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$2\Delta_0/k_BT_c$ Ratio and Temperature Dependence of the Superfluid Density in Overdoped $La_{2-x}Sr_xCuO_4$

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Abstract

Using band structure parameters extracted from photoemission data by Yoshida *et al.*, Phys. Rev. B., 2006, we have analyzed the temperature dependence of the superfluid density $n_s \propto 1/\lambda_{ab}^2$ in overdoped $La_{2-x}Sr_xCuO_4$. We point out that the temperature behavior $1/\lambda_{ab}^2$ is very sensitive to the ratio $2\Delta_0/k_BT_c$ and have estimated this quantity, using experimental data obtained previously by Panagopoulos *et al.*, Phys. Rev. B., 1999. We compare the results with those obtained from NMR/NQR (Nuclear Magnetic Resonance/Nuclear Quadrupole Resonance) (Mayer *et al.*, J. Phys, Cond. Mat., 2007) and scanning tunneling spectroscopy (Kato *et al.*, Physica C, 2007) data.

Keywords: Supefluid Density, Overdoped $La_{2-x}Sr_xCuO_4$, Short Range Pairing

1. Introduction

Up to now there is no consensus about the nature of superconductivity in layered cuprates. In this context it is important to study the temperature dependencies of the superfluid density because this quantity is directly related to superconductivity. At the same time, its analysis requires preliminary information about the Fermi surface (band structure parameters), the symmetry and temperature dependence of the superconducting gap and a transparent description of the London penetration depth within tight binding approximation. At the moment all this information is available for $La_{2-x}Sr_xCuO_4$ superconductor and in what follows we analyze the temperature dependence of superfluid density and compare the results with available experimental data by Panagopoulos [1]. Won and Maki [2] were first who pointed out that in case d-wave pairing the ratio $2\Delta_0/k_BT_c$ can be different from its standard BCS value 3.52. However their calculations were performed in week coupling approximation which in the strict sense is not appreciable for layered cuprates.

Our main result is that the temperature behavior of $1/\lambda_{ab}^2$ depends sensitively on the ratio $2\Delta_0/k_BT_c$ and we estimate this quantity for $La_{2-x}Sr_xCuO_4$ with for the doping concentration x = 0.20, x = 0.22 and 0.24.

2. Calculations of Superfluid Density

The superconducting current density is proportional to a vector potential and written as (London's equation):

$$j_s = -\frac{c}{4\pi\lambda^2}A. \quad (1)$$

Here λ – is a so-called London's penetration depth of external magnetic field into a superconductor (magnetic penetration depth). This quantity is measured by various experimental techniques [1,3]. Obviously its temperature and doping dependencies contain important information about fundamental microscopic properties of a superconductor. The microscopic expression for superfluid density $n_s \sim 1/\lambda_{ab}^2$ for layered cuprates is discussed in detail in [4] (and Refs there in). It is written as follows:

$$\frac{1}{\lambda_x^2} = 4\pi\left(\frac{e}{c\hbar}\right)^2 \left\{ \sum_k \frac{\partial\varepsilon_k}{\partial k_x} \left[\begin{array}{l} \frac{|\Delta_k|^2}{E_k^2}\frac{\partial\varepsilon_k}{\partial k_x} - \\ -\frac{(\varepsilon_k-\mu)}{2E_k^2}\frac{\partial|\Delta_k|^2}{\partial k_x} \end{array} \right] \times \left[\frac{1}{E_k} - \frac{\partial}{\partial E_k}\right] \tanh\left(\frac{E_k}{2k_BT}\right) \right\}. \quad (2)$$

Here it is assumed that the magnetic field is applied along the x-axis in CuO_2-plane (ab). ε_k is the energy dispersion of quasiparticles in the normal state, μ is the chemical potential,

$$E_k = \sqrt{(\varepsilon_k-\mu)^2 + |\Delta_k|^2} \quad (3)$$

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is Bogolubov's quasiparticle energy in the superconducting state, Δ_k is the superconducting energy gap, which depends on the momentum and temperature. At the first glance, (2) has to be averaged over all possible orientations of the sample with respect to the external field, because the Fermi surfaces of $La_{2-x}Sr_xCuO_4$ are not cylinders. However, it is easy to prove analytically, that in the case of a tetragonal symmetry (2) yields the same result for any orientation of the external field in the ab plane. Therefore one can safely write

$$1/\lambda_x^2 = 1/\lambda_{ab}^2 . \quad (4)$$

Rich information is available about the energy dispersion of quasiparticles in the normal state of $La_{2-x}Sr_xCuO_4$ [5]. Angle-resolved photoemission spectra are well fitted by the tight binding energy dispersion of the following form:

$$\begin{aligned} \varepsilon_k = \mu - 2t_1\left(\cos k_x a + \cos k_y a\right) \\ - 4t_2 \cos k_x a \cos k_y a \\ - 2t_3\left(\cos 2k_x a + \cos 2k_y a\right) \end{aligned}, \quad (5)$$

where k is a wave vector, a – lattice parameter.

Parameters of the conduction band t_1, t_2 and t_3 correspond to the effective hopping integrals between first, second and third neighbors on a square sublattice of Cu sites in CuO_2 plane. For different doping index (x) they are different [5]. Because the nature of pseudogap phenomenon in uderdoped samples $La_{2-x}Sr_xCuO_4$ is not clear yet, we focus on the overdoped compounds only. Following the approximation adopted in [5] we also neglect the small orthorhombic distortions in $La_{2-x}Sr_xCuO_4$. In this case the superconducting gap function corresponds to the d-wave symmetry

$$\Delta_k = \frac{\Delta_0(T)}{2}\left(\cos k_x a - \cos k_y a\right), \quad (6)$$

where T is temperature.

The temperature dependence of $\Delta_0(T)$ was studied previously in the analyses of the NMR data (Knight shift and relaxation rate) in [6]. It was found that for the optimally doped $YBa_2Cu_3O_7$ and $Bi_2Sr_2CaCu_2O_8$ it can be approximated as:

$$\Delta_0(T) = \Delta_0 \tanh\left[1.75\left(\frac{T_c}{T} - 1\right)^{0.5}\right]. \quad (7)$$

The value of Δ_0 is considered as an independent fitting parameter for each doping index (x). The results of our calculations are summarized in **Figure 1** in comparison with the experimental data taken from [1]. The set of the hopping integrals and value of the chemical potential are given in **Table 1**.

Table 1. Chemical potential and effective hopping integrals (in eV), based on ARPES data [5].

$La_{2-x}Sr_xCuO_4$	μ	t_1	t_2	t_3
x = 0.20	0.215	0.25	-0.034	0.017
x = 0.22	0.22	0.25	-0.325	0.0162
x = 0.24	0.227	0.25	-0.032	0.0159

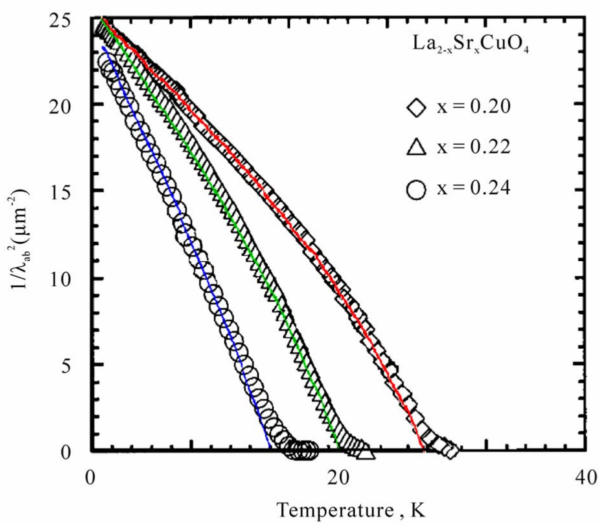

Figure 1. Temperature dependencies of superfluid densities in overdoped $La_{2-x}Sr_xCuO_4$. Symbols (rhomb, triangles and circles) – experimental data [1], lines – our calculations using (2). The $2\Delta_0/k_BT_c$ ratios are: (6.1 ± 0.1), (5.9 ± 0.1) and (5.3 ± 0.1), for x = 0.20, x = 0.22 and x = 0.24, correspondently.

It is important to stress that a curvature of the superfluid density versus temperature curve is quite sensitive to the ratio $2\Delta_0/k_BT_c$. This fact allows us to identify their values for each of the experimental curves.

3. Discussion

The ratios for $2\Delta_0/k_BT_c$ obtained by us are in agreement with findings for this quantity from the experimental data. Indeed, according to angle-resolved photoemission [7] and scanning tunneling spectroscopy [8], the averaged value for optimally doped $La_{2-x}Sr_xCuO_4$ is about 5.5. However, the error bars are quite large which is perhaps related to quality of the surface. In particular, according to [9], the gap distribution in the overdoped $La_{2-x}Sr_xCuO_4$ with x = 0.22 studied by scanning tunneling spectroscopy is $2\Delta_0/k_BT_c$ = 5.5 ± 2.5. Uncertainly in our case is much smaller. Moreover our calculations have revealed an important trend in the evolution of this ratio. It gradually decreases with doping (in overdoped region). This trend can be interpreted as a weakening of the

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strong correlation effect in overdoped side of phase diagram for $La_{2-x}Sr_xCuO_4$. It is interesting to compare the ratio $2\Delta_0/k_BT_c$ extracted by us with those obtained from a solution of the BCS equation

$$\Delta_k = \frac{1}{N}\sum_{k'} J(k-k')\frac{\Delta_{k'}}{2E_{k'}}\tanh\left(\frac{E_{k'}}{2k_BT}\right). \quad (8)$$

Here $J(q) = 2J(cos(q_xa)+cos(q_ya))$ is a Fourier transform of the short-range pairing potential, which included all possible interactions between nearest neighbors in CuO_2 plane (for example superexchange, screened Coulomb repulsion and phonon mediated interaction). The solutions of (8) are presented in **Table 2**. Note that the momentum and temperature dependencies fairly well correspond to (6-7).

The analysis of the temperature dependencies of the superfluid density $n_s \sim 1/\lambda_{ab}^2$ in $YBa_2Cu_3O_{6+x}$ and single layer tetragonal compound $Tl_2Ba_2CuO_{6+\delta}$ (Tc = 78 K), performed in [4], showed that in the overdoped compound $Tl_2Ba_2CuO_{6+\delta}$ (Tc = 78 K) the ratio $2\Delta_0/k_BT_c$ decreases with respect to that of an optimally doped $YBa_2Cu_3O_{6+x}$ (Tc = 92 K). In present paper we have found the same trend for overdoped $La_{2-x}Sr_xCuO_4$. This fact seems to reflect a quite general property of layered cuprates.

This conclusion is also supported by the fact that the temperature dependence of the superconducting energy-gap (7) in overdoped $La_{2-x}Sr_xCuO_4$ is described fairly well by (8).

4. Conclusion

In summary, we have analyzed the temperature dependence of the superfluid density in overdoped $La_{2-x}Sr_xCuO_4$. Our calculations have revealed important trend of changes the ratio $2\Delta_0/k_BT_c$. It gradually decreases with doping in overdoped region. The temperature dependence of the energy gap is fairly well described by simple BCS equation with short range pairing potential yielding a d-wave symmetry of the superconducting gap.

Table 2. Solutions of Equation (8) for short range interaction.

$La_{2-x}Sr_xCuO_4$	T_c, K	J, meV	$2\Delta_0/k_BT_c$
x = 0.20	34	85	4.02 ± 0.01
x = 0.22	25	91	4.21 ± 0.01
x = 0.24	18	93.7	4.36 ± 0.01

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