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Biānchi Type-II, VIII & IX Perfect Fluid Cosmological Models in Brans Dicke Theory of Gravitation

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Abstract

Field equations in the presence of perfect fluid distribution are obtained in a scalar tensor theory of gravitation proposed by Brans and Dicke [1] with the aid of Bianchi type-II, VIII & IX metrics. Exact prefect fluid Bianchi type- IX cosmological model is presented since other models doesn't exist in Brans-Dicke scalar tensor theory of gravitation. Some physical and geometrical properties of the models are also discussed.

Keywords: Bianchi Type-II, VIII & IX, Brans Dicke Scalar Tensor Theory, Perfect Fluid Distribution

1. Introduction

Brans and Dicke [1] theory of gravitation is well known modified version of Einstein's theory. It is a scalar tensor theory in which the gravitational interaction is mediated by a scalar field ϕ as well as the tensor field g_{ij} of Einstein's theory. In this theory the scalar field ϕ has the dimension of the inverse of the gravitational constant.In recent years, there has been a renewed interest of the gravitational constant. The latest inflationary models (Mathiazhagan and Johri [2]), possible "graceful exit" problem (Pimental [3]) and extended chaotic inflations (Linde [4]) are based on Brans and Dicke theory of gravitation.

Brans-Dicke field equations for combined scalar and tensor field are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}\left(\phi_{i;j} - g_{ij}\phi_{,k}{}^{;k}\right) \quad (1.1)$$

and

$$\phi = \phi_{;k}{}^{;k} = 8\pi(3+2\omega)^{-1}T \quad (1.2)$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is an Einstein tensor,

T_{ij} is the stress energy tensor of the matter, ω is the dimensionless coupling constant and comma and semicolon denote partial and covariant differentiation respectively.

The equation of motion

$$T^{ij}{}_{;j} = 0 \quad (1.3)$$

is a consequence of the field Equations (1.1) and (1.2).

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. The work of Singh and Rai [5] gives a detailed discussion of Brans-Dicke cosmological models. In particular, spatially homogeneous Bianchi models in Brans-Dicke theory in the presence of perfect fluid with or with out radiation are quite important to discuss the early stages of evolution of the universe.

Nariai [6], Belinskii and Khalatnikov [7], Reddy and Rao [8], Banerjee and Santos [9], Singh *et al.* [10], Shriram [11], Shriram and Singh [12], Berman *et al.* [13], Reddy [14], Reddy *et al.* [15], Adhav *et al.* [16] and Rao *et al.* [17,18] are some of the authors who have investigated several aspects of this theory.

Chakraborty [19], Raj Bali and Dave [20], Raj Bali and Yadav [21] studied Bianchi type IX string as well as viscous fluid models in general relativity. Reddy, Patrudu and Venkateswarlu [22] studied Bianchi type-II, VIII & IX models in scale covariant theory of gravitation. Shanthi and Rao [23] studied Bianchi type-VIII & IX models in Lyttleton-Bondi Universe. Also Rao and Sanyasi Raju [24] and Sanyasi Raju and Rao [25] have studied Bianchi type-VIII & IX models in Zero mass scalar fields and self creation cosmology. Rahaman *et al.* [26] have investigated Bianchi type-IX string cosmological model in a scalar-tensor theory formulated by Sen [27] based on Lyra [28] manifold. Rao *et al.* [29-31] have studied Bianchi type-II, VIII & IX string cosmological models, perfect fluid cosmological models in SaezBallester scalar-tensor theory of gravitation and string cosmological models in general relativity as well as self creation theory of gravitation respectively.

In this paper we discuss Bianchi type-II, VIII & IX perfect fluid cosmological models in a scalar-tensor theory proposed by Brans and Dicke [1].

2. Metric and Energy Momentum Tensor

We consider a spatially homogeneous Bianchi type-II, VIII and IX metrics of the form

$$ds^2 = dt^2 - R^2\left[d\theta^2 + f^2(\theta)d\phi^2\right] - S^2\left[d\psi + h(\theta)d\phi\right]^2 \quad (2.1)$$

where (θ,ϕ,ψ) are the Eulerian angles, R and S are functions of t only. It represents

Bianchi type-II if $f(\theta)=1$ and $h(\theta)=\theta$

Bianchi type-VIII if $f(\theta)=\cosh\theta$ and $h(\theta)=\sinh\theta$

Bianchi type-IX if $f(\theta)=\sin\theta$ and $h(\theta)=\cos\theta$

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij} = (\rho+p)u_i u_j - p g_{ij} \quad (2.2)$$

where ρ is the density and p is the pressure.

Also

$$g_{ij}u^i u^j = 1 \quad (2.3)$$

In the co moving coordinate system, we have from Equations (2.2) and (2.3)

$$T_1^1 = T_2^2 = T_3^3 = -p,\ T_4^4 = \rho \text{ and } T_i^i = 0 \text{ for } i \neq j \quad (2.4)$$

The quantities ρ and p are functions of "t" only.

3. Bianchi Type-II, VIII & IX Perfect Fluidcosmological Models in Brans-Dicke Theory of Gravitation

The field Equations (1.1), (1.2) & (1.3) for the metric (2.1) with the help of Equations (2.2), (2.3) and (2.4) can be written as

$$\frac{\ddot{R}}{R}+\frac{\ddot{S}}{S}+\frac{\dot{R}\dot{S}}{RS}+\frac{S^2}{4R^4}+\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{\ddot{\phi}}{\phi}+\frac{\dot{R}\dot{\phi}}{R\phi}+\frac{\dot{S}\dot{\phi}}{S\phi}=\frac{-8\pi p}{\phi} \quad (3.1)$$

$$\frac{2\ddot{R}}{R}+\frac{\dot{R}^2+\delta}{R^2}-\frac{3S^2}{4R^4}+\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{\ddot{\phi}}{\phi}+\frac{2\dot{R}\dot{\phi}}{R\phi}=\frac{-8\pi p}{\phi} \quad (3.2)$$

$$\frac{2\dot{R}\dot{S}}{RS}-\frac{S^2}{4R^4}+\frac{\dot{R}^2+\delta}{R^2}-\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{2\dot{R}\dot{\phi}}{R\phi}+\frac{\dot{S}\dot{\phi}}{S\phi}=\frac{8\pi\rho}{\phi} \quad (3.3)$$

$$\left(\frac{\dot{S}}{S}-\frac{\dot{R}}{R}\right)\frac{h(\theta)\dot{\phi}}{\phi}=0 \quad (3.4)$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{2\dot{R}}{R}+\frac{\dot{S}}{S}\right)=\frac{8\pi}{3+2\omega}(\rho-3p) \quad (3.5)$$

and

$$\dot{\rho}+\left(\frac{2\dot{R}}{R}+\frac{\dot{S}}{S}\right)(\rho+p)=0 \quad (3.6)$$

where "$\cdot$" denotes differentiation with respect to "t".

When δ = 0, –1 & +1, the field Equations (3.1)-(3.6) correspond to the Bianchi type-II, VIII & IX universes respectively.

Using the transformation $R=e^{\alpha}$, $S=e^{\beta}$, $dt=R^2 S dT$, where α and β are functions of "T" only.

The field Equations (3.1) to (3.6) reduce to

$$\alpha''+\beta''-\alpha'^2-2\alpha'\beta'+\frac{e^{4\beta}}{4}+\frac{\omega\phi'^2}{2\phi^2}-\frac{\alpha'\phi'}{\phi}+\frac{\phi''}{\phi}=\frac{-8\pi p}{\phi}e^{(4\alpha+2\beta)} \quad (3.7)$$

$$2\alpha''-\alpha'^2-2\alpha'\beta'+\delta e^{(2\alpha+2\beta)}-\frac{3}{4}e^{4\beta}+\frac{\omega\phi'^2}{2\phi^2}-\frac{\beta'\phi'}{\phi}+\frac{\phi''}{\phi}=\frac{-8\pi p}{\phi}e^{(4\alpha+2\beta)} \quad (3.8)$$

$$2\alpha'\beta'+\alpha'^2+\delta e^{(2\alpha+2\beta)}-\frac{1}{4}e^{4\beta}-\frac{\omega\phi'^2}{2\phi^2}+\frac{\beta'\phi'}{\phi}+\frac{2\alpha'\phi'}{\phi}=\frac{8\pi\rho}{\phi}e^{(4\alpha+2\beta)} \quad (3.9)$$

$$(\alpha'-\beta')\frac{h(\theta)\phi'}{\phi}=0 \quad (3.10)$$

$$\phi''=\frac{8\pi}{3+2\omega}(\rho-3p)e^{(4\alpha+2\beta)} \quad (3.11)$$

$$\rho'+(2\alpha'+\beta')(\rho+p)=0 \quad (3.12)$$

where "$'$" denotes differentiation with respect to "T".

Since we are considering the Bianchi type-II, VIII and IX metrics, we have $h(\theta)=\theta$, $h(\theta)=\sinh\theta$ & $h(\theta)=\cos\theta$ for Bianchi type-II, VIII and IX metrics respectively. Therefore, from the Equation (3.10), we will consider the following possible cases with $h(\theta)\neq 0$.

1) $\alpha'-\beta'=0$ and $\phi'\neq 0$
2) $\alpha'-\beta'\neq 0$ and $\phi'=0$
3) $\alpha'-\beta'=0$ and $\phi'=0$

CASE (1): $\alpha' - \beta' = 0$ and $\phi' \neq 0$:

Here, we get $\alpha = \beta + c$

Without loss of generality by taking the constant of integration $c = 0$, we get

$$\alpha = \beta \quad (3.13)$$

By using (3.13), the field Equations (3.7) to (3.12) will reduce to

$$2\beta'' - 3\beta'^2 + \frac{e^{4\beta}}{4} + \frac{\omega\phi'^2}{2\phi^2} - \frac{\beta'\phi'}{\phi} + \frac{\phi''}{\phi} = \frac{-8\pi p}{\phi} e^{6\beta} \quad (3.14)$$

$$2\beta'' - 3\beta'^2 + \delta e^{4\beta} - \frac{3}{4} e^{4\beta} + \frac{\omega\phi'^2}{2\phi^2} - \frac{\beta'\phi'}{\phi} + \frac{\phi''}{\phi} = \frac{-8\pi p}{\phi} e^{6\beta} \quad (3.15)$$

$$3\beta'^2 + \delta e^{4\beta} - \frac{1}{4} e^{4\beta} - \frac{\omega\phi'^2}{2\phi^2} + \frac{3\beta'\phi'}{\phi} = \frac{8\pi\rho}{\phi} e^{6\beta} \quad (3.16)$$

$$\phi'' = \frac{8\pi}{3 + 2\omega} (\rho - 3p) e^{6\beta} \quad (3.17)$$

$$\rho' + (\rho + p) 3\beta' = 0 \quad (3.18)$$

where "$'$" denotes differentiation with respect to "T".

From (3.14) and (3.15), we have

$$\delta e^{4\beta} - e^{4\beta} = 0 \quad (3.19)$$

From (3.19), we observe that, we can't find Bianchi type-II ($\delta = 0$) and VIII ($\delta = -1$) perfect fluid cosmological models in Brans-Dicke theory of gravitation. But we can get Bianchi type-IX ($\delta = 1$) perfect fluid cosmological model in Brans-Dicke theory of gravitation.

For $\delta = 1$, the field Equations (3.14)-(3.18) reduce to

$$2\beta'' - 3\beta'^2 + \frac{e^{4\beta}}{4} + + \frac{\omega\phi'^2}{2\phi^2} - \frac{\beta'\phi'}{\phi} + \frac{\phi''}{\phi} = \frac{-8\pi p}{\phi} e^{4\beta} \quad (3.20)$$

$$3\beta'^2 + \frac{3}{4} e^{4\beta} - \frac{\omega\phi'^2}{2\phi^2} + \frac{3\beta'\phi'}{\phi} = \frac{8\pi\rho}{\phi} e^{4\beta} \quad (3.21)$$

$$\phi'' = \frac{8\pi}{3 + 2\omega} (\rho - 3p) e^{6\beta} \quad (3.22)$$

$$\rho' + 3(\rho + p)\beta' = 0 \quad (3.23)$$

From (3.20), (3.21) & (3.23), we get

$$\beta'' - \beta'^2 + \frac{e^{4\beta}}{4} + \frac{\omega\phi'^2}{6\phi^2} - \frac{\omega\phi''}{3\phi} = 0 \quad (3.24)$$

Then from (3.24), we get

$$\phi = (aT + b)^{-2} \quad (3.25)$$

$$e^{\beta} = (aT + b)^{-1/2} = e^{\alpha} \quad (3.26)$$

with the relation $16a^2\omega - 3a^2 - 3 = 0$, where $a \,\&\, b$ are arbitrary constants.

Using (3.25) & (3.26) in (3.20) & (3.21), we get

$$8\pi p = \frac{(11a^2 - 8a^2\omega - 1)(aT + b)^{-1}}{4} \quad (3.27)$$

$$8\pi\rho = \frac{(15a^2 + 3 - 8a^2\omega)(aT + b)^{-1}}{4} \quad (3.28)$$

The corresponding metric can be written in the form

$$ds^2 = (aT + b)^{-3} dT^2 - (aT + b)^{-1} \left[d\theta^2 + \sin^2\theta d\varphi^2 \right] - (aT + b)^{-1} \left[d\psi + \cos\theta d\varphi \right]^2 \quad (3.29)$$

Thus (3.29) together with (3.27) and (3.28) constitutes an exact Bianchi type-IX perfect fluid cosmological model in Brans-Dicke scalar-tensor theory of gravitation.

PHYSICAL AND GEOMETRICAL PROPERTIES:

The volume element of the Bianchi type-IX perfect fluid cosmological model is given by

$$V = (-g)^{\frac{1}{2}} = (aT + b)^{\frac{-3}{2}} \sin\theta$$

We can observe that the spatial volume V decreases as time "T" increases, *i.e.*, the model is contracting. Also the model has initial singularity at $T = -b/a$, $a \neq 0$

The scalar expansion θ and shear σ are given by

$$\theta = u^{i}_{;i} = \frac{-3a}{2(aT + b)}$$

$$\sigma^2 = \frac{9a^2}{24(aT + b)^2}$$

for Bianchi type-IX perfect fluid cosmological model in Brans-Dicke theory of gravitation. The scalar expansion $\theta \to 0$ as $T \to \infty$ and $\theta \to \infty$ as $T \to 0$. So, the rate of expansion is rapid as time decreases and it becomes slow as time increases. The shear scalar $\sigma^2 \to 0$ as $T \to \infty$ and $\sigma^2 \to \infty$ as $T \to 0$. Thus the shape of universe changes uniformly. The deceleration parameter q is obtained as $q = -3$. The negative value of q indicates that the model is inflationary. Since $\lim_{T\to\infty} \left(\frac{\sigma}{\theta} \right) \neq 0$ which confirms that the universe remains anisotropic throughout the evolution.

CASE (2): $\alpha' - \beta' \neq 0$ and $\phi' = 0$:

In this case $\phi = c_1$, where c_1 is a constant of integration, without loss of generality we can take $c_1 = 1$.

Hence the field Equations (3.7) to (3.12) reduce to general relativity field equations with $\alpha \neq \beta$.

$$\alpha'' + \beta'' - \alpha'^2 - 2\alpha'\beta' + \frac{e^{4\beta}}{4} = -8\pi p e^{(4\alpha + 2\beta)} \quad (3.30)$$

$$2\alpha'' - \alpha'^2 - 2\alpha'\beta' + \delta e^{(2\alpha+2\beta)} - \frac{3}{4}e^{4\beta} = -8\pi p e^{(4\alpha+2\beta)} \quad (3.31)$$

$$2\alpha'\beta' + \alpha'^2 + \delta e^{(2\alpha+2\beta)} - \frac{1}{4}e^{4\beta} = 8\pi \rho e^{(4\alpha+2\beta)} \quad (3.32)$$

$$0 = \frac{8\pi}{3+2\omega}(\rho - 3p)e^{(4\alpha+2\beta)} \quad (3.33)$$

$$\rho' + (2\alpha' + \beta')(\rho + p) = 0 \quad (3.34)$$

From (3.33), we get

$$\rho = 3p \quad (3.35)$$

Since $\rho = 3p$, we will get only radiating universe in this case.

The field Equations (3.30) to (3.34) reduce to

$$\alpha'' + \beta'' - \alpha'^2 - 2\alpha'\beta' + \frac{e^{4\beta}}{4} = -8\pi p e^{(4\alpha+2\beta)} \quad (3.36)$$

$$2\alpha'' - \alpha'^2 - 2\alpha'\beta' + \delta e^{(2\alpha+2\beta)} - \frac{3}{4}e^{4\beta} = -8\pi p e^{(4\alpha+2\beta)} \quad (3.37)$$

$$2\alpha'\beta' + \alpha'^2 + \delta e^{(2\alpha+2\beta)} - \frac{1}{4}e^{4\beta} = 24\pi p e^{(4\alpha+2\beta)} \quad (3.38)$$

$$\rho' + \frac{4}{3}\rho(2\alpha' + \beta') = 0 \quad (3.39)$$

From (3.36) to (3.38), we have

$$\alpha'' + 2\beta'' - \alpha'^2 - 2\alpha'\beta' + \frac{3e^{4\beta}}{4} = 0 \quad (3.40)$$

Then from (3.40), we get

$$e^{\alpha} = (aT + b)^m \quad (3.41)$$

$$e^{\beta} = (aT + b)^{1/2} \quad (3.42)$$

where m, a and b are arbitrary constants satisfying $4a^2(m^2 - 1) = 3$, $a \neq 0$ & $m^2 \neq 1$.

FOR BIANCHI TYPE- II METRIC $(\delta = 0)$:

From (3.36)-(3.38), we get

$$8\pi p = \frac{(2m^2a^2 + 4ma^2 + 1)}{4(aT + b)^{4m+1}} \quad (3.43)$$

$$8\pi \rho = \frac{(4m^2a^2 - 4ma^2 - 1)}{4(aT + b)^{4m+1}} \quad (3.44)$$

The corresponding metric can be written in the form

$$ds^2 = (aT + b)^{4m-1} dT^2 - (aT + b)^{2m}\left[d\theta^2 + d\varphi^2\right] - (aT + b)^{-1}\left[d\psi + \theta d\varphi\right]^2 \quad (3.45)$$

Thus (3.45) together with (3.43) & (3.44) constitutes Bianchi type-II Perfect fluid radiating cosmological models in general theory of relativity.

FOR BIANCHI TYPE-VIII METRIC $(\delta = -1)$:

From (3.36)-(3.38), we get

$$8\pi p = \frac{\left(4m^2a^2 + 4ma^2 + 4(aT + b)^{2m+1} + 3\right)}{4(aT + b)^{4m+1}} \quad (3.46)$$

$$8\pi \rho = \frac{\left(4m^2a^2 - 4ma^2 - 4(aT + b)^{2m+1} - 1\right)}{4(aT + b)^{4m+1}} \quad (3.47)$$

The corresponding metric can be written in the form

$$\begin{aligned} ds^2 &= (aT + b)^{4m-1} dT^2 \\ &\quad - (aT + b)^{2m}\left[d\theta^2 + \cosh^2\theta d\phi^2\right] \\ &\quad - (aT + b)^{-1}\left[d\psi + \sinh\theta d\phi\right]^2 \end{aligned} \quad (3.48)$$

Thus (3.48) together with (3.46) & (3.47) constitutes Bianchi type-VIII Perfect fluid radiating cosmological models in general theory of relativity.

FOR BIANCHI TYPE-IX METRIC $(\delta = 1)$:

From (3.36)-(3.38), we get

$$8\pi p = \frac{\left(4m^2a^2 + 4ma^2 - 4(aT + b)^{2m+1} + 3\right)}{4(aT + b)^{4m+1}} \quad (3.49)$$

$$8\pi \rho = \frac{\left(4m^2a^2 - 4ma^2 + 4(aT + b)^{2m+1} - 1\right)}{4(aT + b)^{4m+1}} \quad (3.50)$$

The corresponding metric can be written in the form

$$\begin{aligned} ds^2 &= (aT + b)^{4m-1} dT^2 \\ &\quad - (aT + b)^{2m}\left[d\theta^2 + \sin^2\theta d\phi^2\right] \\ &\quad - (aT + b)^{-1}\left[d\psi + \cos\theta d\phi\right]^2 \end{aligned} \quad (3.51)$$

Thus (3.51) together with (3.49) & (3.50) constitutes Bianchi type-IX Perfect fluid radiating cosmological models in general theory of relativity.

PHYSICAL AND GEOMETRICAL PROPERTIES:

The volume element of the above three models [(3.45), (3.48) & (3.51)] are given by

$$V = (-g)^{\frac{1}{2}} = (aT + b)^{\frac{4m-1}{2}} f(\theta)$$

where $f(\theta) = 1$, sinhθ and sinθ respectively.

In the above expressions, the volume decreases as time increases if $m < 1/4$ *i.e.*, the models are contracting, the volume increases as time increases if $m > 1/4$ *i.e.*, the models are expanding and the volume is independent of time T if $m = 1/4$. Also the models have initial singular-- ity at $T = -b/a$, $a \neq 0$.

The expansion θ and shear σ are equal for all Bianchi type-II, VIII & IX perfect fluid radiating cosmological models in general relativity. Which are given by

$$\theta = u^i_{;i} = \frac{a(4m-1)}{2(aT+b)}$$

$$\sigma^2 = \frac{a^2(4m-1)^2}{12(aT+b)^2}$$

The deceleration parameter

$$q = -3\theta^{-2}\left(\theta_{;i}u^i + \frac{1}{3}\theta^2\right) = \left(\frac{7-4m}{4m-1}\right)$$

It can be seen that for large "T" the quantities θ and σ will become zero if $m \neq 1/4$. Also the quantities θ and σ tends to $+\infty$ as $T \to 0$ if $4m-1>0$ and tends to $-\infty$ if $4m-1<0$.Thus the rate of expansion is rapid as time decreases, it becomes slow as time increases and the shape of universe changes uniformly. In the case of $4m-1=0$, we can see that the Spatial Volume "V" is independent of time "T" and θ, σ will become zero. Also, since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$, the models are not isotropic for large T. The negative value of the deceleration parameter q shows that the models inflate except for $m = 1$.

CASE (3): $\alpha' - \beta' = 0$ and $\phi' = 0$

Here, we get $\alpha = \beta + c$

Without loss of generality by taking the constant of integration $c = 0$, we get

$$\alpha = \beta \tag{3.52}$$

Since $\phi' = 0 \Rightarrow \phi = c$,

where c is a constant of integration, without loss of generality we can take $c = 1$.

Hence the field equations (3.7) to (3.12) reduce to general relativity field equations with $\alpha = \beta$.

$$2\beta'' - 3\beta'^2 + \frac{e^{4\beta}}{4} = -8\pi p e^{6\beta} \tag{3.53}$$

$$2\beta'' - 3\beta'^2 + \delta e^{4\beta} - \frac{3}{4}e^{4\beta} = -8\pi p e^{6\beta} \tag{3.53}$$

$$3\beta'^2 + \delta e^{4\beta} - \frac{1}{4}e^{4\beta} = 8\pi\rho e^{6\beta} \tag{3.55}$$

$$0 = \frac{8\pi}{3+2\omega}(\rho - 3p)e^{6\beta} \tag{3.56}$$

$$\rho' + (\rho + p)3\beta' = 0 \tag{3.57}$$

where " ′ " denotes differentiation with respect to "T".

From (3.53) and (3.54), we have

$$\delta e^{4\beta} - e^{4\beta} = 0 \tag{3.58}$$

From (3.58), we observe that, we can't find Bianchi type II ($\delta = 0$) and VIII ($\delta = -1$) perfect fluid cosmological models of general relativity. But we can get only Bianchi type IX ($\delta = 1$) perfect fluid cosmological model of general relativity.

For $\delta = 1$, the field equations (3.53)-(3.57) reduce to

$$2\beta'' - 3\beta'^2 + \frac{e^{4\beta}}{4} = -8\pi p e^{4\beta} \tag{3.59}$$

$$3\beta'^2 + \frac{3}{4}e^{4\beta} = 8\pi\rho e^{4\beta} \tag{3.60}$$

$$0 = \frac{8\pi}{3+2\omega}(\rho - 3p)e^{6\beta} \tag{3.61}$$

$$\rho' + (\rho + p)3\beta' = 0 \tag{3.62}$$

From (3.61), we get

$$\rho = 3p \tag{3.63}$$

Since from $\rho = 3p$, we will get only radiating universe in this case.

Now from (3.59), (3.60) and (3.61), we have

$$\beta'' - \beta'^2 + \frac{e^{4\beta}}{4} = 0 \tag{3.64}$$

From (3.64), we get

$$e^{2\beta} = 4\left[(aT+b)^2 + 4\right]^{-1} \tag{3.65}$$

Using (3.65) in (3.59) & (3.60), we get

$$8\pi\rho = 24\pi p = \frac{3(aT+b)^4 + 24(aT+b)^2 + 48}{64} \tag{3.66}$$

The corresponding metric can be written in the form

$$\begin{aligned} ds^2 &= 64\left[(aT+b)^2+4\right]^{-3} dT^2 \\ &\quad -4\left[(aT+b)^2+4\right]^{-1}\left[d\theta^2 + \sin^2\theta d\phi^2\right] \\ &\quad -4\left[(aT+b)^2+4\right]^{-1}\left[d\psi + \cos\theta d\phi\right]^2 \end{aligned} \tag{3.67}$$

Thus (3.67) together with (3.66) constitutes Bianchi type-IX radiating perfect fluid cosmological model in general theory of relativity.

PHYSICAL AND GEOMETRICAL PROPERTIES:

The volume element of the model (3.67) is given by

$$V = (-g)^{\frac{1}{2}} = 64\left[(aT+b)^2+4\right]^{-3}\sin\theta$$

Now the expression for expansion θ and shear σ are given by

$$\theta = u^i_{;i} = \frac{-6a(aT+b)}{\left((aT+b)^2+4\right)}$$

$$\sigma^2 = \frac{6a^2\left(aT+b\right)^2}{\left(\left(aT+b\right)^2+4\right)^2}$$

for Bianchi type-IX perfect fluid radiating cosmological model in Brans-Dicke theory of gravitation. The spatial volume tends to zero as $T \to \infty$. Thus the model is contracting with the increase of time and also the model has no real singularity. The deceleration parameter q is obtained as $q = -2$. The negative value of q indicates that the model is inflationary. Also, since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$ which confirms that the universe remains anisotropic throughout the evolution.

4. Conclusions

Bianchi type space-times play a vital role in understanding and description of the early stages of evolution of the universe. In particular, the study of Bianchi type-II, VIII & IX universes are important because familiar solutions like FRW universe with positive curvature, the desitter universe, the Taub-Nut solutions etc correspond of Bianchi type-II, VIII & IX space-times. In view of the importance of Bianchi type-II, VIII & IX space- times and also since exact solutions offer an alternative and complementary approach to study various cosmological models, in this paper we have presented Bianchi type-II, VIII & IX perfect fluid cosmological models in Brans-Dicke theory of gravitation.

In case of $\alpha' - \beta' = 0$ and $\phi' \neq 0$, we can observe that the only Bianchi type-IX perfect fluid cosmological model exists in Brans-Dicke theory of gravitation. The model is anisotropic, inflationary and has initial singularity at $T = -b/a$, $a \neq 0$. Also established the non-existence of Bianchi type-II & VIII perfect fluid cosmological models in this theory. Since "*a*" is an arbitrary constant and "*ω*" is a coupling constant, it is always possible to assign specific values to "*a*" and "*ω*" to keep the pressure "*p*" (3.27) and density "*ρ*" (3.28) be always positive.

In case of $\alpha' - \beta' \neq 0$ and $\phi' = 0$, we can observe that Bianchi type-II, VIII & IX perfect fluid radiating cosmological models of general relativity exist in this theory. The models have initial singularity at $T = -b/a$, $a \neq 0$ and remain anisotropic throughout the evolution.

In case of $\alpha' - \beta' = 0$ and $\phi' = 0$, we have obtained only Bianchi type-IX anisotropic radiating perfect fluid cosmological model of general relativity with $\alpha = \beta$. In this case also we have observed that Bianchi type-II & VIII cosmological models doesn't exist in this theory.

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