can contribute to the acceleration of our Universe expansion. While each galaxy compresses its own DM within its gravitation field, the DM bordering neighboring galaxies, far from their centers, is pulled apart. It is shown that, although the DM pressure tends to zero at such locations, the DM compressibility tends to infinity. This allows the DM to expand between galaxies without gravitation hindrance. The model is consistent with the coupled distributions of baryonic and dark matters, with black hole formation at the centers of large galaxies, with galactic flat rotation curves, with a Tully-Fisher relation, and with Milgrom's MOND relation. Results are discussed.

Keywords: Dark Energy, Dark Matter

1. Introduction

A main problem in astrophysics is the discovery, about two decades ago, that our Universe expansion is accelerating, instead of slowing down as predicted by the Big Bang theory [1] Scientists hypothesize the existence of an anti-gravity energy field dubbed Dark Energy (DE). It has been claimed that the identification and understanding of that DE would be the greatest accomplishment of the century [2].

A less recognized problem is the Big Bang itself, the explosion of the biggest of black holes, which the present theory and model of black holes cannot accommodate. Instead, the Big Bang theory deals with the evolution of our Universe after the initial Bang [1]. Inclusion of the DE proposed here could destabilize black holes sufficiently to help resolve that problem.

An elastically compressible DM might contribute to the accelerated expansion of our Universe and to the destabilization of black holes. In the following, we show that such model of DM is consistent with the observed DM phenomena. Then, we derive a possible mechanism of acceleration of our Universe expansion.

1.1. Dark Matter or New Physics?

The prevailing scientific view today is that DM is constituted of weakly interacting massive particles (WIMPs). At this time, such view must be considered a hypothesis because, so far, a constitutive particle of DM has not been detected, and computer simulations based on the predicted microscopic properties of WIMPs have not succeeded in modeling the observed behavior of DM [1-15].

An alternative hypothesis is that DM is not a substance at all and that the DM-phenomena result from a "new physics." Some scientists proposed a modification of Newton dynamics (MOND) for weak acceleration as such alternative [16-23]. Milgrom's version of MOND [16-22] proposes at very low accelerations, corresponding to very large distances R, a relation that departs from Newton laws as follows: defining γ_o as a small constant (= about 10^{-8} cm·s^{-2}) of acceleration dimensions, then for $\gamma \ll \gamma_o$ MOND departs from Newton laws by postulating:

$$\gamma_{small} = F/M = -(GM\gamma_o)^{1/2}R^{-1} = -(V_c)^2R^{-1}. \quad (1.1.1)$$

From Equation (1.1.1):

$$V_c = \left(GM\gamma_o\right)^{1/4}, \quad (1.1.2)$$

at very low accelerations, corresponding to very large distances R_{large}.

Although MOND's relation does not define the dynamics in the intermediate range where $\gamma \approx \gamma_o$, it is tailored to produce the galactic rotation curve plateau in a form (Equation (1.1.2)) fitting a Tully-Fisher relation (TFR). However, MOND relates the luminous mass of the galaxy with the limiting speed (Equation (1.1.2)) while the TFR implies a relation with the maximum, or peak speed.

While very successful at modeling much of DM behavior, serious reasons preclude the scientific acceptance of MOND [16-21,24,25]. In particular, Milgrom's derivation of MOND, based on a departure from Newton laws of gravitation, though speeds are small compared with the speed of light, conflicts with general relativity and makes that derivation unacceptable. However, because some doubt that DM particles will be substantiated, it has been suggested to take a new look at MOND [26]. On the other hand, the recent observation of dissociation between DM and baryonic clouds during a collision of galactic clusters [27] tends to tip the scales, at this time, in favor of the existence of DM.

Here, we explore another "new physics" consisting of an elastically compressible DM, with a mass density distribution ρ. The exploration of such "new physics" appears fruitful, leading to the derivation of an equation of state (EoS) for such DM in a gravitation field, of an equation governing the coupled distributions of baryonic and dark matters around galaxies, of a possible mechanism of black hole formation at the centers of large galaxies, of galactic flat rotation curves, of a Tully-Fisher relation, of Milgrom's MOND relation, and of a possible mechanism for the accelerated expansion of our universe.

1.2. Alternative Model of Dark Matter

a) DM is a non-baryonic substance, consistent with the observation of a collision between galactic clusters [27].

b) DM distribution is spherically symmetric around some spiral galaxies, without flattening at the poles [3,6,10,28].

c) Consistent with assumption (b), DM does not necessarily partake in galaxy rotation [1,3,4,10,28]. The concept that DM is constituted of particles conflicts with this assumption.

d) Dark matter has no centrifugal force to balance the centripetal gravitation pull, consistent with assumption (c).

e) Instead, we assume that DM is elastically compressible, without energy dissipation, and develops a DM-pressure P, which balances the gravitation pull on DM.

Because DM-WIMPs have no self interaction sufficient to develop DM-pressure high enough to balance the gravitation pull, assumption (e) excludes the possibility that DM be constituted of WIMPs, consistent also with the absence of success, so far, of efforts to prove the existence of DM-WIMPs [1-4,7,8,11-15].

f) Newton dynamics may be used as a good approximation of general relativity in the limits of speeds small relative to the speed of light and of weak gravitation.

2. Coupled Distributions of Baryonic and Dark Matters

To simplify, we consider only systems of spherical symmetry.

Including DM mass, Newton's equations applied to a spherically symmetric galaxy yield:

$$F = -G(M + m)M'R^{-2} \tag{2.1}$$

where M and m (g) are the baryonic (mostly luminous) mass and the DM-mass, respectively, both within a radius R from the galaxy center, and F is the gravitation force applied by the sum of M and m on a gravitating small test mass M' at radial distance R from the galaxy center.

The gravitational acceleration γ of M' at R then is:

$$\gamma = -(V_c)^2/R = F/M' = -G(M + m)R^{-2}, \tag{2.2}$$

from which:

$$(V_c)^2 = -G(M + m)R^{-1} \tag{2.3}$$

and

$$V_c = [G(M + m)R^{-1}]^{1/2}. \tag{2.4}$$

The mass m may be expressed as:

$$m = 4\pi\int \rho R^2 \mathrm{d}R \tag{2.5}$$

from which:

$$\mathrm{d}m = 4\pi\rho R^2 \mathrm{d}R \tag{2.6}$$

where ρ is the DM density at location R.

2.1. Equation of State of Dark Matter in a Gravitation Field

Staying with the simplicity of spherical symmetry, let us consider a spherically symmetric distribution of baryonic mass, its associated DM, and their gravitation field of center O, at infinite distance from any other mass.

Because DM does not necessarily rotate around galaxies (assumption c), it lacks the centrifugal force that might balance the gravitation force on DM (assumption d). Instead, the gravitational field elastically compresses DM to a pressure distribution P (g cm$^{-1}\cdot$s^{-2}), function of radial distance R, without energy dissipation (assumption e). Therefore, the pressure P is a function only of location R and of other functions of R. The DM pressure balances the gravitation force and may be expressed in differential form as the ratio of the gravitation differential force $\mathrm{d}F_m$ exerted by the sum of M and m on the differential mass dm of the DM confined in a spherical shell of radius R, and differential thickness dR, to the surface area S (= $4\pi R^2$) of that shell:

$$\mathrm{d}P = -G(M + m)R^{-2}\mathrm{d}m(4\pi)^{-1}R^{-2} = \gamma(4\pi)^{-1}R^{-2}\mathrm{d}m = \rho \mathrm{d}R < 0, \tag{2.1.1}$$

in which Equation (2.6) was used in the last step.

Equation (2.1.1) shows that the pressure P is expressible in terms of the location R, the DM mass density at location R, and the gravitation acceleration γ (of a test mass M') at that location. The pressure might be expressed as:

$$P - P_{oo} = -A\rho\gamma R = A\rho(V_c). \quad (2.1.2)$$

where P_{oo} is the DM pressure at infinite distance, and A is a dimensionless constant. When the value of A is determined, Equation (2.1.2) provides the EoS for DM in the spherically symmetric gravitation field.

To determine the value of the integration constant A, we derive an expression for the distribution of DM mass-density ρ around a galaxy and then compare it to the boundary condition at very large radii R_{large}. The procedure is as follows:

Form the ratio of Equation (2.1.1) to Equation (2.1.2), side by side:

$$\frac{\mathrm{d}P}{P-P_\infty} = \frac{\rho\gamma \mathrm{d}R}{-A\rho\gamma R} = -\frac{\mathrm{d}R}{AR} \quad (2.1.3)$$

and by integration:

$$\mathrm{Log}(P - P_{oo}) + \mathrm{A}^{-1}\mathrm{Log}\ (R) = \mathrm{Log(constant)} \quad (2.1.4)$$

or:

$$(P - P_{oo})R^{(1/\mathrm{A})} = -\mathrm{A}\rho\gamma R^{[1+(1/\mathrm{A})]} = \mathrm{A}\rho(V_c)^2R^{(1/\mathrm{A})} = \text{constant} = \mathrm{A}k \quad (2.1.5)$$

where the integration constant is conveniently written as Ak to yield:

$-\rho\gamma R^{[1+(1/\mathrm{A})]} = \rho(V_c)^2R^{(1/\mathrm{A})} = k$, another constant specific to the galaxy. (2.1.6)

The constant k is specific to the galaxy because it is a function of the total luminous mass M and of its distribution in the galaxy, so that 2 galaxies of equal total luminous mass but with different distributions can have different constants k.

For a galactic flat rotation curve, V_c becomes constant at very large radial distances R_{large}, which requires that the DM mass density be distributed there as:

$\rho(R_{\text{large}})^2 = K$, another constant specific to the galaxy. (2.1.7)

From Equation (2.1.6):

$$\rho R^{(1/A)} = k/(V_c)^2. \quad (2.1.8)$$

Equation (2.1.8) shows that the necessary and sufficient condition for V_c to remain constant in any range of radial locations R, is that $\rho R^{(1/A)}$ remain constant in that range. Comparison with Equation (2.1.7), shows that Equation (2.1.8) has the required form, and that the constant A = 1/2.

With the determination of A = 1/2, Equation (2.1.2) becomes the sought EoS:

$$P - P_\infty = -\rho\gamma R/2 = \rho V_c^2/2\,. \quad (2.1.9)$$

Because of the restriction to spherical symmetry, Equation (2.1.9) is the EoS derived for DM outside any galaxy and inside any spherically symmetric galaxy, except perhaps at sufficiently high DM-pressures where DM might undergo phase changes with corresponding changes of properties. Such changes might occur sufficiently close to some galaxy centers.

However, being a material property of DM, the validity of the EoS (Equation 2.1.9) is not limited to spherical symmetry and would be valid outside and inside any galaxy, except perhaps at sufficiently high DM pressures where DM might undergo phase changes with corresponding changes of DM properties. Such changes might occur sufficiently close to some galaxy centers.

2.2. Equation Governing the Coupled Distributions of Baryonic and of Dark Matter

With the determination of the constant A = 1/2, Equation (2.1.8) becomes:

$$-\rho\gamma R^3 = \rho(V_c)^2R^2 = k. \quad (2.2.1)$$

Because V_c depends on the distribution of both the baryonic mass and the DM mass, its participation in Equation (2.2.1) means that this Equation (2.2.1) governs both distributions.

Equation (2.2.1) satisfies the known trivial condition that for V_c to become constant at sufficiently large distances R_{large}, the expression $\rho R^2_{l\arg e}$ must become constant there (Equation (2.1.7)).

Still, it remains to show that both V_c and $\rho R^2_{l\arg e}$ do become constant at sufficiently large distances R_{large}. We now derive that result and, therefore, the flat galactic rotation curves.

2.3. Derivation of Galactic Flat Rotation Curves

Combine Equations (2.2 and 2.5) and rearrange to yield:

$$RV_c^2 - GM = Gm = 4\pi G\int_a^b \rho R^2 \mathrm{d}R\,. \quad (2.3.1)$$

The validity of Equation (2.3.1) is restricted to spherical symmetry.

Form the differential on both sides of Equation (2.3.1), to obtain:

$$(V_c)^2\mathrm{d}R + R\mathrm{d}(V_c)^2 - \mathrm{G}\mathrm{d}M = 4\pi G\rho R^2\mathrm{d}R. \quad (2.3.2)$$

Substitute for ρR^2 from Equation (2.2.1) to yield:

$$(V_c)^2\mathrm{d}R + R\mathrm{d}(V_c)^2 - G\mathrm{d}M = 4\pi Gk[\mathrm{d}R/(V_c)^2]. \quad (2.3.3)$$

Express dM in terms of the luminous mass density ρ_b:

$$\mathrm{d}M = 4\pi\rho_b R^2\mathrm{d}R. \quad (2.3.4)$$

Substitute into Equation (2.3.3), multiply throughout by $(V_c)^2/R$, and re-arrange to yield:

$(V_c)^4 dR/R + (V_c)^2 d(V_c)^2 - G4\pi\rho_b(V_c)^2 R dR = 4\pi Gk[dR/R]$.

or:

$$G4\pi k(\rho_b/\rho)dR/R + 4\pi Gk[dR/R] - (V_c)^4 dR/R - (V_c)^2 d(V_c)^2 = 0. \quad (2.3.5)$$

Solution of Equation (2.3.5) yields the flat galactic curves. Various approaches are possible. One approach is, for any particular galaxy, to determine ρ_b from luminosity measurements, then solve Equations (2.2.1 and 2.3.5) simultaneously, for V_c and ρ as functions of R.

Another approach is described in the Appendix. The results are as follows:

a) V_c reaches a limit at sufficiently large R:

$$V_{c,\lim} = (4\pi Gk)^{1/4} \quad (2.3.6)$$

b) For large dense galaxies, V_c asymptotically reaches the limit from higher values according to:

$$V_c^4 = 4\pi Gk + (GM/R)^2. \quad (2.3.7)$$

c) For small galaxies, V_c asymptotically rises to the limit from lower values according to:

$$(V_c)^4 = (V_{c,\lim})^4 - \text{constant}/R^2 \quad (2.3.8)$$

2.4. A Tully-Fisher Relation

In keeping with the simplicity of spherical symmetry, we consider a hypothetical galaxy of large luminous (baryonic) mass but small radius, and the region outside its small luminous radius.

Substituting into Equation (2.5) for ρr^2 from equation (2.2.1) and for V_c^2 from Equation (2.3.12), and then integrating, yields:

$$m = 4\pi k\int_{a<<R}^{R} \frac{dr}{V_c^2} = 4\pi k\int_{a<<R}^{R} \left\{ \frac{dr}{\left[4\pi Gk + (GM/r)^2\right]^{1/2}} \right\}_{a<<R}^{R}$$

$$= \left\{ \frac{\left[(GM)^2 + 4\pi GkR^2\right]^{1/2}}{G} \right\}_{a<<R}^{R} .$$

$$= \frac{\left[(GM)^2 + 4\pi GkR^2\right]^{1/2}}{G} - M \quad (2.4.1)$$

2.4.1. At Sufficiently Large Distance from Mass Center: Galactic Flat Rotation Curve

For sufficiently large distances R_{large} from the center of M, Equation (2.4.1) yields:

$$m \to \left(4\pi kG^{-1}\right)^{1/2} R_{\text{large}}. \quad (2.4.1.1)$$

Equations (2.4.1 and 2.4.1.1) show that for $R_{large}^2 \gg (GM)^2/4\pi Gk$, the DM mass m increases linearly with R_{large}, consistent with galactic flat rotation curves.

2.4.2. At Sufficiently Small Distances from Mass Center

For sufficiently small distances R_{small} from the center of M, Equation (2.4.1) yields:

$$m = \left\{\left[(GM)^2 + 4\pi GkR^2\right]^{1/2} - GM\right\}G^{-1}$$

$$= \frac{\left[(GM)^2 + 4\pi GkR^2\right]^{1/2} - GM}{G} \cdot \left\{\frac{\left[(GM)^2 + 4\pi GkR^2\right]^{1/2} + GM}{\left[(GM)^2 + 4\pi GkR^2\right]^{1/2} + GM}\right\}$$

$$= \frac{(GM)^2 + (4\pi Gk)R^2 - (GM)^2}{G\left\{\left[(GM)^2 + 4\pi GkR^2\right]^{1/2} + GM\right\}} \to \frac{2\pi k}{GM}R^2. \quad (2.4.2.1)$$

Equation (2.4.2.1) shows that for:

$$R_{\text{small}}^2 \ll (GM)^2/4\pi Gk, \quad (2.4.2.2)$$

the DM-mass m around M increases in proportion to $(R_{\text{small}})^2$, contributing an added constant acceleration-term:

$$\gamma_m = -GmR^{-2} = -G\frac{2\pi kR^2}{GM}R^{-2} = -2\pi k/M, \quad (2.4.2.3)$$

that is, directed towards the center of M.

Equation (2.4.2.3) is valid around a star like the Sun and would explain very nicely all the observations about the Pioneer Anomaly except for the Viking ranging data [29-31].

2.4.3. Tully-Fisher Relation

Expressing k from Equation (2.4.2.3) as:

$$k = (-\gamma_m/2\pi)M \quad (2.4.3.1)$$

and then substituting into Equation (2.1.13) yields:

$$V_{c,\text{limit}}^4 = 4\pi Gk = 4\pi G(-\gamma_m/2\pi)M > 0 \quad (2.4.3.2)$$

The above results apply outside any galaxy, where DM is spherically symmetric[1,3,4,10] and $Dm = 0$. In particular, they apply outside spiral galaxies so that Equation (2.4.3.2) expresses a TFR. However, just like Milgrom's MOND relation, Equation (2.4.3.2) expresses a relation of the baryonic mass M with the limit velocity $V_{c,\text{limit}}$, while the TFR is with the maximum (peak) circular velocity $V_{c,\max}$.

2.5. Derivation of Milgrom's Relation

Milgrom's MOND relation may be derived from the properties of DM, without departing from Newton laws by substituting for k from Equation (2.4.3.1) into Equation (2.3.8):

$V_{c,\lim} = -(2G\gamma_m M)^{1/4} > 0$ for sufficiently small acceleration. (2.5.1)

Equation (2.5.1) is the MOND correlation (1.1.8) at sufficiently small acceleration. Comparison with Equation (1.1.8) yields:

$$\gamma_o = -2\gamma_m > 0 \quad (2.5.2)$$

Substituting for $-2\gamma_m$ from equation (2.5.1):

$$\gamma_o = -2\gamma_m = 4\pi k M^{-1} = V_{c\lim}^4 G^{-1} M^{-1} \quad (2.5.3)$$

According to Equation (2.5.3), γ_o is not strictly identical for all galaxies but should exhibit some variation over different galaxies, because even galaxies of equal mass have slightly different k-values for different distributions of their masses.

Assuming the Sun gravitates around our Galaxy center at about the limiting speed, Equation (2.5.3) yields for our Galaxy:

$\gamma_o = (2.2 \times 10^7\ \text{cm}\cdot\text{s}^{-1})^4\ (6.67 \times 10^{-8}\ \text{cm}^3\cdot\text{s}^{-2}\cdot\text{g}^{-1})^{-1}$ $[(6 \times 10^{10})(1.989 \times 10^{33}\ \text{g})]^{-1} = 2.9 \times 10^{-8}\ \text{cm}\cdot\text{s}^{-2}$, (2.5.4)

which is of the order of magnitude of the value prescribed by Milgrom, thus contributing some credibility to our treatment.

According to this derivation, Milgrom's relation applies to equilibrium or quasi-equilibrium conditions, and therefore is not applicable to collision of galaxy clusters.

Therefore, all the results obtained with MOND derive also from the elastically compressible model of DM. This result is important because MOND's relation has been shown to fit very well galactic rotation curves and the Tully-Fisher relation [32-34].

2.6. Elastic Compressibility of Dark Matter

The elastic compressibility of DM plays a crucial role in establishing a DM pressure, a DM EoS, and perhaps an understanding of DM behavior. It behooves us to derive an expression for that elastic compressibility κ:

$$\kappa = -\left(\frac{1}{v}\right)\left(\frac{\mathrm{d}v}{\mathrm{d}P}\right) = \left(\frac{1}{\rho}\right)\left(\frac{\mathrm{d}\rho}{\mathrm{d}P}\right), \quad (2.6.1)$$

where v is the volume of a fixed mass m of DM at pressure P.

Forming the derivatives on both sides of Equation (2.1.4) and substituting for ρV_c^2 from Equation (2.2.1):

$$\mathrm{d}P = \mathrm{d}\left(\frac{k}{2R^2}\right) = -k\frac{\mathrm{d}R}{R^3} \quad (2.6.2)$$

Differentiating Equation (2.2.1):

$$\frac{\mathrm{d}\rho}{\rho} = -2\left(\frac{\mathrm{d}R}{R} + \frac{\mathrm{d}V_c}{V_c}\right) \quad (2.6.3)$$

Substituting from Equations (2.6.2 and 2.6.3) into Equation (2.6.1) yields:

$$\kappa = \left(\frac{1}{\rho}\right)\frac{\mathrm{d}\rho}{\mathrm{d}P} = 2\frac{\frac{\mathrm{d}R}{R} + \frac{\mathrm{d}V_c}{V_c}}{k\frac{\mathrm{d}R}{R^3}} = \frac{2R^2}{k}\left[1 + \frac{R}{V_c}\frac{\mathrm{d}V_c}{dR}\right] = \frac{2R^2}{k}\left[1 + \frac{\mathrm{d}(LogV_c)}{\mathrm{d}(LogR)}\right]. \quad (2.6.4)$$

When R becomes sufficiently large, then

$V_c \to (4\pi kG)^{1/4}$ and $\dfrac{\mathrm{d}(LogV_c)}{\mathrm{d}(LogR)} \to 0$ yielding:

$$\kappa \to \frac{2R^2}{k} = \frac{2}{\rho V_c^2} = \frac{1}{P - P_\infty} \to \infty\,. \quad (2.6.5)$$

When R becomes sufficiently small, then V_c also becomes very small and $\dfrac{R}{V_c} \to \dfrac{\mathrm{d}R}{\mathrm{d}V_c}$ so that

$$\frac{R}{V_c}\frac{\mathrm{d}V_c}{\mathrm{d}R} \to 1 \text{ and } \kappa \to 4\frac{R^2}{k} \to 0 \quad (2.6.6)$$

2.7. Accelerated Expansion of Our Universe

In a Universe where all mass is embodied in particles, after the Big Bang, gravitation tends to slow down the initial expansion of our Universe. In such a Universe, even DM contributes to the gravitation forces and, therefore, to slowing down the expansion of the Universe of particles.

For simplicity, consider two galaxies, their center-to-center line, and the distance between the two galaxies expanding due to an initial big bang impetus. The gravitation attraction between the two galaxies tends to slow down that expansion. Intergalactic DM adds to the galaxies mass and therefore to the gravitation thus contributing to further slowing down the expansion.

However, in our Universe an elastically compressible DM can produce a different result: with the two galaxies pulling the DM in opposite directions, somewhere along that line, there is a point X where the pulls of the two galaxies balance each other. The DM on each side of X is pulled away from X thus expanding it even though its pressure tends to zero. The infinite compressibility/decompressibility (Equation (2.6.5)) facilitates that expansion. This contribution to the expansion of the intergalactic DM is above, and in addition to, the contribution from the initial impetus. The decompression of DM between galaxies might contribute to the accelerated expansion of our Universe. Thus, the elastically compressible DM under pressure could be a DE candidate.

3. Discussion and Conclusions

3.1. Nature of Dark Matter

The non-particulate, massive, and elastically compressible DM model is an alternative "new physics" because it is not constituted of particles.

The hypotheses that DM is constituted of WIMPs or that DM is elastically compressible are in conflict with each other because the self-interactions of WIMPs are not strong enough to produce the elastic compressibility and pressure sufficient to balance the gravitational pull.

If DM is not constituted of particles, we are left with a fluid continuum, which makes it unlike any known other substance, and some of its properties must be unlike any property of known other substances.

From observations we know that it does not emit electromagnetic radiation, neither does it absorb nor reflect any: it is perfectly transparent. Its only known interaction with baryonic matter is gravitational. Similarly, with electromagnetic radiation that is deflected gravitationally in lensing.

At this time, we do not know if DM exhibits a refractive index. In the affirmative, we do not know if it does vary, and perhaps how it varies, with DM density ρ.

If it were not for its distributed mass density, DM could be identified with space, which raises the question: does space have a distributed mass density? That would make space a substance! Some might object that this would be a kind of revival of ether, which has been definitely disqualified more than a hundred years ago.

Others may hold that the ether was never proven wrong, only unnecessary, given the knowledge of the time, but is needed now. That the existence of measured physical properties of space: electrical permittivity, magnetic permeability, both in concordance with the speed of light, support the material nature of space.

If this model of DM is allowed to be published, it would be expected to become controversial: some will hold that the success of the consequences derived from the model do not prove its validity. In particular, anything not made of particles is unthinkable. Others may hold, as is generally accepted in science, that the success speaks in favor of the model, that the existence of a DM particle is not proven at all and that to reject a non-particulate DM because of the unsubstantiated belief in a DM particle is tantamount to dogmatism.

However, physicists and astrophysicists are entitled to a fair hearing of that model as much as to the concepts of DM WIMPs, or to other alternative "new physics." Whether they compromise and accept to explore the model of non-particulate DM remains to be seen.

3.2. Tully-Fisher Relation

Like Milgrom's MOND relation, the relation derived here is between the luminous mass of a galaxy and the limiting speed of the galactic rotation curve, while the Tully-Fisher relation is with the maximum (peak) speed. However, Milgrom tailored his relation, ad hoc, and in conflict with general relativity, while the derivation in this paper is based on the properties of DM without conflict with general relativity.

3.3. Relation between Baryonic and Dark Matter Distributions

Equations (2.2.1 and 2.3.1) provide the relation between the baryonic matter and the DM distributions, sometimes dubbed the halo-disc conspiracy (for spiral galaxies). This is the first known derivation of such relation, and can be tested against data from existing galactic rotation curves.

3.4. Derivation of Milgrom's Relation

Because the Milgrom relation at weak acceleration fits very well galactic rotation curves and the Tully-Fisher relation, the derivation of those relations from the DM model supports that model. However, as already stated, according to our derivation, Milgrom's relation applies only to equilibrium and quasi-equilibrium conditions and is not valid for collisions.

3.5. Power of the Model

The power of the model is demonstrated in part by its ability to derive:

a) An equation of state for DM.

b) A relation between DM and baryonic matter distributions.

c) Galactic flat rotation curves,

d) A Tully-Fisher relation.

e) Milgrom's MOND relation.

f) A possible mechanism for black hole formation at the centers of large galaxies.

g) A possible mechanism for the acceleration of our Universe expansion.

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Appendix

In section 2.3 of the present paper, it was stated that solution of Equation (2.3.5) yields the flat galactic rotation curves. One suggested approach is, for any particular galaxy, to determine the luminous mass density ρ_b from luminosity measurements, then solve Equations (2.2.1 and 2.3.5) simultaneously, for the galactic rotation speed V_c and for the DM density ρ, as functions of R.Equations (2.2.1 and 2.3.5) are reproduced here as Equations (A1 and A2, respectively) for the reader's convenience:

$$-\rho R^3 = \rho(V_c)^2R^2 = k. \quad \text{(A1)}$$

$$G4\pi k(\rho_b/\rho \mathrm{d}R/R + 4\pi Gk[\mathrm{d}R/R] - (V_c)^4\mathrm{d}R/R - (V_c)^2\mathrm{d}(V_c)^2 = 0. \quad \text{(A2)}$$

Another approach consists of the consideration of two simpler cases:

a) Equation (A2) is solved first for the case of large galaxies where the rotation speed rises fast with R, reaches speeds larger than the plateau, reaches a maximum speed, then beyond the luminous region, slows down and asymptotically reaches the plateau speed limit.

Beyond the luminous radius R_0 ($0 < R_0 < R$) of a galaxy, the luminous mass M becomes constant so that $\mathrm{d}M = 0$ and Equation (A2) becomes

$$4\pi Gk[\mathrm{d}R/R] = (V_c)^4\mathrm{d}R/R + (V\mathrm{c})^2\mathrm{d}(V\mathrm{c})^2.$$

or

$$2\left(V_c^4 - 4\pi Gk\right)\left(\mathrm{d}R/R\right) + \mathrm{d}V_c^4 = 0\,. \quad \text{(A3)}$$

A singular solution of Equation (A3) is:

$$V_c = (4\pi Gk)^{1/4} \quad \text{(A4)}$$

When $RV_c^2\left(V_c^4 - 4\pi Gk\right) \neq 0$, then from Equation (A3): $2\left(\mathrm{d}R/R\right) + \left[\left(\mathrm{d}V_c^4\right)/\left(V_c^4 - 4\pi Gk\right)\right] = 0$ and, upon integration, $Ln\left(R^2\left|V_c^4 - 4\pi Gk\right|\right) = Ln\left(k_c\right)$ or:

$$R^2\left|V_c^4 - 4\pi Gk\right| = k_c > 0\,. \quad \text{(A5)}$$

Equation (A5) yields:

$$V_c^4 = 4\pi Gk \pm \left(k_c/R^2\right). \quad \text{(A6)}$$

Equations (A5 and A6) are valid outside the luminous radius of any galaxy, where $\mathrm{d}M = 0$ and DM mass density is spherically symmetric.

Equation (A6) shows that, as R_{large} becomes sufficiently large, the speed V_c tends to a limit $V_{c,\lim}$ given by:

$$V_{c,lim} = (4\pi Gk)^{1/4}, \quad \text{(A7)}$$

thus establishing the galactic flat rotation curve and the DM mass density distribution of Equation (A1).

To determine the value of k_c, first we consider the gravitation outside the luminous region, that is for $R > R_0$ where M is constant, $\gamma = -G\left(M + m\right)R^{-2} = -V^2R^{-1}$ from which:

$$RV_c^2 - GM = Gm = 4\pi G\int_a^b \rho R^2 \mathrm{d}R \quad \text{(A8)}$$

Second, we substitute under the integral for ρR^2 from equation (A1) and then for V_c^2 from Equation (A6) to obtain:

$$\begin{aligned} &RV_c^2 - GM \\ &= 4\pi Gk\int \frac{\mathrm{d}R}{V_c^2} = 4\pi Gk\int\left[4\pi Gk \pm \left(k_c/R^2\right)\right]^{-1/2}\mathrm{d}R \\ &= \int \frac{\mathrm{d}\left(4\pi GkR^2 \pm k_c\right)}{2\left[4\pi Gk \pm \left(k_c/R^2\right)\right]^{1/2}} = \left[\sqrt{4\pi GkR^2 \pm k_c}\right]_o^R \\ &= \sqrt{4\pi GkR^2 \pm k_c} - \sqrt{\pm k_c} = RV_c^2 - \sqrt{\pm k_c}\,, \end{aligned}$$

from which $\sqrt{\pm k_c} = GM$. Because $k_c > 0$, the negative sign under the square root is unacceptable and Equations (A5 and A6) yield only one branch, for $R > R_0$:

$$V_c^4 = 4\pi Gk + \left(GM/R\right)^2 = V_{c,\lim}^4 + \left(\frac{GM}{R}\right)^2. \quad \text{(A9)}$$

Equation (A9) shows the speed V_c asymptotically approaching the speed limit from higher speeds, as is the case for large dense galaxies where the circular speed initially rises fast, then slows down, reaches a maximum, and comes down to a plateau. It does not represent the cases of small dense galaxies that behave differently.

From Equation (A1), along the rotation speed plateau:

$$\rho R^2 = \frac{k}{V_{c,\lim}^2} = K\,, \text{ another constant.} \quad \text{(A10)}$$

Comparing Equation (A9) to Equation (2.3.10) yields:

$$K = \rho R_{\text{large}}^2 = kV_{c,\lim}^{-2} = V_{c\lim}^2\left(4\pi G\right)^{-1}, \quad \text{(A11)}$$

where the constant k is expressed from Equation (A7) as:

$$k = V_{c,\lim}^4\left(4\pi G\right)^{-1} \quad \text{(A12)}$$

The above results are similar to those of Milgrom's MOND.

We now derive the galactic flat plateau in cases of smaller galaxies where the rotation curve asymptotically rises to the plateau from lower speeds.

b) Within the luminous region, for $R < R_0$, equation (A2) is valid, $G4\pi k\rho_b/\rho \mathrm{d}R/R + 4\pi Gk[\mathrm{d}R/R] - (V_c)^4\mathrm{d}R/R - (V_c)^2\mathrm{d}(V_c)^2 = 0$, and substituting $(V_{c,\lim})^4$ for $4\pi Gk$, we obtain:

$$\begin{aligned}[(V_{c,\lim})^4 - (V_c)]^4](\mathrm{d}R/R) = (V_c)^2\mathrm{d}(V\mathrm{c})^2] \\ - (V_{c,\lim})^4(\rho_b/\rho)\,\mathrm{d}R/R\end{aligned}$$

or:

$$2[(V_{c,\lim})^4\ (1 - \rho_b/\rho) - (V_c)]^4](\mathrm{d}R/R) = \mathrm{d}(V\mathrm{c})^4 \quad \text{(A13)}$$

As R becomes sufficiently large, the luminous matter mass density becomes negligible and Equation (A13) becomes:

$$2[(V_{c,\lim})^4 - (V_c)]^4](\mathrm{d}R/R) = \mathrm{d}(V\mathrm{c})^4 \qquad \text{(A14)}$$

The solution of Equation (A14) is
$2\mathrm{Log}\,(\mathrm{R}) + \mathrm{Log}[(V_{c,\lim})^4 - (V_c)^4] = \mathrm{Log}\,(\text{constant})$ from which:

$$(V_c)^4 = (V_{c,\lim})^4 - \text{constant}/\mathrm{R}^2 \qquad \text{(A15)}$$

Equation (A15) shows $(V_c)^4$ rising linearly with R^{-2} at a decreasing slope and asymptotically reaching the speed limit $V_{c,\lim}$.

