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Finite Element Modeling of Stress Strain Curve and Micro Stress and Micro Strain Distributions of Titanium Alloys—A Review

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ABSTRACT

Most of the alloys like titanium, steel, brass, copper, etc., are used in engineering applications like automobile, aerospace, marine etc., consist of two or more phases. If a material consists of two or more phases or components it is very difficult to predict the properties like mechanical and other properties based on simple laws such as rule of mixtures. Titanium alloys are capable of producing different microstructures when it subjected to heat treatments, so much of money and time are squandering to study the effect of microstructure on mechanical properties of titanium alloys. This squandering can be reduced with the help of modeling and optimization techniques. There are many modeling techniques like Finite element method, Mat lab, Mathematical modeling etc. are available. But Finite element method is widely used for prediction because of capable of producing distributions of stresses and strains at any different loads. From the literature it is observed that there is a good agreement between the calculated and measured stress strain curves. This review paper describes the effect of volume fraction and grain size of alpha phase on the stress strain curve of the titanium alloys. It also can predict the effect of strength ratio on stress strain curve by using FEM. This information will be of great use in designing and selecting the titanium alloys for various engineering applications.

Keywords: Titanium Alloys; Finite Element Modeling; Stress-Strain Curve

1. Introduction

Titanium alloys are considered as an important material because of its excellent combination of properties such as elevated strength to weight ratio, high fatigue life, toughness and excellent resistance to corrosion. It is heat treatable and hot or cold deformed [1,2] and has gained more and more applications in many fields like aerospace, marine etc. [3,4]. Titanium alloys are broadly classified into three types based on the chemical composition of the alloys and each of these families serves a specific role. This classification consists of α and near-α alloys, α/β alloys and β alloys. Low temperature allotrope form of titanium is α, and the microstructure of α and near-α alloys consists predominantly of the α-phase. The α/β alloys consist of mostly α phase and they do have more β phase. β is the high temperature allotropic form of titanium. Mostly β-alloys consist not fully β phase, but in very general terms, they are capable of retaining 100% β when quenched from the β-phase field [5-7].

Diffusion and diffusion less transformations taking place during heat treatment are important factors for determining the functional characteristics of these materials. These transformations are controlled by means of heat treatment selection and the chemical composition of the phases that are present in these alloys, enable advancement in operational properties [8,9]. Mechanical properties of titanium alloys are important criteria for both in aerospace as well as industrial applications. Microstructure of the alloy is one of the important factors controlling both the tensile strength and the fatigue strength [10,11]. The properties of titanium alloys can be varied over a wide range by heat treatment or thermo-mechanical processing [12-17]. The microstructure of the alloy can be changed from equiaxial through bi-modal to fully lamellar. A bi-modal microstructure is reported to have advantages in terms of yield stress, tensile stress and ductility and fatigue stress. A fully lamellar structure is characterized by high fatigue crack propagation resistance and high fracture toughness. The important parameters for a lamellar structure with respect to mechanical properties are the β-grain size, size of the colonies of α-phase lamellae, thickness of the α-lamellae and the nature of the inter lamellar interface (β-phase) [18].

1.1. Modelling of Titanium Alloys

Titanium alloys exhibits different morphologies and volume fractions of α when it is subjected to heat treatments. The way is to know the effect of these parameters on the properties of titanium alloys is experimentation, thus squandering of money, time and material. To reduce this squandering, modeling techniques are used in order to get the effect of parameters on properties. Modeling of material properties can be done by using different techniques such as finite element method (FEM), mathematical modeling, artificial neural networking etc.

1.2. Artificial Neural Network (ANN) Modelling

Malinov *et al*. [19] developed a model of artificial neural network for simulation of time-temperature—transformation (TTT) diagrams for titanium alloy. This model predicted the influence of aluminium, vanadium, molybdenum and oxygen on transformation kinetics in titanium alloys. The results are in good agreement with the theory of phase transformation. Using the model, TTT diagrams for some commercial alloys were predicted. Some other authors predicted the tensile properties, correlation between the processing parameters on the properties of titanium alloys, etc. by using artificial neural network modeling [20-23].

1.3. Mathematical Modelling

Masaharu *et al*. [24] discussed the effects of shape and volume fraction of a second phase on stress states and deformation behavior of two-phase materials with the help of empirical relations. They embedded inhomogeneous spheroidal (second phase) inclusions in a matrix. Analytical expressions to describe the stress states in elastically and plastically deformed two-phase materials are obtained with the Eshelby method and the Mori-Tanaka concept of the "average stress". Considering that the second phase is also plastically deformable, the overall deformation behavior of the two-phase materials is discussed with the results obtained by the evaluation of the stress and strain distributions in the materials. Some of the authors predicted the stress strain curve with the help of empirical relations [25-28] and phase transformations of titanium alloys [29,30].

1.4. Finite Element (FE) Modelling

Finite element method (FEM) is one of the most used modeling technique in worldwide, different software packages are available in the market like ANSYS, NASTRAN based on the FEM. Jindrich Jinoch *et al*. [31] calculated the stress strain curve of α-β Ti-8Mn Alloy by using FEM, there is a small error between the calculated curve and measured curve but the fit is acceptable. S. Neti [32,33] modeled the deformation behavior of titanium alloy and the effect of strength ration on the stress strain curve. The author used NASTRAN software for the modeling. Sreeramamurthy Ankem *et al*. [34] calculated the effect of volume fraction of second phase on the tensile properties of titanium alloys by using FEM. Some other authors calculated the mechanical properties of titanium alloys, fracture surface, porosity effects by using FEM [35-40].

The aim of this review paper is to give idea on the importance of finite element modeling and the application of Finite element modeling to predict or model the mechanical properties of titanium alloys. It also explains how the advancements taking place in the modeling from past 20 years.

2. Finite Element Modeling (FEM) and Methodology

2.1. Modelling of Stress Strain Curve

For the modeling of stress strain curve, a FE model can be developed in such a way that it should equivalent to the microstructure and this FE model consist of elements which are linked with nodes. In 1978 Jindrich Jinoch *et al*. [31] calculated the stress strain curve of α-β Ti-8Mn Alloy by using FEM. The calculated curve lies below the experimental curve, and this may be due to finer grain sizes in the Ti-8Mn alloy and the contribution of the interface phase, which were not considered in modeling. **Figure 1** shows the microstructure of Ti-8Mn alloy at volume fraction of α is 17%. **Figure 2** is the corresponding FE model, after application of different loads the stress strain curve is calculated and the **Figure 3** shows the comparison between the calculated and measured stress strain curve.

2.2. Effect of Grain Size of Secondary Phase (α) on the Stress Strain Curve

In 1982 Sreeramamurthy Ankem *et al*. [34] calculated the

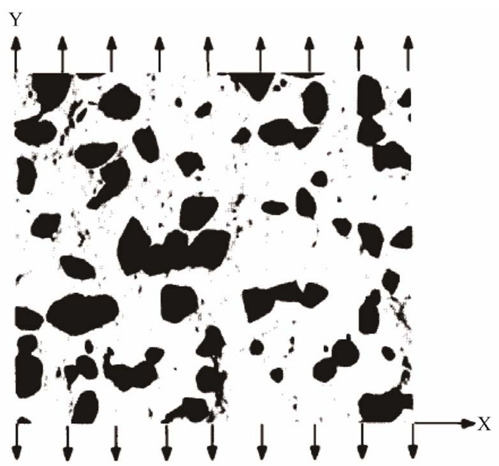


Figure 1. Microstructure of α-β Ti-8Mn alloy at 17% volume fraction α.

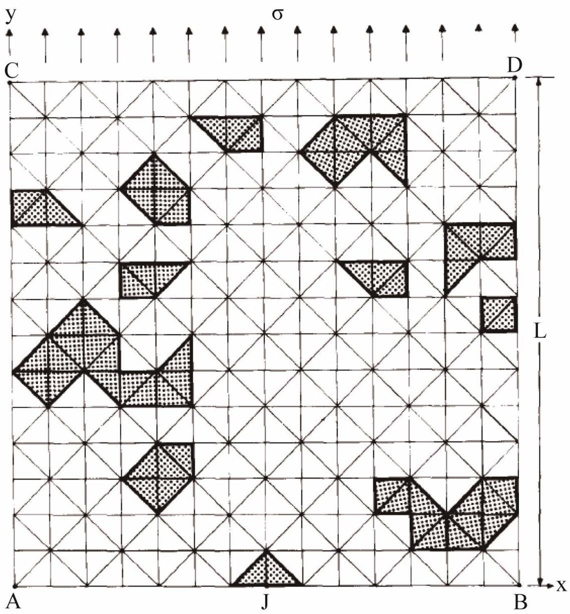


Figure 2. FEM Model.

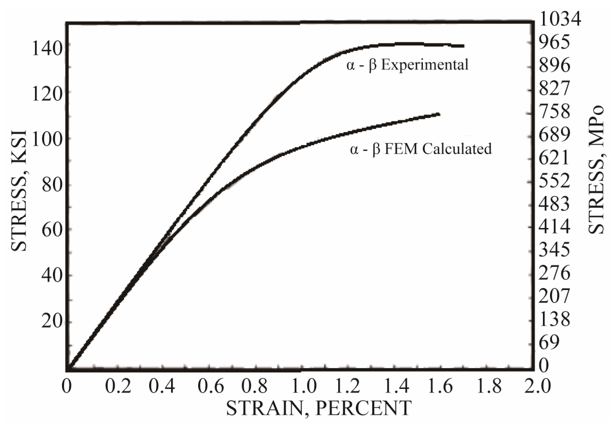


Figure 3. Comparison of stress strain curves between the measured and calculated one.

effect of particle size, matrix, and volume fraction on the stress-strain relations of α-β titanium alloys by using FEM. It was observed that for a given volume fraction, the calculated stress-strain curve was higher for a finer particle size than for a coarse particle size within the range of the strains considered, and this behavior was seen for all the different volume fraction alloys considered. For a 50:50 volume percent α-β alloy, the stress-strain curve with β, the stronger phase, as the matrix was higher than that with α, the softer phase, as the matrix. The calculated stress-strain curves for four different volume percent α alloys were compared with their corresponding measured curves, and good agreement was found. **Figure 4** shows the FEM models of the titanium alloys at different grain sizes for a particular volume fraction of α at 16.3% volume percent.

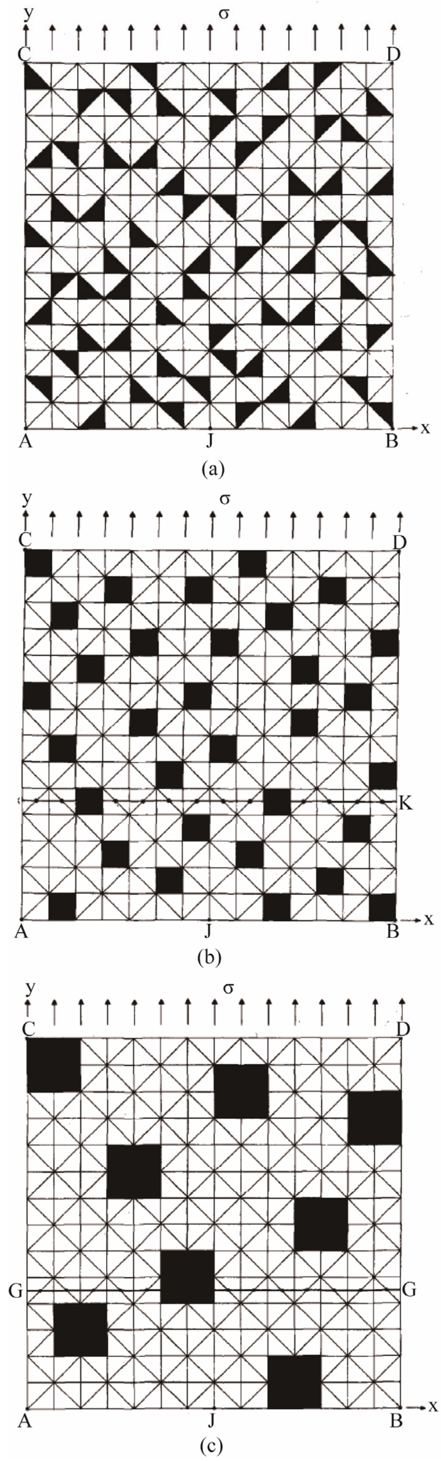


Figure 4. FEM Model (a) Finer mesh; (b) Medium mesh; (c) Coarser mesh.

2.3. Effect of Strength Ratio and Volume Fraction of Secondary Phase (α) on the Stress Strain Curve

In 1991 S. Neti [32,33] has studied the effect of the strength ratio on the stress-strain behavior of various two-phase materials by the finite element method where the volume per cent of the second phase was varied from 20 to 80 vol.%. The strength ratio of the harder β phase to the softer α phase was varied from approximately 2 to 5 by keeping the α phase strength (0.2% YS) was constant at 368 MPa. From his study it was observed that the flow stress of any given two-phase material did not vary linearly with the strength ratio. In addition to that, for a particular strength ratio, the flow stress of the two-phase material did not vary linearly with volume per cent. For materials with less than 40 vol.% β phase, the increase in the strength of the two-phase materials with either volume per cent of β or strength ratio was very small. This was attributed to the fact that the softer α phase, being the matrix, could deform relatively freely without the phase β undergoing plastic deformation up to a plastic strain of 0.5%. When the volume per cent of β was much greater than 50 vol.%, the softer a phase could not deform freely without the harder β phase undergoing plastic deformation, resulting in increased flow stresses with increased strength ratios. **Figure 5** shows the different stress-strain curves used for simulation. **Figure 6** shows the FE models with different percentages of α and β. **Figure 7** indicates the β stress strain curves at different ranges in order to get different strength ratios which are used in modeling.

2.4. Distributions of Stress and Strain

Recently most of the authors used ANSYS software for the analysis of stress strain curve and its distributions [39,41,42]. In the year 2008 Zhao Xiqing *et al*. [39] did work on the distributions of stresses and strains within the phases at different loads. By comparing the calculated stress-strain curve with the measured one, it can be seen that the fit is acceptable. Thus, the FE model built in this work is effective. According to the above mentioned model, the distributions of stress and strain in the α and β phases were simulated. It is observed from the author work that the stress gradients exist in both α and β phases, and the distributions of stress are non-homogeneous. The stress inside the phase is generally higher than the near interface. Meanwhile, the stress in the α phase is lower

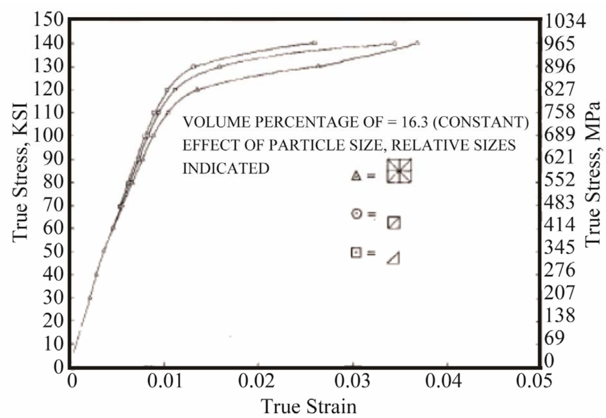


Figure 5. Comparison between the stress strain curves at different grain sizes of α.

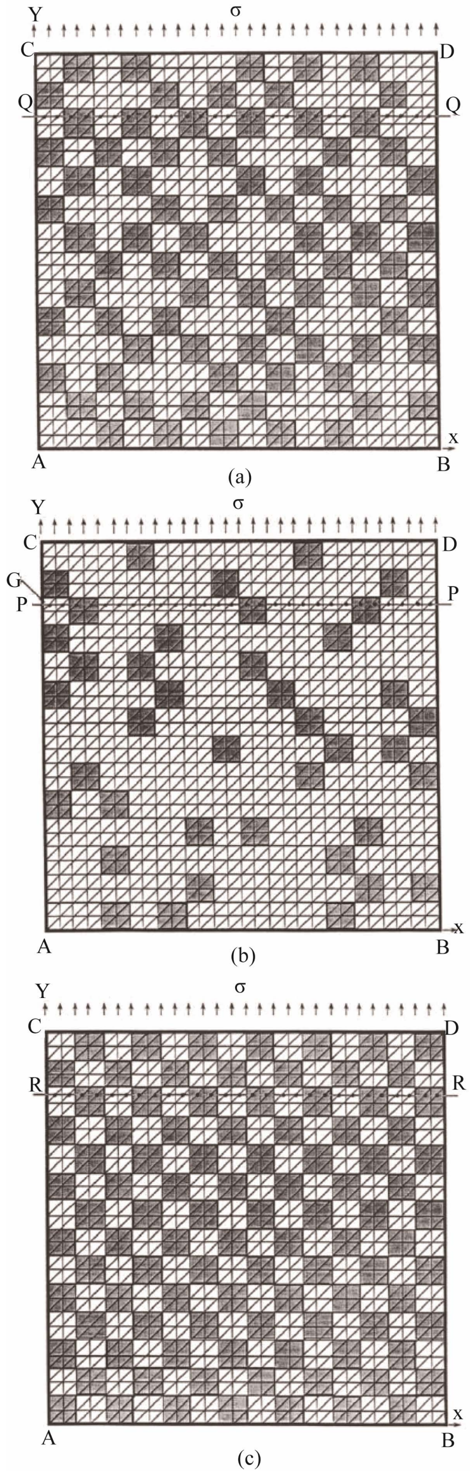


Figure 6. FE Model (a) 20% β and 80% α; (b) 60% β and 40% α; (c) 50% β and 50% α.

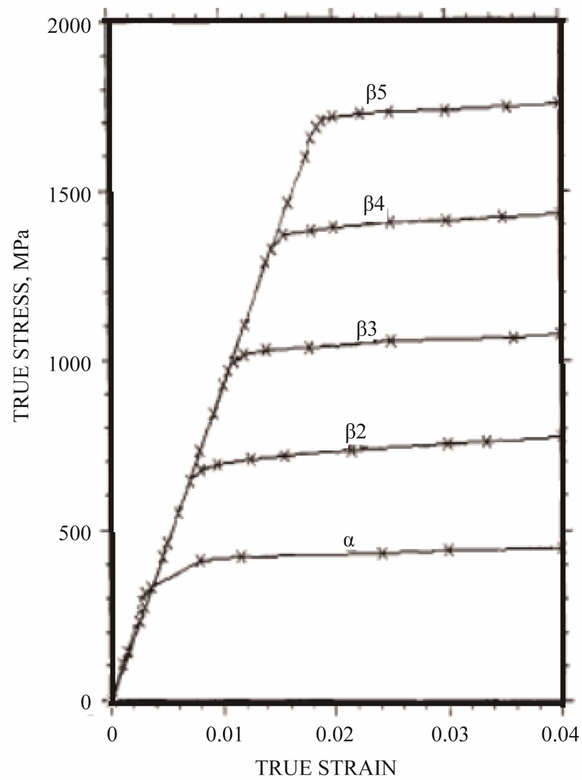


Figure 7. Stress-strain curves of *α* and *β* used for FEM calculations; note that four different *β* curves are used to determine the effect of strength difference.

than that in the *β* phase, whereas the strain in the *α* phase is higher than that in the *β* phase. **Figures 10** and **11** shoes the vonmises distributions of stress and strain at different loads. These figures are showing the variations of stresses and strains with the application of different loads.

Finite element modeling is used in so many other fields like heat transfer in furnaces, and buckling of GRP etc. [43,44]. FEM calculated stress-strain curves for four different *β* to *α* ratio at (a) 80% *β* and 20% *α* (b) 20% *β* and 80% *α* are depicted in **Figure 8**. A variation of 0.2% flow stress with strength ratio for different volume per cents of α and β phases are depicted in **Figure 9**.

3. Summary

Titanium alloys play an important role in the aerospace industry. To maintain the prominent position in the industry efforts must be directed towards cost reduction of titanium structure. So in this cost reduction process, numerical methods play an important role in characterization of titanium alloys. The finite element modeling one of the useful numerical techniques in predicting mechanical properties for titanium alloys, because of the small error between the measured and predicted results. It is very much useful in case of titanium alloys because

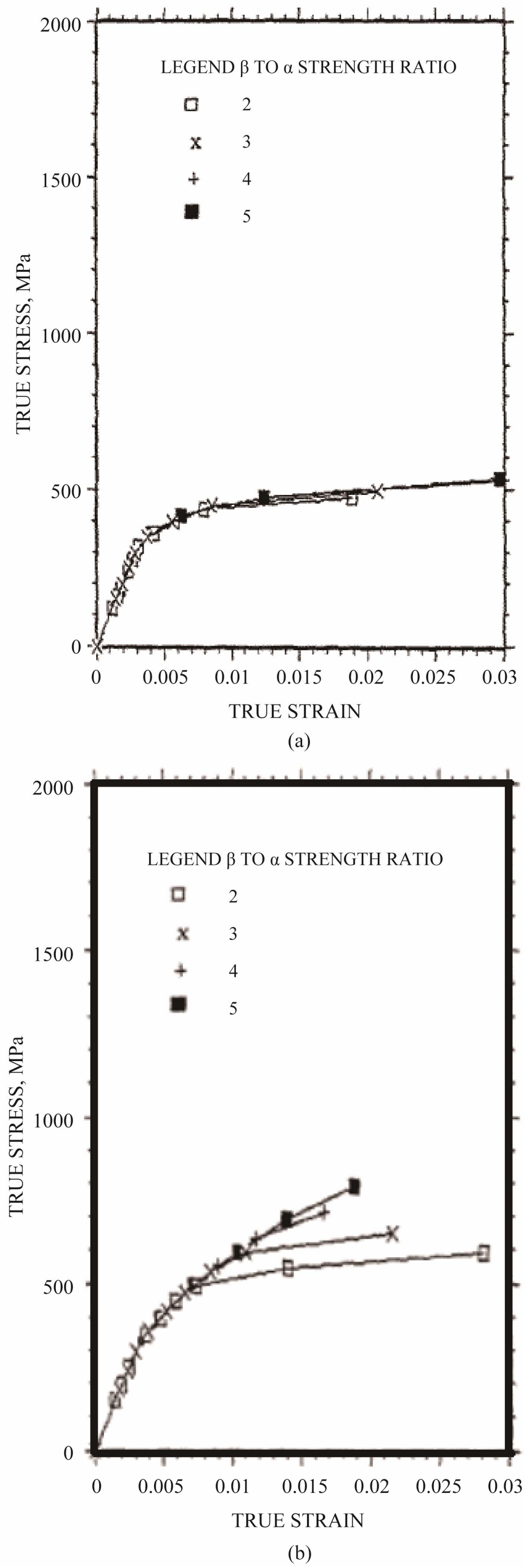


Figure 8. FEM calculated stress-strain curves for four different *β* to *α* ratio at (a) 80% *β* and 20% *α* (b) 20% *β* and 80% *α*.

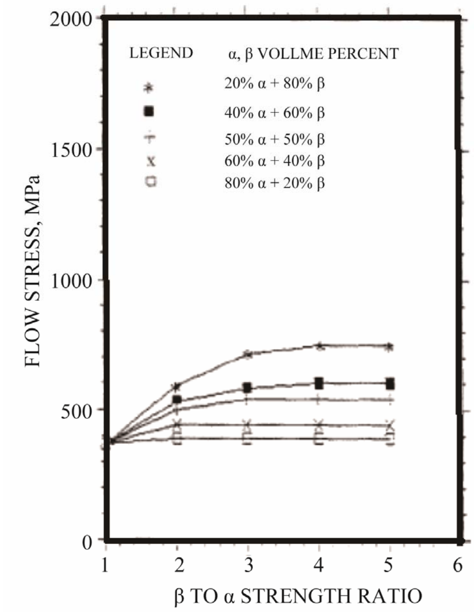
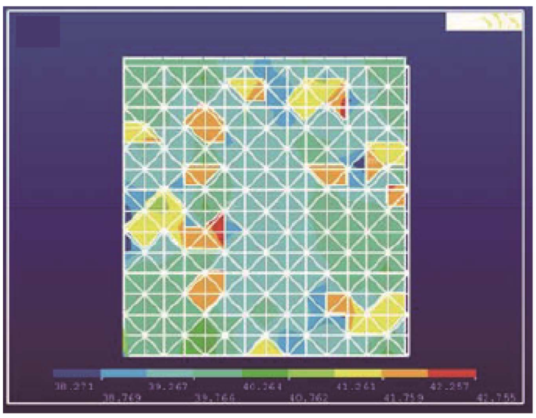
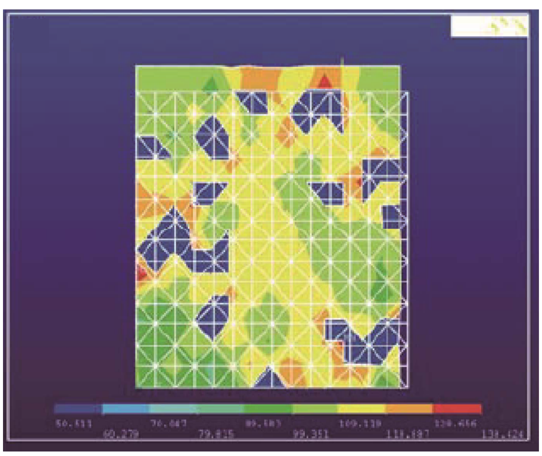


Figure 9. Variation of 0.2% flow stress with strength ratio for different volume per cents of *α* and *β* phases.

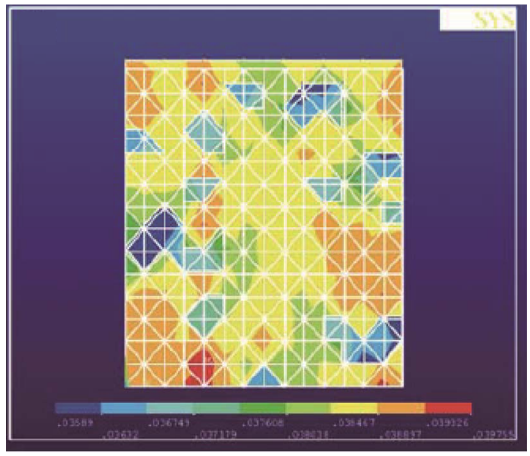


(a)

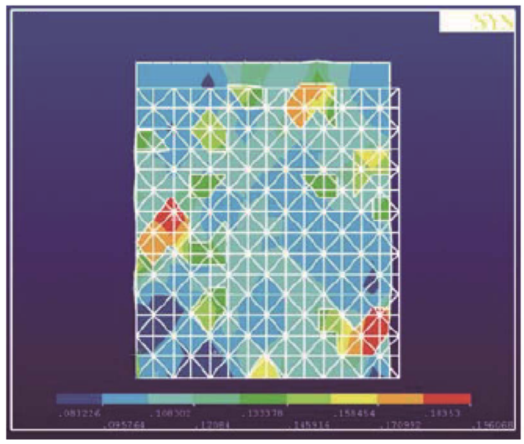


(b)

Figure 10. Distributions of von Mises stress with different loads: (a) 276 MPa; (b) 690 MPa.



(a)



(b)

Figure 11. Distributions of von Mises strain with different loads: (a) 276 MPa; (b) 690 MPa.

it exhibits different microstructures. It is know that titanium is very expensive material, by using these modeling techniques a lot of amount and time can be saved. This finite element modeling will be great useful for the designing and selection of titanium alloys for different engineering applications.

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