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Quasi-Degenerate Neutrino Masses with Normal and Inverted Hierarchy

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Abstract

The effects of CP-phases on the three absolute quasi-degenerate Majorana neutrino (QDN) masses are studied with neutrino mass matrices obeying $\mu - \tau$ symmetry for normal as well as inverted hierarchical mass patterns. We have made further investigations on 1) the prediction of solar mixing angle which lies below tri-bimaximal mixing value in consistent with neutrino oscillation observational data, 2) the prediction on absolute neutrino mass parameter (m_{ee}) in $0\nu\beta\beta$ decay, and 3) cosmological bound on the sum of the three absolute neutrino masses $\sum_i^3 m_i$. The numerical analysis is carried out through the parameterization of neutrino mass matrices using only two unknown parameters (ε, η) within $\mu - \tau$ symmetry. The results show the validity of QDN mass models in both normal and inverted hierarchical patterns. These models are far from discrimination and hence not yet ruled out. The results presented in this article are new and have subtle effects in the discrimination of neutrino mass models.

Keywords: QDN Models, Absolute Neutrino Masses, CP-Phases

1. Introduction

The presently available neutrino oscillation observational data [1-5] on two neutrino mass-squared differences ($\Delta m_{21}^2, |\Delta m_{23}^2|$), is found to be insufficient to predict the three absolute neutrino masses in case of quasi-degenerate neutrino (QDN) mass models [6-14], as three independent equations are required for solving three unknown values of absolute neutrino masses. The values of the absolute neutrino mass scale ranging from 0.1 eV to 0.4 eV have been taken as input parameters in most of the theoretical calculations [6-15] in the literature. However, since the present tightest cosmological upper bound of the sum of three absolute neutrino masses, has gone down to the lowest value, $\sum m_i \leq 0.28$ eV [16,17], a larger value of neutrino mass $m_i \geq 0.1$ eV in QDN models, has to be ruled out. Further the upper bound value of neutrino mass parameter $m_{ee} \leq 0.27$ eV appeared in the neutrinoless double beta decay ($0\nu\beta\beta$) experiments [18], also disfavours larger values of neutrino mass eigenvalues with same CP-parity. Investigations on QDN models for both normal hierarchical (NH) and inverted hierarchical (IH) patterns of the three absolute neutrino masses, require detailed numerical analysis to check whether such QDN models can really accommodate lower values of absolute neutrino masses $m_i \leq 0.09$ eV which is consistent with the above cosmological bound. In addition, the solar mixing angle lying below the tri-bimaximal mixing (TBM) [19-21], which is consistent with present observational data [1-5] and the effects of CP-phases on three absolute neutrino masses, are also important ingredients for further analysis of QDN models. We address all these issues in the present work and show the validity of the quasi-degenerate models for lower values of neutrino masses $m_i \leq 0.09$ eV. This constitutes the main part of our work which differs from others on the mass scale rather than on the resulting lower solar mixing angles. We first introduce a general classification of QDN models based on their CP-parity patterns in the three absolute neutrino masses, and then parameterize the mass matrices using only two unknown parameters (ε, η) for achieving a practical numerical solution within $\mu - \tau$ symmetry. Finally we present a detailed numerical analysis and results.

2. Parameterizations of Neutrino Mass Matrix

A general $\mu - \tau$ symmetric neutrino mass matrix [22-24] with its four unknown independent matrix elements, requires at least four independent equations for a realistic numerical solution,

$$m_{LL} = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix}. \quad (1)$$

The three mass eigenvalues m_i and solar mixing angles θ_{12}, are given by

$$m_1 = m_{11} - \sqrt{2}\tan\theta_{12} m_{12},$$
$$m_2 = m_{11} + \sqrt{2}\cot\theta_{12} m_{12},$$
$$m_3 = m_{22} - m_{23}.$$
$$\tan 2\theta_{12} = \frac{2\sqrt{2}m_{12}}{m_{11} - m_{22} - m_{23}}. \quad (2)$$

The observed mass-squared differences are then calculated as

$$\Delta m_{12}^2 = m_2^2 - m_1^2 > 0$$
$$\Delta m_{32}^2 = \left|m_3^2 - m_2^2\right|. \quad (3)$$

In the basis where charged lepton mass matrix is diagonal, we have the leptonic mixing matrix $U_{PMNS} = U$ of the form [23,24],

$$U_{PMNS} = \begin{pmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (4)$$

The neutrino mass parameters m_{ee} appeared in the neutrinoless double beta decay ($0\nu\beta\beta$) and the sum of the three absolute neutrino masses in WMAP cosmological bound $\sum_i^3 m_i$, are given respectively by,

$$m_{ee} = \left|m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2\right|,$$
$$m_{cosmos} = m_1 + m_2 + m_3. \quad (5)$$

A general classification for the three types of quasi-degenerate neutrino mass models [22] with respect to the Majorana CP-phases in their three mass eigenvalues, is adopted here. Diagonalisation of the left-handed Majorana neutrino mass matrix m_{LL} in Equation (1) is given by $m_{LL} = UDU^T$, where U is the diagonalising matrix in Equation (4) and $D = Diag\left(m_1, m_2 e^{i\phi_1}, m_3 e^{i\phi_2}\right)$ is the diagonal matrix with two unknown Majorana phases (ϕ_1, ϕ_2). In the basis where charged lepton mass matrix is diagonal, the leptonic mixing matrix is given by $U = U_{PMNS}$ [23,24]. We then adopt the following classification according to their CP-parity patterns in the mass eigenvalues m_i, namely, Type IA: $(+-+)$ for $D = Diag(m_1, -m_2, m_3)$; Type IB: $(+++)$ for $D = Diag(m_1, m_2, m_3)$ and Type IC: $(++-)$ for $D = Diag(m_1, m_2, -m_3)$ respectively. We now introduce the following parameterization of $\mu - \tau$ symmetric neutrino mass matrices m_{LL} which satisfy the above classifications [22].

3. Numerical Analysis and Results

We first estimate the numerical values of the three absolute neutrino masses. As discussed before, we need to introduce the neutrino mass scale m_3 as input parameter in addition to the observational data [1-5] on solar and atmospheric neutrino mass-squared differences (Δm_{21}^2 and $\left|\Delta m_{32}^2\right|$). For numerical analysis we use the best-fit values of the global neutrino oscillation observational data [1-5]

$$\Delta m_{12}^2 = \left(m_2^2 - m_1^2\right) = 7.60 \times 10^{-5}\ \text{eV}^2,$$

and

$$\left|\Delta m_{32}^2\right| = \left|m_3^2 - m_2^2\right| = 2.40 \times 10^{-3}\ \text{eV}^2,$$

and also define the following parameters $\alpha = \frac{\left|\Delta m_{23}^2\right|}{m_3^2}$ and $\beta = \frac{\Delta m_{21}^2}{\left|\Delta m_{23}^2\right|}$, where m_3 is the input quantity. For quasi-degenerate model with normal hierarchy (NH-QD), the other two mass eigenvalues are estimated from,

$$m_2 = m_3\sqrt{1-\alpha};$$
$$m_1 = m_3\sqrt{1-\alpha(1+\beta)} \quad (6)$$

and for quasi-degenerate model with inverted hierarchy (IH-QD) the mass eigenvalues are extracted from,

$$m_2 = m_3\sqrt{1-\alpha};$$
$$m_1 = m_3\sqrt{1+\alpha(1-\beta)}. \quad (7)$$

For suitable input value of m_3, one can estimate the values of m_1 and m_2 for both NH-QD and IH-QD models, using the observational values of Δm_{21}^2 and $\left|\Delta m_{23}^2\right|$. **Table 1** gives the calculated numerical values for both models.

In the next step we parameterize the mass matrix Equation (1) into three types: Type IA with

$D = Diag(m_1, -m_2, m_3)$: The mass matrix of this type [22, 25,26] can be parameterized using two parameters (ε, η):

$$m_{LL} = \begin{pmatrix} \varepsilon - 2\eta & -c\varepsilon & -c\varepsilon \\ -c\varepsilon & \frac{1}{2} - d\eta & -\frac{1}{2} - \eta \\ -c\varepsilon & -\frac{1}{2} - \eta & \frac{1}{2} - d\eta \end{pmatrix} m_3 . \quad (8)$$

This predicts the solar mixing angle,

$$\tan 2\theta_{12} = -\frac{2c\sqrt{2}}{1 + (d-1)\eta/\varepsilon} . \quad (9)$$

When we choose the values $c = d = 1.0$, we get the tri-bimaximal mixings (TBM), $\tan 2\theta_{12} = -2\sqrt{2}$ (which is $\tan^2 \theta_{12} = 0.50$) and the values of ε and η are calculated for both NH-QD and IH-QD models, by using the values of **Table 1** in these two eigenvalue expressions: $m_1 = (2\varepsilon - 2\eta)\, m_3$ and $m_2 = (\varepsilon - 2\eta)\, m_3$. The results are given in **Table 2** for $\tan^2 \theta_{12} = 0.50$. The solar angle can be further lowered by taking the values $c < 1$ and $d > 1$ while using the earlier values of ε and η extracted using TBM. For $\tan^2\theta_{12} = 0.45$ case, the results are given in **Table 3** Type IB with$D = (m_1, m_2, m_3)$: This type [22,25, 26] of quasidegenerate mass pattern is given by the mass matrix,

$$m_{LL} = \begin{pmatrix} 1 - \varepsilon - 2\eta & c\varepsilon & c\varepsilon \\ c\varepsilon & 1 - d\eta & -\eta \\ c\varepsilon & -\eta & 1 - d\eta \end{pmatrix} m_3 \quad (10)$$

This predicts the solar mixing angle,

$$\tan 2\theta_{12} = \frac{2c\sqrt{2}}{1 + (1-d)\eta/\varepsilon} , \quad (11)$$

which gives the TBM solar mixing angle with the input values $c = 1.0$ and $d = 1.0$. Like in Type-IA, here ε and η values are computed for NH-QD and IH-QD, by using **Table 1** in the corresponding two eigenvalue expressions: $m_1 = (1 - 2\varepsilon - 2\eta)\, m_3$ and $m_2 = (1 + \varepsilon - 2\eta)\, m_3$. Type IC with $D = (m_1, m_2, -m_3)$: It is not necessary to treat this model [22] separately as it is similar to Type IB except

Table 1. The absolute neutrino masses in eV estimated from oscillation data [1] (Using calculated value of β = 0.03166).

Input	Calculated	NH-QD		IH-QD	
m_3	α	m_1	m_2	m_1	m_2
0.400	0.015	0.3968929	0.39699	0.4028945	0.4029888
0.10	0.24	0.086741	0.08718	0.111035	0.1113552
0.08	0.375	0.0626418	0.06326	0.0934023	0.0938083

Table 2. Prediction for $\tan^2\theta_{12}$ = 0.50.

Parameters	NH-QD		IH-QD	
	Type-IA	Type-IB	Type-IA	Type-IB
c	1.0	1.0	1.0	1.0
d	1.0	1.0	0.10	0.10
m_3 (eV)	0.10	0.10	0.08	0.08
ε	0.57972	0.0015	0.78004	0.00169
η	0.14602	0.0649	0.19628	–0.0855
m_1 (eV)	0.08674	0.08672	0.09340	0.09340
m_2 (eV)	–0.0872	0.08717	–0.0938	0.09380
m_3 (eV)	0.10	0.10	0.08	0.08
$\Sigma\lvert m_i\rvert$ eV	0.273	0.273	0.267	0.267
Δm^2_{21} eV^2	7.6×10^{-5}	7.6×10^{-5}	7.6×10^{-5}	7.6×10^{-5}
$\lvert\Delta m^2_{23}\rvert$ eV^2	2.4×10^{-3}	2.4×10^{-3}	2.4×10^{-3}	2.4×10^{-3}
$\tan^2\theta_{12}$	0.50	0.50	0.50	0.50
$\lvert m_{ee}\rvert$ eV	0.0869	0.0869	0.0869	0.0869

Table 3. Prediction for $\tan^2\theta_{12} = 0.45$

Parameters	NH-QD		IH-QD	
	Type-IA	Type-IA	Type-IA	Type-IB
c	0.86	0.945	0.868	0.96
d	1.025	0.998	1.0	1.002
m_3 (eV)	0.10	0.10	0.08	0.08
ε	0.6616	0.00145	0.88762	0.00169
η	0.1655	0.06483	0.22317	–0.0855
m_1 (eV)	0.08754	0.08676	0.09392	0.09341
m_2 (eV)	–0.0879	0.08717	–0.0943	0.09381
m_3 (eV)	0.0996	0.10002	0.08	0.08001
$\Sigma\lvert m_i\rvert$ eV	0.275	0.274	0.268	0.267
Δm^2_{21} eV^2	7.7×10^{-5}	7.3×10^{-5}	7.6×10^{-5}	7.4×10^{-5}
$\lvert\Delta m^2_{23}\rvert$ eV^2	2.2×10^{-3}	2.4×10^{-3}	2.4×10^{-3}	2.4×10^{-3}
$\tan^2\theta_{12}$	0.45	0.45	0.45	0.45
$\lvert m_{ee}\rvert$ eV	0.0877	0.0869	0.0936	0.0935

for the interchange of two matrix elements {22} and {23} in the mass matrix in Equation (10), and this effectively imparts an additional odd CP-parity on the third mass eigenvalue m_3 in Type IC. Such change does not alter the predictions of Type IB. A self explanatory results for two models of Types-IA and IB, for $\tan^2\theta_{12} = 0.50$ and $\tan^2\theta_{12} = 0.45$, are presented in **Tables 2** and **3** respectively.

4. Conclusions

We have studied the effects of the Majorana phases on the prediction of absolute neutrino masses in three different types of QDN models having both normal and inverted hierarchical patterns within $\mu - \tau$ symmetry. These predictions are consistent with observational data on the mass-squared difference derived from various neutrino oscillation experiments, and with the upper bound on absolute neutrino mass parameter in $0\nu\beta\beta$ decay as well as upper bound of $\sum_i m_i \leq 0.28$ eV obtained from cosmological observations. It has been found that the QDN models with $m_i \leq 0.09$ eV, are still far from discrimination and hence not yet ruled out. The prediction on solar mixing angle is also found to be lower than TBM value viz, $\tan^2\theta_{12} = 0.45$ which coincides with the best-fit in the neutrino oscillation data [1]. The result shows the validity of NH-QD and IH-QD models. The results presented in this article are new and have important implications in the discrimination of neutrino mass models.

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