Journal of Mathematical Finance, 2019, 9, 591-615
https://www.scirp.org/journal/jmf
ISSN Online: 2162-2442
ISSN Print: 2162-2434

Modelling Volatility Dynamics of Cryptocurrencies Using GARCH Models

Anthony Ngunyi, Simon Mundia, Cyprian Omari*

Department of Statistics and Actuarial Science, Dedan Kimathi University of Technology, Nyeri, Kenya
Email: antony.ngunyi@dkut.ac.ke, simon.mundia@dkut.ac.ke, *cyomari@dkut.ac.ke

How to cite this paper: Ngunyi, A., Mundia, S. and Omari, C. (2019) Modelling Volatility Dynamics of Cryptocurrencies Using GARCH Models. *Journal of Mathematical Finance*, **9**, 591-615.
https://doi.org/10.4236/jmf.2019.94030

Received: January 2, 2019
Accepted: October 14, 2019
Published: October 17, 2019

Abstract

Cryptocurrencies have become increasingly popular in recent years attracting the attention of the media, academia, investors, speculators, regulators, and governments worldwide. This paper focuses on modelling the volatility dynamics of eight most popular cryptocurrencies in terms of their market capitalization for the period starting from 7th August 2015 to 1st August 2018. In particular, we consider the following cryptocurrencies; Bitcoin, Ethereum, Litecoin, Ripple, Moreno, Dash, Stellar and NEM. The GARCH-type models assuming different distributions for the innovations term are fitted to cryptocurrencies data and their adequacy is evaluated using diagnostic tests. The selected optimal GARCH-type models are then used to simulate out-of-sample volatility forecasts which are in turn utilized to estimate the one-day-ahead VaR forecasts. The empirical results demonstrate that the optimal in-sample GARCH-type specifications vary from the selected out-of-sample VaR forecasts models for all cryptocurrencies. Whilst the empirical results do not guarantee a straightforward preference among GARCH-type models, the asymmetric GARCH models with long memory property and heavy-tailed innovations distributions overall perform better for all cryptocurrencies.

Keywords

Bitcoin, Backtesting, Cryptocurrencies, GARCH, Volatility, Value-at-Risk

1. Introduction

The cryptocurrency market has experienced exponential growth in recent years within a short period of its existence. Cryptocurrencies have become increasingly popular attracting wide coverage from the media and drawing the attention of academia, investors, speculators, regulators, and governments worldwide. A cryptocurrency is a digital asset initially designed to work as a medium of exchange using cryptography [1]. Since the invention of Bitcoin in January 2009, approximately over 1600 other cryptocurrencies have been developed and existed at some point.

For the period from January 2017 to December 2017, the market capitalization of the cryptocurrency market increased exponentially. The cryptocurrency market crossed the $100 billion market capitalization for the first time in June 2017, following months of consistent growth [2]. Bitcoin's price jumped from about 998 to 19,497 US dollars reaching its all-time high price of 19,891 US dollars on 17th December 2017 on the Bitfinex exchange[1]. The remarkably exponential growth was also noticible for other cryptocurrencies like Ripple, Ethereum, Litecoin, Moreno, Dash, Stellar and others during this period. According to CoinMarketCap [3], the total value of all cryptocurrencies hit the all-time high market capitalization in January 2018 of approximately $830 billion.

The cryptocurrency market also experienced its fair share of ups and downs in the year 2018 with events like exchange hacks, market surges and major developments on networks. The hacking of Japan's largest cryptocurrency OTC market on 26th of January 2018 and the subsequent loss of 530 million US dollars worth of the NEM is the largest ever event of cryptocurrency theft in the history of cryptocurrency markets. The price of Bitcoin lost about 65 percent of its price in a month reaching about 6000 US dollars between January 26, 2018 and February 6, 2018. In March 2018, Coinbase launched the Coinbase Index Fund which tracks the overall performance of the digital assets listed by Coinbase weighted by market capitalization. Late in March 2018, social media giants Facebook, Twitter and search engine Google banned all advertisements related to cryptocurrencies and for initial coin offerings (ICO) and token sales. By the end of the first quarter of 2018, the cryptocurrencies' market lost about 342 billion US dollars [3]. In other developments, Bitflyer[2] a cryptocurrency exchange became the first regulated exchange in Japan, US and Europe in June 2018.

By September 2018, cryptocurrencies collapsed 80% of their market capitalization from their highest point in January 2018. This cryptocurrency crash (also known as the Bitcoin Crash) is the worst in the history of cryptocurrencies. By November 15, 2018, Bitcoin's market capitalization recorded less than 100 billion dollars for the first time since October 2017. Bitcoin being the world's most widely traded cryptocurrency reflects mounting investor uncertainty over the future of digital currencies. As of 22 December 2018, there were 2067 cryptocurrencies with market value and actively traded in 16,055 cryptocurrency markets and OTC trading desks across the world that are listed on coinmarketcap[3]. The market capitalization of all the cryptocurrencies stands at $128 billions according to figures from CoinMarketcap.com. The top ten cryptocurrencies represent approximately 85% of the total market value, with Bitcoin dominating with

[1]https://www.bitfinex.com
[2]http://www.bitflyer.com/
[3]http://www.coinmarketcap.com/

about 53% of the market capitalization. Bitcoin is currently the largest blockchain network, followed by Ripple, Ethereum and Bitcoin cash respectively [3].

The rest of the paper is structured as follows: Section 2 reviews the cryptocurrency literature. Section 3 presents the GARCH modelling framework including the Maximum Likelihood (ML) estimation of the models with the selected innovations distribution assumptions, VaR estimation and backtesting procedures. Section 4 presents data and some preliminary summary descriptive statistics. Section 5 provides estimation results and empirical results of the VaR backtesting tests and Section 6 concludes the paper.

2. Literature Review

Cryptocurrencies are generally characterized by high volatility dynamics and extremely erratic price jumps. The cryptocurrency markets still remains a potential source of financial instability and the impact of the unprecedented growth of cryptocurrencies to the financial markets still remains uncertain. Unlike the financial securities like stocks and commodities with regulators and conventional currencies with central banks, cryptocurrencies are completely decentralized and also lack any formal regulation of their markets. There is also limited understanding of the cryptocurrencies as investments assets. Governments and financial market regulatory bodies are particularly concerned about the lack of a formal regulatory framework to regulate the creation of new cryptocurrencies, as well as trading mechanisms in the cryptocurrency markets.

Empirical evidence suggests that cryptocurrencies share most of the stylized facts with financial time series, such as stocks and currencies returns. For example, just like stock prices, cryptocurrency prices also exhibit; time-varying volatility, volatility clustering, asymmetric response to the sign of historical observations of the volatility process (*i.e.* leverage effects), heavy-tailed distributions and long memory. Cryptocurrencies are also known to be highly volatile and exhibit extreme price jumps compared to traditional financial securities like currencies and are leptokurtic. Osterrieder and Lorenz [4] suggests that Bitcoin returns not only exhibit higher volatility than conventional fiat currencies but also non-normal and heavy-tailed characteristics. Another important feature of cryptocurrencies is that as opposed to sovereign currencies in a one-money economy there are several types of such cryptocurrencies available in the market.

Over the last few years, there has been increased interest in Bitcoin and other cryptocurrencies generally. With the ever increasing interest in cryptocurrencies and their importance in the financial world, there is need for a comprehensive analysis to study volatility dynamics and out-of-sample forecasting behaviour of the cryptocurrencies. However, despite the growing interest, acceptance and integration of cryptocurrencies to the global financial markets, there is limited research on modelling cryptocurrencies' volatility dynamics. Most of the previous studies have mostly focussed on the Bitcoin market (see e.g. [5]-[14]).

However, there have also been several studies on modelling volatility dynamics of the cryptocurrency market recently, for instance, Dyhrberg [15] estimated the volatility of the Bitcoin, Gold and the US Dollar using the GARCH and asymmetric EGARCH models and concludes that they have similarities and respond the same way to variables in the GARCH model, arguing that it can be used for hedging. Katsiampa [16] analyzed the Bitcoin volatility using a range of GARCH-type models assuming normally distributed errors and concludes that AR (1)-CGARCH (1, 1) is the best model to estimate Bitcoin returns volatility. Charles and Darn [17] replicate the study of Katsiampa considering the presence of extreme observations and using jump-filtered returns and the AR (1)-GARCH (1, 1) model is selected as the optimal model. Pichl and Kaizoji [18] study the time-varying realized volatility of Bitcoin and conclude that it is significantly bigger compared to that of fiat currencies. Bariviera [19] investigate the time-varying volatility the behaviour of long memory on Bitcoin returns using the Hurst exponent analysis. Urquhart and Zhang [20] model a range of GARCH volatility models and analysis the hedging ability of the crypto-coin against other currencies. In terms of different innovations distributions, Liu and Tsyvinski [21] compare the performance of the normal reciprocal inverse Gaussian (NRIG) with the normal distribution and the Student's t error distributions under the GARCH framework and concludes that the GARCH-type model with Student's, t distributed innovations outperform the new heavy-tailed distribution in modelling the Bitcoin returns. Chu *et al.* [22] estimated the volatility of seven cryptocurrencies using GARCH-type models with different innovations distributions and conclude that the IGARCH (1, 1) model is the most appropriate in estimating Bitcoin volatility.

Unfortunately, the majority of recent studies have focused entirely on the Bitcoin behaviour or a few other cryptocurrencies and specifically on the in-sample modelling framework. Trucios [23] estimated the one step-ahead-ahead volatility forecast using several GARCH-type models and also estimate Value-at-Risk taking into consideration the presence of outliers. Naimy and Hayek [24] compare the one-step-ahead volatility forecasting ability of the GARCH, the EWMA, and the EGARCH models with normal, Student's t and generalized error distributions. The forecasted volatility is compared with the realized volatility using the mean absolute error (MAE), mean absolute percentage error (MAPE) and the root mean square error (RMSE) and concludes that the EGARCH model performs best amongst the models considered. Peng *et al.* [25] compare the volatility forecast estimated by GARCH, EGARCH and GJRGARCH models assuming symmetric and asymmetric Gaussian and Student-t errors against the Support Vector Regression GARCH model and they concluded that the later results in more accurate forecasts. Moreover, most of out-of-sample comparisons focusing on the time-varying volatility dynamics of the cryptocurrency market available in the literature are restrictive since they only consider few models leaving out several GARCH-type models, and several innovations distributions.

This paper focusses on analyzing conditional volatility dynamics over eight most popular cryptocurrencies, *i.e.* Bitcoin, Ethereum, Litecoin, Ripple, Moreno, Dash, Stellar and NEM by market capitalization. The aim is to determine the most appropriate GARCH-type model as well as the best fitting distribution to model the volatility of the major cryptocurrencies returns. This study contributes and extends existing literature on modelling cryptocurrencies volatility dynamics by employing a wider range of GARCH-type models, nine different innovations term distributions and a longer time period to try and fill a gap in the literature. First, a comprehensive in-sample volatility modelling is implemented and their goodness of fit is checked in terms of information selection criteria. The most appropriate GARCH-type models are used to estimate the out-of-sample Value at Risk (VaR) forecasts. The conditional and unconditional coverage tests are used to backtest the accuracy of VaR forecasts. Finally, a comprehensive out-of-sample comparison is implemented to investigate the effects of long memory in the volatility process as well as the asymmetric responses to historical values of the return series to forecast volatility.

3. Methodology

This section illustrates the theoretical GARCH modelling framework. First, we outline the alternative Generalized Autoregressive Conditionally Heteroscedastic (GARCH)-type specifications that are used to model time-varying volatility in cryptocurrencies return series and also provide an overview of the set of innovations distributions. Secondly, the selection criteria that will be used to determine the most appropriate GARCH-type specifications are also described. Finally, we describe the estimation of one-day-ahead Value-at-Risk (VaR) forecasts and backtesting procedures.

3.1. The GARCH Models

The GARCH-type models are commonly employed in modelling conditional volatility often present in financial time series. Let P_t denote the price of an asset (*i.e.* cryptocurrency exchange rates) at time *t*, $r_t = \ln\left(P_t / P_{t-1}\right)$ is the continuously compounded return series, for $t = 1, \cdots, n$. The return series of interest, r_t, can be decomposed as follows;

$$r_t = \mu_t + \varepsilon_t,$$
$$\varepsilon_t = \sigma_t z_t, \tag{1}$$

where $\mu_t = E\left(r_t \mid F_{t-1}\right)$ is the conditional mean given the information set F_{t-1}, $\{\varepsilon_t\}$ are the return innovations, $\sigma_t^2 = \operatorname{Var}\left(r_t \mid F_{t-1}\right) = E\left[\left(r_t - \mu_t\right)^2 \mid F_{t-1}\right]$ is the conditional variance of the process $\{\varepsilon_t\}$ and $\{z_t\}$ are independent and identically distributed (i.i.d.) innovations with zero mean and unit variance.

The conditional variance equation for standard GARCH (1, 1) model introduced by Bollerslev [26] is given by

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{2}$$

where $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ are unknown parameters. The restrictions on parameters ensure that the conditional variance is always positive. The necessary and sufficient condition for 2 to be uniquely stationary is $\alpha + \beta < 1$ and the unconditional variance is given by $\omega/\left(1-(\alpha+\beta)\right)$, thus higher order moments exist. If the GARCH model is correctly specified it will converge to this long term variance as the forecast horizon is increased.

In this paper, twelve GARCH-type specifications are employed in modelling the volatility behaviour of cryptocurrencies, namely: SGARCH, IGARCH, EGARCH, GJR-GARCH, TGARCH, APARCH, CSGARCH, AVGARCH, NGARCH, NAGARCH, FGARCH, and FIGARCH models. All the GARCH-type models implemented follow the same specification in Equation (1); however, in each case, the models are distinguished by the evolution of the volatility process σ_t^2 over time. The GARCH extensions involve different specifications for the conditional variance component. For brevity we consider only the first order lags in all GARCH models, *i.e.* $p = q = 1$, since empirical evidence suggests that higher order models rarely performed better than the lower order models in the out-of-sample analysis [27]. The conditional variance equations for all the GARCH-type specifications implemented are summarized in **Table 1**. The necessary conditions for stationarity (weak or strong) for most GARCH-type models are well documented in the literature.

Additionally, for all GARCH-type models, the innovation term $\{z_t\}$ follow one of the nine distributions; Normal distribution, Skew-Normal distribution, (Skew)-Student's t distribution, (Skew)-GED, (Skew)-Student (GH), Normal Inverse Gaussian (NIG), Generalized Hyperbolic (GH) and the Johnson's reparametrized SU (JSU) distribution; see Ghalanos [28] for details of the distributions and GARCH-type models considered in this paper. These distributions are selected to account for skewness, excess kurtosis and heavy-tailedness in financial return series. However, it is important to note that assuming a parametric distribution for the return innovations may lead to mis-specification errors which can compromise the estimate and forecast of volatility.

The parameters of all GARCH-type models are generally estimated using the Quasi-maximum likelihood estimation (QMLE) method. The Quasi-maximum likelihood estimator (QMLE) is preferred since, according to Bollerslev and Wooldridge [29], it is generally consistent, has a normal limiting distribution and provides asymptotic standard errors that are valid under non-normality. The selection of the optimal GARCH model is based on three information criteria: Akaike information criterion (AIC), Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC). The model with the lowest AIC, BIC and HQIC score is assumed to be the most appropriate model with the best fit. The purpose of selecting these optimal GARCH-type models is to forecast the one-day-ahead conditional variance (volatility) that is used to estimate VaR forecasts.

Table 1. The conditional variance of GARCH-type models.

Model	Conditional variance equation	Proposed by
IGARCH	$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + (1-\alpha)\sigma_{t-1}^2$	Engle and Bollerslev [30]
EGARCH	$\ln(\sigma_t^2) = \omega + \alpha\varepsilon_{t-1}^2 + \gamma(\lvert\varepsilon_{t-1}\rvert - E(\lvert\varepsilon_{t-1}\rvert)) + \beta\ln(\sigma_{t-1}^2)$	Nelson [31]
GJR	$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma I(\varepsilon_{t-1} < 0)\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$	Glosten *et al.* [32]
APARCH	$\sigma_t^\delta = \omega + \alpha(\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1})^\delta + \beta\sigma_{t-1}^\delta$	Ding *et al.* [33]
CSGARCH	$\sigma_t^2 = q_t + \alpha(r_{t-1}^2 + q_{t-1}) + \beta(\sigma_{t-1}^2 + q_{t-1})$	
	$q_t = \omega + \rho q_{t-1} + \phi\varepsilon_{t-1}^2 - \sigma_{t-1}^2$	Engle and Lee [34]
TGARCH	$\sigma_t = \omega + \alpha\sigma_{t-1}(\lvert\varepsilon_{t-1}\rvert - \eta_1\varepsilon_{t-1}) + \beta\sigma_{t-1}$	Zakoian [35]
AVGARCH	$\sigma_t = \omega + \alpha\sigma_{t-1}(\lvert\varepsilon_{t-1} - \eta_2\rvert - \eta_1(\varepsilon_{t-1} - \eta_2)) + \beta\sigma_{t-1}$	Schwert and Seguin [36]
NGARCH	$\sigma_t^\delta = \omega + \alpha\sigma_{t-1}^\delta(\lvert\varepsilon_{t-1}\rvert)^\delta + \beta\sigma_{t-1}^\delta$	Higgins and Bera [37]
NAGARCH	$\sigma_t^2 = \omega + \alpha\sigma_{t-1}^2(\lvert\varepsilon_{t-1} - \eta_2\rvert)^2 + \beta\sigma_{t-1}^2$	Engle and Ng [38]
FGARCH	$\sigma_t^\delta = \omega + \alpha\sigma_t^\delta(\lvert\varepsilon_{t-1} - \eta_2\rvert - \eta_1(\varepsilon_{t-1} - \eta_2))^\delta + \beta\sigma_{t-1}^\delta$	Hentschel *et al.* [39]
FIGARCH	$\phi(L)(1-L)^d\varepsilon_t^2 = \alpha_0 + [1-\beta(L)]v_t$	Baillie *et al.* [40]

3.2. VaR Forecast Using GARCH Models

Value-at-Risk (or VaR) is a standard risk measure that is commonly used in risk management which summarizes the downside risk into a single value. It is defined as the maximum loss expected due to a change in the investment position with a given probability over a specific period of time. The VaR forecast for the GARCH-type models relies on the one-day-ahead conditional variance forecast, σ_{t+1}^2 of the volatility model. To this extent, one-step ahead forecasts of the conditional variance of returns is recursively obtained as:

$$\hat{\sigma}_{t+1}^2 = E(\sigma_{t+1}^2 \mid F_t), \quad (3)$$

where F_t is the information set at time *t*, and σ_t^2 is defined as in **Table 2**. The rolling-fixed-window estimation procedure is used to evaluate the out-of-sample performance of the GARCH-type models. In each window, the parameters of the GARCH-type models are estimated and then used to determine the one-step-ahead forecasts of the conditional mean, conditional variance and standardized residuals.

For each GARCH-type model, under the assumption of different innovations term distribution the one-day-ahead VaR forecast at $\alpha\%$ confidence level is obtained as:

$$\widehat{\text{VaR}}_{t+1}(\alpha) = \hat{\mu}_{t+1} + F^{-1}(\alpha)\hat{\sigma}_{t+1} \quad (4)$$

where $F^{-1}(\alpha)$ is the α-quantile of the cumulative distribution function of the innovations distribution. All the twelve GARCH-type models proposed in the previous section are used calculate the econometric VaR assuming the nine innovations distributions for all the cryptocurrencies.

Table 2. Descriptive statistics and statistical tests for daily cryptocurrencies returns for the entire sample period starting from 7th August 2015 to 1st August 2018.

	Bitcoin	Ethereum	Monero	Litecoin	Dash	Ripple	Stellar	NEM
Nobs	1090.000	1090.000	1090.000	1090.000	1090.000	1090.000	1090.000	1090.000
Min	−0.202077	−1.373989	−0.291734	−0.391050	−0.243432	−0.601706	−0.333422	−0.430828
Max	0.223513	0.403457	0.567670	0.518452	0.383096	1.010963	0.704038	1.068486
Mean	0.003053	0.004617	0.004699	0.002739	0.003789	0.003665	0.004314	0.006469
Std.Dev	0.040218	0.080835	0.072959	0.058599	0.059552	0.077085	0.086585	0.094524
Skewness	−0.160668	−4.207377	1.023870	1.351149	0.875008	3.091153	2.083472	2.179137
Kurtosis	4.568298	78.519090	7.087234	13.339778	5.097968	38.634223	14.509124	19.372076
JB	958.2979	284,299.01	2484.144	8450.867	1326.721	69,798.432	10,394.084	17,981.302
p– *value*	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
				Skewness Test statistic				
Statistic	−3.756248	−98.22221	23.90249	31.54289	20.42727	72.16369	48.63914	50.87247
p-value	(0.00009)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
				ADF Test statistics				
Statistic	−2.7535	−9.2397	−9.6014	−9.36	−8.8872	−8.343	−8.827	−9.2518
P-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
				Ljung-Box Test statistics at various lags				
Q(5)	3.9483	17.086	14.409	4.0676	7.5975	28.37	18.857	22.819
p-value	(0.5569)	(0.004339)	(0.01321)	(0.5397)	(0.1799)	(0.00001)	(0.002044)	(0.000366)
Q(10)	9.4171	23.716	34.91	24.166	16.638	46.097	29.805	29.725
p-value	(0.493)	(0.008391)	(0.0001294)	(0.007172)	(0.08277)	(1.377e-06)	(0.0009219)	(0.0009501)
Q(15)	11.065	27.348	37.383	31.254	21.286	51.782	32.263	29.725
p-value	(0.748)	(0.02602)	(0.001112)	(0.008123)	(0.128)	(6.138e-06)	(0.005928)	(0.0009501)
Q(20)	24.583	43.69	43.322	37.097	38.457	67.26	39.917	45.247
p-value	(0.2178)	(0.001655)	(0.001853)	(0.01139)	(0.007783)	(5.069e-07)	(0.005118)	(0.001022)
				ARCH-LM Test statistics at various lags				
LM (5)	76.774	252.35	43.329	40.299	42.939	109.96	189.39	29.864
p-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.00002)
LM (10)	85.385	144.71	107.36	67.757	67.394	120.83	191.84	32.942
p-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.00028)
LM(20)	106.63	123.8	124.14	81.364	78.707	123.99	197.94	33.56
p-value	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.02926)

3.3. Testing the Accuracy of VaR Forecasts

The accuracy of the volatility models in predicting VaR is assessed using statistical backtesting methods. The starting point is normally to compare the out-of-sample VaR forecasts with the actual realized returns in the next time period and this is summarized in terms of a hit ratio. $\{I_t\}$ is a sequence of violations, where it takes the value one if the ex-post loss exceeds the VaR predicted at time $t+1$ and the value zero otherwise. Mathematically, the hit function which is also referred to as the indicator function is defined as:

$$I_t(\alpha) = \begin{cases} 1 & \text{if } r_{t+1} < \text{VaR}_{t+1|t}(\alpha) \\ 0 & \text{if } r_{t+1} \geq \text{VaR}_{t+1|t}(\alpha) \end{cases} \tag{5}$$

where $\{r_1, r_2, \cdots, r_N\}$ is the sequence of daily return, α is the quantile level of coverage defined by its confidence level.

For a VaR model to be accurate in its predictions, then the average hit ratio or the failure rate over the full sample should be equal α for the $(1-\alpha)$th quantile VaR (*i.e.*, for 97.5% VaR, $\alpha = 2.5\%$). As expected, the closer the hit ratio is to the expected value $(1-\alpha)$, the better the forecasts of the risk model. If the hit ratio is greater than the expectation, then the model underestimates the risk; with a hit ratio smaller than $(1-\alpha)$, the model overestimates risk. In this study, two accuracy measure tests: Kupiec [41] unconditional coverage test and Christoffersen [42] conditional coverage test are used to perform the back-testing of the GARCH model for the correct number of exceedances. The unconditional coverage test checks whether the violation ratio or failure rate, during the selected time interval, are in accordance with the chosen confidence level. This implies that the probability of realizing a VaR violation should be equal to $\alpha \times 100\%$. On the other hand, conditional coverage tests examine whether the hits are serially independent of each other over time.

3.3.1. Unconditional Coverage Test

Kupiec [41] proposed the unconditional coverage test which is a likelihood ratio test for testing the model accuracy. Let $N = \sum_{t=1}^{T} I_t$ be the number of observed exceedances over a sample period of length, *T*, *i.e.*, the number of days when realized loss exceeds the VaR forecast. The number of exceedances follows a binomial distribution where the expected failure rate is $\hat{p} = N/T$. Under the Null hypothesis, the probability of failure for each trial ($\hat{p}$) should be equals to (*p*). The appropriate likelihood ratio statistic is:

$$\text{LR}_{\text{uc}} = -2\ln\left[\frac{p^N (1-p)^{T-N}}{\left(\frac{N}{T}\right)^N \left(1-\frac{N}{T}\right)^{T-N}}\right] \tag{6}$$

The Kupiec's unconditional coverage test has a chi-square distribution, asymptotically, with one degree of freedom. The test can be employed to test whether the sample point estimate is statistically consistent with the VaR model's prescribed confidence level. This can reject a model that either overestimates or underestimates the true but unobservable VaR, however, it cannot scrutinize whether the exceptions are randomly distributed.

3.3.2. Conditional Coverage Test

According to Christoffersen [42], it is important that the VaR violations should be spread out over time such that an adequate risk model would not yield VaR violation clusters. In order for the VaR forecast model to be accurate, the hit sequence has to satisfy the two properties of correct failure rate and independence of exceptions. This implies that both the independence and unconditional coverage tests based on the evaluation of interval forecasts must be simultaneously considered when comparing GARCH-type models for VaR forecasting. Christoffersen [42] thus proposed a conditional coverage test (LR_{cc}) to jointly test the correct unconditional coverage and serial independence. The LR_{cc} test is a joint test of these two properties and the corresponding test statistic is the sum of the individual test statistics for the properties; *i.e.*, $\mathrm{LR}_{cc} = \mathrm{LR}_{uc} + \mathrm{LR}_{ind}$ when conditioned on the first observation. The LR_{ind} test denotes the likelihood ratio statistic that tests whether exceptions are independent, and the LR_{uc} is defined in the previous subsection. Thus, under the null hypothesis of the expected proportion of exceptions equals p and the failure process is independent, the appropriate likelihood ratio test statistic is expressed as follows:

$$\mathrm{LR}_{cc} = -2\ln\left[\frac{p^N (1-p)^{T-N}}{\hat{\pi}_{01}^{n_{01}} (1-\hat{\pi}_{01})^{n_{00}} \hat{\pi}_{11}^{n_{11}} (1-\hat{\pi}_{11})^{n_{10}}}\right] \tag{7}$$

where n_{ij} denotes the number of observations with value i followed by value j $(i, j = 0,1)$, $\pi_{ij} = P\{I_t = j \mid T_{t-1} = i\}$ $(i, j = 0,1)$, $\hat{\pi}_{01} = n_{01}/(n_{00} + n_{01})$, and $\hat{\pi}_{11} = n_{11}/(n_{10} + n_{11})$. The Christoffersen's conditional coverage test has an asymptotically chi-square distribution, with two degrees of freedom.

4. Data and Descriptive Statistics

The sample data used in this empirical study was extracted from http://www.investing.com/. Specifically, the data consists of the daily closing prices of cryptocurrencies starting from 7th August 2015 until the 1st August 2018. The full sample data yields a total of 1091 daily observations, including weekends since trading in cryptocurrencies is not restricted to business days or the trading hours of stock exchanges. A starts date of 7th August 2015 was purposely chosen so that we can analyze eight of the top fifteen cryptocurrencies, ranked according to their market capitalization, as of 7th August 2018 (see [3]) for the latest rankings of cryptocurrencies by market capitalization. A total of eight cryptocurrencies are selected to be part of our sample data: Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Stellar (XLM), Litecoin (LTC), Monero (XMR), Dash (DASH) and NEM. The eight are the most popular cryptocurrencies in terms of their market capitalization representing about 85% of total market capitalization as of 7th August 2018 (CoinMarketCap 2018). However, due to the non-availability of data, several cryptocurrencies were omitted from our sample sets such as Bitcoin Cash, EOS, Cardano, IOTA, TRON, LISK, and NEO. On the other hand, Tether (USDT) was also eliminated since it did not conform to the stylized characteristics of financial time series data. The daily prices of the crypto currencies are transformed into continuously compounded returns; $r_t = \log\left(P_t / P_{t-1}\right)$, where P_t is the daily closing cryptocurrency price at time *t*.

Figure 1 presents time series plots of daily prices for the eight cryptocurrencies. All cryptocurrencies illustrate instances of high price volatility and the highest extreme price jumps is recorded in January 2018. The returns plots of the nine cryptocurrencies are also presented in Figure 2. The returns are characterized by patterns of time-varying volatility clustering where periods of high (low) volatility are followed by periods of high (low) volatility. The time-varying behaviour of cryptocurrencies returns suggests the presence of stylized characteristics normally exhibited by financial time series data.

The summary descriptive statistics and statistical tests results for the daily returns of each cryptocurrency are presented in Table 2. The statistics include the maximum, minimum, mean, standard deviation, skewness, kurtosis, Jarque-Bera statistics, Ljung-Box statistics for raw and squared returns. During the sample period, the average return for all cryptocurrencies is positive and relatively close to zero indicating that future prices tend to increase with time. The sample standard deviations indicate high volatility for all cryptocurrencies returns. The excess kurtosis values reported indicate that all cryptocurrencies are heavy tailed

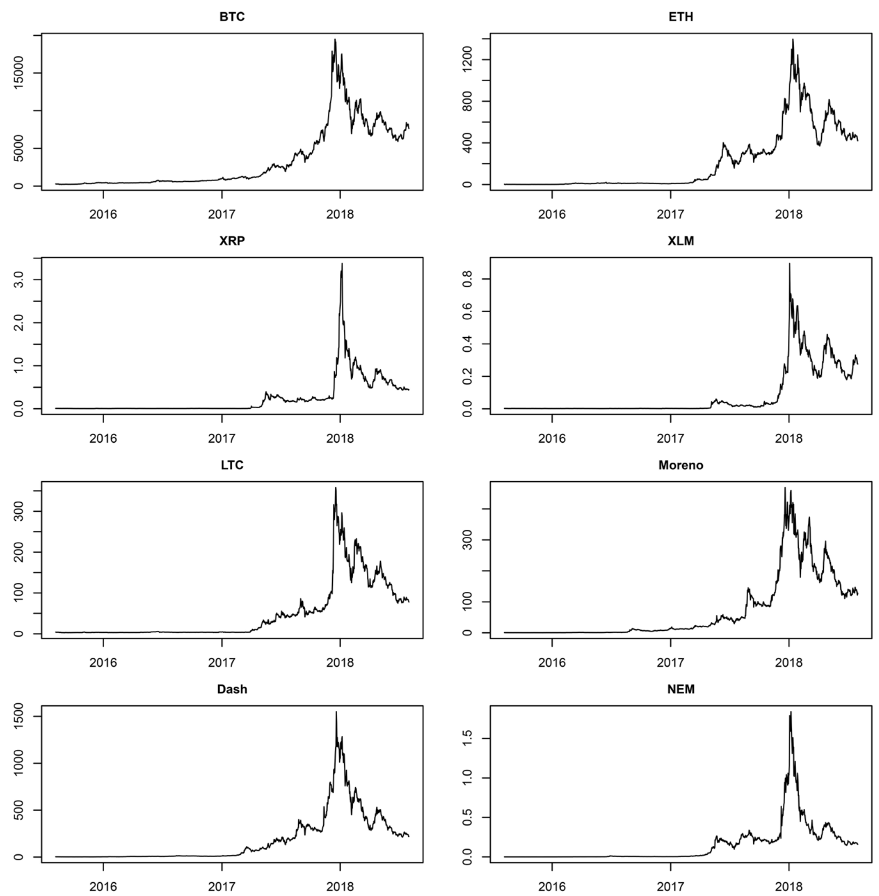

Figure 1. Daily closing prices of cryptocurrencies (period from August 7, 2015 to August 1, 2018).

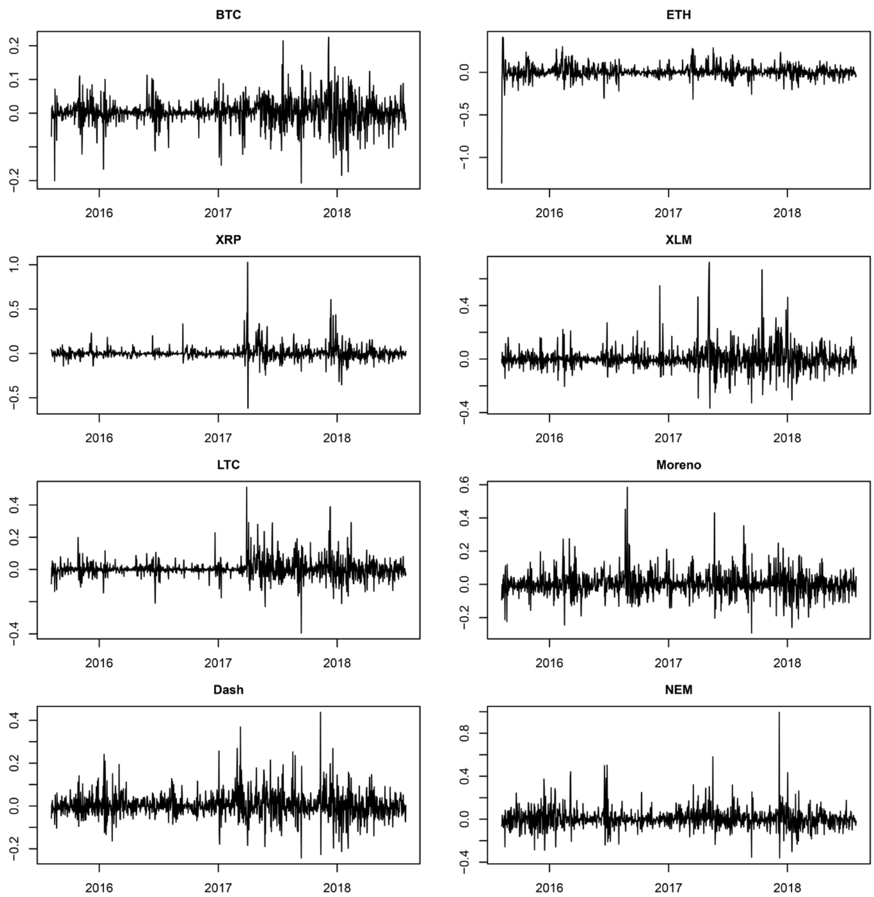

Figure 2. Returns plot between August 7, 2015 and January 1, 2019 of nine cryptocurrencies.

and exhibit leptokurtic behaviour beyond that of the normal distribution, with the most peaked being those of Ethereum and Ripple. Moreover, only Bitcoin and Ethereum are negatively skewed while other cryptocurrencies are positively skewed. Additionally, the Jarque-Bera statistic confirms that all cryptocurrencies are not normally distributed. The Augmented Dickey Fuller (ADF) test results reject unit root hypothesis for all cryptocurrencies series, implying that the series are assumed to be stationary. Ljung-Box (Q) statistic for raw returns series reject the null hypothesis that all correlation coefficients up to lag 20 are equal to zero in the majority of cases, except for Bitcoin. Therefore we conclude that some return series present some linear dependence. The significant serial correlations reported in the squared returns imply that there is non-linear dependence in the return series. Finally, the ARCH-LM test rejects the no ARCH effect hypothesis, thus indicating the presence of volatility clustering, long memory and a GARCH-type specification should be considered in the modelling of cryptocurrencies.

5. Empirical Results and Analysis

In this study, twelve GARCH-type models: the SGARCH, IGARCH, EGARCH,

GJR-GARCH, TGARCH, APARCH, CSGARCH, AVGARCH, NGARCH, NAGARCH, FGARCH, and FIGARCH models are utilized to model the conditional volatility and estimate one-step-ahead VaR forecast of the eight cryptocurrencies. Further, two backtesting measures: the conditional and unconditional coverage tests are used to evaluate the out-of-sample VaR forecasts performance of the twelve GARCH models. Prior to implementing the comparative performance of VaR forecast for the above twelve GARCH models, the fitting of the implemented twelve models is explored via the empirical results of the parameter estimates for the competing models.

5.1. Parameter Estimates for Fitted GARCH Models

First, the best fitting ARMA models for the mean components are selected via the information criteria: the Akaike Information Criterion (AIC). The ARMA (p, q) specification for Bitcoin, Ethereum, Monero, Litecoin, and Dash are assumed to be equal to zero, based on the BIC. This indicates that even the AR (1) model is not necessary since there is no significant degree of serial autocorrelation in cryptocurrencies returns. The most appropriate models for Stellar and NEM are MA (1) and MA (2) respectively, while AR (1) is selected for Ripple. For brevity in modelling and forecasting the cryptocurrencies volatility, we assume that mean component is not significant for all the cryptocurrencies return series.

The distribution of the innovations term is also an important component in modelling a GARCH process. For purposes of selecting the most appropriate innovations distribution for all cryptocurrencies, the GARCH (1, 1) model is utilized. The information criteria and log-likelihood results for the fitted GARCH (1, 1) model assuming the nine different innovations distributions are reported in Table 3. Based on the empirical results, we note that the use of skewed and heavy-tailed innovations distributions are justified, as they give better results based on the log-likelihood, AIC and BIC information criteria compared to other innovations distributions like the normal, skewed normal and normal inverse Gaussian (NIG) distributions for all cryptocurrencies returns data. Particularly, the generalized error distribution (GED) has the highest log-likelihood value, as well as the lowest AIC and BIC values respectively among the innovation distributions for Bitcoin and Ethereum. The Johnson's SU distribution is selected for the Litecoin, Dash, Stellar and NEM cryptocurrencies, while the Student-t distribution and the skewed generalized error distributions are selected for Moreno and Ripple cryptocurrencies respectively.

The most appropriate GARCH-type model is selected from the different specifications (GARCH, IGARCH, EGARCH, GJRGARCH, APARCH, TGARCH, NGARCH, NAGARCH, AVGARCH, FIGARCH and HGARCH) fitted to the eight cryptocurrencies with their respective best fitting innovations distribution. Table 4 presents the results for three information criteria: Akaike (AIC) and Schwartz (BIC) and Hannan-Quinn (HQC) for the fitted GARCH-type models. The IGARCH (1, 1) is selected as the most appropriate model for Bitcoin, Moreno and Dash with GED, Student-t and JSU innovations distributions respectively while CSGARCH (1, 1) model selected for Ethereum and Stellar with GED and JSU innovations distributions. The FIGARCH (1, 1) model is selected for Ripple and NEM with SGED and JSU innovations distributions. Finally, TGARCH (1, 1) model with JSU innovations distribution is selected for Litecoin.

Table 3. The AIC, BIC and LLF values for GARCH (1, 1) model for the entire sample period assuming nine different innovations term distributions.

Error Distn	Norm	Skewed Norm	Student t	Skewed Student t	ged	sged	nig	ghyp	jsu
					BitCoin				
AIC	−3.9038	−3.9076	−4.1193	−4.1189	−4.1373	−4.1380	−4.1395	−4.1412	−4.1316
BIC	−3.8763	−3.8755	−4.0872	−4.0823	**−4.1052**	−4.1013	−4.1029	−4.1000	−4.0950
LLF	2133.59	2136.63	2252.02	2252.80	2261.81	2263.19	2264.04	2265.95	2259.75
					Ethereum				
AIC	−2.4230	−2.4596	−2.8341	−2.8377	−2.8346	−2.8348	−2.8304	−2.8302	−2.8381
BIC	−2.3955	−2.4275	−2.8020	−2.8010	**−2.8025**	−2.7982	−2.7938	−2.7889	−2.8014
LLF	1326.51	1347.48	1551.58	1554.53	1551.87	1552.99	1550.57	1551.43	1554.76
					Monero				
AIC	−2.5109	−2.5304	−2.7169	−2.7191	−2.7029	−2.7089	−2.7176	−2.7166	−2.7192
BIC	−2.4834	−2.4984	**−2.6848**	−2.6824	−2.6708	−2.6723	−2.6809	−2.6754	−2.6825
LLF	1374.42	1386.09	1487.69	1489.89	1480.09	1484.36	1489.07	1489.54	1489.96
					Litecoin				
AIC	−3.0858	−2.3136	−3.7286	−3.7305	−3.7298	−3.7300	−3.7429	−3.7410	−3.7460
BIC	−3.0583	−2.2815	−3.6965	−3.6939	−3.6977	−3.6934	−3.7062	−3.6998	**−3.7094**
LLF	1687.77	1267.89	2039.09	2041.13	2039.75	2040.88	2047.87	2047.84	2049.59
					Dash				
AIC	−2.9812	−3.0190	−3.1359	−3.1458	−3.1210	−3.1347	−3.1464	−3.1466	−3.1474
BIC	−2.9537	−2.9869	−3.1039	−3.1091	−3.0889	−3.0981	−3.1098	−3.1054	**−3.1107**
LLF	1630.75	1652.36	1716.08	1722.45	1707.92	1716.41	1722.79	1723.89	1723.32
					Ripple				
AIC	−2.9755	−3.0230	−3.4227	−3.4264	−3.4151	−3.4278	−3.4317	−3.4370	-
BIC	−2.9480	−2.9909	−3.3907	−3.3897	−3.3831	**−3.4278**	−3.3950	−3.3958	-
LLF	1627.67	1654.51	1872.39	1875.37	1868.25	1876.17	1878.27	1882.18	-
					Stellar				
AIC	−2.4069	−2.4698	−2.7620	−2.7724	−2.7429	−2.7526	−2.7778	−2.7775	−2.7784
BIC	−2.3794	−2.4378	−2.7300	−2.7357	−2.7109	−2.7159	−2.7412	−2.7362	**−2.77417**
LLF	1317.78	1353.06	1512.31	1518.94	1501.90	1508.15	1521.92	1522.71	1522.22
					NEM				
AIC	−2.1393	−2.1532	−2.3528	−2.3573	−2.3390	−2.3443	−2.3562	−2.3568	−2.3583
BIC	−2.1118	−2.1212	−2.3208	−2.3206	−2.3069	−2.3076	−2.3195	−2.3156	**−2.3217**
LLF	1171.94	1180.52	1289.30	1292.71	1281.74	1285.62	1292.12	1293.48	1293.29

Table 4. The information criteria values for GARCH-type models fitted over for the sample period with selected innovations distributions.

	GARCH (1, 1)	EGARCH (1, 1)	GJR-GARCH (1, 1)	CSGARCH (1, 1)	APARCH (1, 1)	IGARCH (1, 1)	TGARCH (1, 1)	AVGARCH (1, 1)	NGARCH (1, 1)	NAGARCH (1, 1)	FIGARCH (1, 1)	ALL-GARCH (1, 1)
BitCoin												
AIC	−4.1379	−4.1480	−4.1415	−4.1360	−4.1458	**−4.1399**	−4.1465	−4.1471	−4.1433	−4.1422	−4.1417	−4.1454
BIC	−4.1104	−4.1159	−4.1095	−4.0994	−4.1092	**−4.1170**	−4.1144	−4.1105	−4.1112	−4.1102	−4.1096	−4.1042
HQC	−4.1275	−4.1358	−4.1294	−4.1222	−4.1319	**−4.1313**	−4.1344	−4.1333	−4.1312	−4.1301	−4.1295	−4.1298
Ethereum												
AIC	−2.8299	−2.8337	−2.8283	**−2.8618**	−2.8306	−2.8315	−2.8172	−2.8154	−2.8324	−2.8289	−2.8312	−2.8300
BIC	−2.8024	−2.8016	−2.7962	**−2.8252**	−2.7940	−2.8086	−2.7852	−2.7787	−2.8003	−2.7968	−2.7992	−2.7888
HQC	−2.8195	−2.8215	−2.8161	**−2.8480**	−2.8168	−2.8228	−2.8051	−2.8015	−2.8202	−2.8167	−2.8191	−2.8144
Monero												
AIC	−2.7141	−2.7122	−2.7150	−2.7105	−2.7132	**−2.7158**	−2.7082	−2.7172	−2.7124	−2.7191	−2.7137	−2.7176
BIC	−2.6866	−2.6802	−2.6829	−2.6738	−2.6765	**−2.6929**	−2.6761	−2.6805	−2.6803	−2.6871	−2.6816	−2.6764
HQC	−2.7037	−2.7001	−2.7029	−2.6966	−2.6993	**−2.7071**	−2.6961	−2.7033	−2.7002	−2.7070	−2.7015	−2.7020
Litecoin												
AIC	−3.7133	−3.7321	−3.7241	−3.7289	−3.7421	−3.7155	**−3.7438**	−3.7427	−3.7374	−3.7170	−3.7270	−3.7413
BIC	−3.6859	−3.7001	−3.6920	−3.6923	−3.7054	−3.6926	**−3.7117**	−3.7060	−3.7053	−3.6849	−3.6949	−3.7001
HQC	−3.7029	−3.7200	−3.7120	−3.7151	−3.7282	−3.7069	**−3.7317**	−3.7288	−3.7253	−3.7048	−3.7149	−3.7257
Dash												
AIC	−3.1483	−3.1495	−3.1476	−3.1553	−3.1463	**−3.1493**	−3.1456	−3.1437	−3.1477	−3.1484	−3.1567	−3.1450
BIC	−3.1209	−3.1174	−3.1155	−3.1186	−3.1097	**−3.1264**	−3.1135	−3.1071	−3.1156	−3.1164	−3.1246	−3.1037
HQC	−3.1379	−3.1374	−3.1355	−3.1414	−3.1325	**−3.1406**	−3.1334	−3.1299	−3.1356	−3.1363	−3.1445	−3.1294
Ripple												
AIC	−3.4161	−3.4223	−3.4131	−3.4007	-	−3.4145	−1.3035	−1.2016	−3.4390	−3.4273	**−3.4629**	−3.4298
BIC	−3.3794	−3.3811	−3.3719	−3.3549	-	−3.3824	−1.2623	−1.1558	−3.3978	−3.3861	**−3.4217**	−3.3794
HQC	−3.4022	−3.4067	−3.3975	−3.3834	-	−3.4024	−1.2879	−1.1843	−3.4234	−3.4117	**−3.4473**	−3.4107
Stellar												
AIC	−2.7801	−2.7847	−2.7784	**−2.8172**	−2.7864	−2.7820	−2.7881	−2.7883	−2.7858	−2.7813	−2.8104	−2.7864
BIC	−2.7480	−2.7480	−2.7417	**−2.7759**	−2.7452	−2.7545	−2.7515	−2.7470	−2.7492	−2.7447	−2.7737	−2.7406
HQC	−2.7679	−2.7708	−2.7645	**−2.8016**	−2.7708	−2.7716	−2.7743	−2.7727	−2.7719	−2.7675	−2.7965	−2.7691
NEM												
AIC	−2.3583	−2.3686	−2.3566	−2.3694	−2.3718	−2.3587	−2.3708	−2.3700	−2.3702	−2.3578	**−2.3716**	−2.3717
BIC	−2.3217	−2.3274	−2.3154	−2.3236	−2.3260	−2.3266	−2.3296	−2.3242	−2.3289	−2.3166	**−2.3304**	−2.3213
HQC	−2.3445	−2.3530	−2.3410	−2.3520	−2.3545	−2.3466	−2.3552	−2.3526	−2.3546	−2.3422	**−2.3560**	−2.3527

The parameter estimates for the most appropriate GARCH-type model selected for each cryptocurrencies together with the specifications tests of residual autocorrelation and conditional heteroscedasticity are given in **Table 5**. The empirical results indicate that the estimated parameters are significant in most cases for all cryptocurrencies. The specification tests carried out after estimation failed to capture serial correlation and there are no ARCH effects remaining in the residuals, suggesting that selected GARCH-type models are adequate for the data. The ARCH-LM test applied to residuals indicates that no ARCH effects are present in the residuals. Ljung-Box test indicates that neither the non-linear dependence nor the long memory dependence is present in residual series at 95% confidence level. The Jarque-Bera statistic also indicates that residuals are not normally distributed.

Table 5. Parameter estimation results for selected optimal GARCH-type models.

	Bitcoin	Ethereum	Monero	Litecoin	Dash	Ripple	Stellar	NEM
α	0.001958	0.001771	0.001546	0.001203	0.002788	0:001548	0:000520	0.001481
	(0.024787)	(0.041057)	(0.378756)	(0.104764)	(0.050867)	(0.011039)	(0.676378)	(0.438475)
ω	0.000013	0.000220	0.000397	0.000580	0.000101	0.000031	0.000000	0.000153
	(0.00028)	(0.03340)	(0.004456)	(0.048486)	(0.026018)	(0.221990)	(0.361084)	(0.150275)
α_1	0.145588	0.035488	0.260066	0.209287	0.171916	0.226872	0.26754	0.374642
	(0.00000)	(0.00000)	(0.00000)	(0.000002)	(0.000050)	(0.001339)	(0.000001)	(0.042643)
β_1	0.854412	0.944190	0.739934	0.868738	0.828084	0.103973	0.484999	0.896461
	(NA)	(0.00000)	(NA)	(0.00000)	(NA)	(0.000066)	(0.000002)	(0.00000)
γ	-	-	-	−0.312124	-			
				(0.011762)				
d	-	-	-	-	-	0.302758		1.000000
						(0.00000)		(0.002696)
δ	-	0.999516					0.999936	
		(0.00000)					(0.00000)	
ϕ	-	0.271006	-	-	-		0.023251	
		(0.000014)					(0.00000)	
$\alpha_1 + \beta_1$	1	0.979678	1	1.078025	1	0.33079326	0.752539	1.645745
Skew	0.969862	1.092519	1.091389	0.116474	0.372318	1.116250	0.316974	0.176591
	(0.00000)	(0.00000)	(0.00000)	(0.012925)	(0.000334)	(0.00000)	(0.000023)	(0.023133)
Shape	0.923403	0.992872	3.336433	0.928236	1.391832	0.807481	1.172223	1.237510
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
ARCH-LM test for heteroscedasticity								
Statistic	2.376	2.463	3.5232	0.16112	0.40051	0.19493	0.39728	0.7637
	(0.6381)	(0.6201)	(0.4206)	(0.9982)	(0.9865)	(0.9972)	(0.9867)	(0.9487)

5.2. Backtesting Results for GARCH Models

The accuracy of the different fitted GARCH-type models considered in the study is assessed by using exceedances percentages at 95%, 97.5%, and 99% confidence levels. The exceedances involve counting the number of actual realized returns that exceed the VaR forecast, and comparing this number with the hypothetically expected number of exceedances for a given probability. Obviously, the closer the observed number of exceedances is to the hypothetically expected number, the more preferable the GARCH model is for estimating accurate forecasts.

Table 6 presents results of exceedances percentages obtained from fitted GARCH-type models at different levels of significance in estimating one-day-ahead VaR forecasts. Generally, we observe that the violation rates are exceptionally high at 95% and 97.5% levels compared to the 99% level. All the GARCH models produce a violation rate above the expected exceedances rates at all levels of significance for all the cryptocurrencies. Based on the proximity of the actual violation ratio to the expected violation ratio, different GARCH-type models give the best fit for different cryptocurrencies at different levels. Specifically, at 95% VaR level, among all of the GARCH type models used for forecasting, the APARCH (1, 1) model gives the best fit for Bitcoin and Ethereum; the NAGARCH (1, 1) model gives the best fit for Moreno; NGARCH (1, 1) model gives the best fit for Litecoin and Stellar; CSGARCH (1, 1) model gives the best fit for Dash and NEM; the IGARCH (1, 1) model gives the best fit for Ripple.

For the 99% VaR forecasts, the violation rates are all relatively close to the expected exceedances rates for most of the GARCH-type models and all cryptocurrencies. Some of the GARCH-models selected at 95% level still perform well at 99% level. The APARCH (1, 1) model still gives the best fit for Bitcoin and Ethereum; EGARCH (1, 1) model for Moreno; CSGARCH (1, 1) for Dash. Litecoin, Stellar and also Bitcoin have several best fitting models including; GARCH (1, 1), CSGARCH (1, 1), AVGARCH (1, 1), APARCH (1, 1) and TGARCH (1, 1). Finally, EGARCH (1, 1) and CSGARCH (1, 1) give the best fit for NEM and GARCH (1, 1) and GJR (1, 1) for Ripple.

We also decided to backtest the GARCH-type model analyzed, since every model has a different distribution of residuals. The forecasting and backtesting procedure is implemented using a fixed-rolling-window scheme. This approach allows us to perform a rolling estimation and forecasting of the GARCH-type model, returning the VaR at specified levels of significance. Notably, it generates the distributional forecast parameters necessary to compute any required measure on the forecast density. The parameters of the fitted GARCH-type models are estimated over a window of length 700 observations and are used to predict the conditional variance process for the following day. Each time the window is shifted forward, the daily returns of the following day are added, the oldest daily returns are dropped from the observation window and the parameters are re-estimated over the new period in order to compute the next set of forecasts. This procedure is iterated until the end of the dataset for a total of 300 one-step ahead forecasts.

Table 6. Violation ratios for VaR estimation of the cryptocurrencies data.

Model	GARCH (1, 1)	EGARCH (1, 1)	GJR-GARCH (1, 1)	CS-GARCH (1, 1)	APARCH (1, 1)	IGARCH (1, 1)	TGARCH (1, 1)	AV-GARCH (1, 1)	NGARCH (1, 1)	NA-GARCH (1, 1)	ALL-GARCH (1, 1)
95% level of significance											
Bitcoin	8.0%	6.7%	7.3%	7.7%	**6.3%**	8.0%	6.7%	7.3%	**6.3%**	7.3%	7.0%
Ethereum	7.7%	**7.0%**	7.3%	8.7%	**7.0%**	7.3%	**7.0%**	7.7%	7.3%	7.3%	8.0%
Monero	9.0%	9.3%	9.0%	9.3%	9.3%	9.0%	9.0%	9.0%	9.0%	**8.7%**	--
Litecoin	10.3%	7.3%	10.0%	9.0%	7.7%	10.3%	8.0%	7.0%	**4.7%**	10.0%	7.0%
Dash	8.0%	8.0%	7.3%	**7.0%**	7.3%	8.0%	8.0%	8.0%	7.7%	--	8.3%
Ripple	7.0%	6.0%	7.3%	6.7%	6.7%	**5.3%**	28%	28%	6.0%	6.7%	8.3%
Stellar	7.0%	7.0%	7.3%	--	6.0%	7.0%	6.7%	6.7%	**5.7%**	7.3%	6.0%
NEM	9.0%	9.0%	8.7%	**8.0%**	9.3%	8.7%	9.3%	9.0%	9.0%	9.0%	10.0%
97.5% level of significance											
Bitcoin	3.3%	**2.7%**	4.0%	4.0%	**2.7%**	3.3%	**2.3%**	2.7%	2.0%	4.7%	2.7%
Ethereum	4.0%	3.7%	4.0%	4.3%	3.7%	4.0%	**3.3%**	3.7%	**3.3%**	4.0%	**3.3%**
Monero	5.3%	5.3%	5.3%	5.3%	5.7%	5.3%	5.0%	6.0%	5.3%	5.0%	--
Litecoin	3.7%	2.7%	4.7%	3.7%	1.7%	3.7%	3.0%	**2.3%**	0.7%	6.7%	3.0%
Dash	5.3%	4.3%	5.3%	5.0%	5.7%	5.3%	4.7%	6.0%	5.7%	--	6.3%
Ripple	3.3%	3.3%	3.3%	3.3%	3.0%	**2.3%**	24.3%	24.7%	2.7%	3.0%	4.0%
Stellar	3.3%	3.0%	3.3%	--	2.7%	3.3%	2.7%	2.3%	2.7%	4.3%	2.3%
NEM	4.3%	3.7%	4.3%	3.3%	3.7%	3.3%	3.7%	4.0%	**3.0%**	4.3%	4.7%
99% level of significance											
Bitcoin	1.3%	**1.0%**	1.3%	**1.0%**	**1.0%**	1.3%	**1.0%**	**1.0%**	**1.0%**	1.7%	**1.0%**
Ethereum	1.7%	1.7%	1.7%	1.7%	**1.3%**	1.7%	**1.3%**	1.7%	1.7%	1.7%	1.7%
Monero	1.3%	**1.0%**	1.3%	1.3%	1.3%	1.3%	0.7%	2.0%	1.3%	1.7%	--
Litecoin	**1.0%**	0.7%	1.7%	**1.0%**	**1.0%**	**1.0%**	**1.0%**	**1.0%**	0.3%	2.7%	1.7%
Dash	1.7%	2.0%	1.7%	**1.3%**	2.7%	1.7%	2.0%	2.7%	3.3%	--	4.0%
Ripple	**1.0%**	0.7%	**1.0%**	0.7%	0.7%	0.3%	19%	19%	0.7%	**1.0%**	1.3%
Stellar	2.0%	**1.3%**	2.0%	--	**1.3%**	2.0%	1.7%	**1.3%**	1.7%	2.3%	**1.3%**
NEM	1.7%	**1.0%**	1.7%	**1.0%**	1.3%	1.7%	0.7%	1.7%	1.7%	1.7%	2.0%

The Kupiec's unconditional and Christoffersen's conditional coverage Value-at-Risk exceedances tests are utilized to assess the VaR forecast performance of the twelve GARCH-type models: the SGARCH, IGARCH, EGARCH, GJR-GARCH, TGARCH, APARCH, CSGARCH, AVGARCH, NGARCH, NAGARCH, FGARCH, and the FIGARCH models at 95%, 97.5%, and 99% confidence levels. In principle, the GARCH-type model with the higher number of passes among the two back-testing procedures bear a better performance than the GARCH-type model with the less number that passes. The most appropriate

GARCH-type model according to conditional and unconditional coverage tests is defined as the one with the highest *p*-value amongst all the fitted models for all cryptocurrencies.

Table 7 presents the results of the Kupiec unconditional (LR_{uc}) test and Christoffersen conditional (LR_{cc}) test for twelve GARCH-type models fitted to the cryptocurrencies under 95%, 97.5%, and 99% confidence level. *p*-values of the unconditional coverage and coverage tests are presented in parentheses. Besides the hypothetical expected percentage of exceedances for the 5%, 2.5%, and 1% level of significance, the percentage of actual exceedances is presented for selected quantiles associated with the distribution. In addition, regarding a specified GARCH-type model, the total number of times that a cryptocurrency pass the LR_{uc} and LR_{cc} types of back-testing are counted respectively at different levels

Table 7. The out-of-sample VaR forecasts performance based on Kupiec and Christoffersen accuracy tests.

α		GARCH (1, 1)	EGARCH (1, 1)	GJR-GARCH (1, 1)	CS-GARCH (1, 1)	APARCH (1, 1)	IGARCH (1, 1)	TGARCH (1, 1)	AV-GARCH (1, 1)	NGARCH (1, 1)	NAGARCH (1, 1)	ALL-GARCH (1, 1)
Bitcoin												
1%	LR_{UC}	0.305	0.000	0.305	0.000	0.000	0.305	0.000	0.000	0.000	1.122	0.000
	p-value	(0.581)	(1.000)	(0.581)	(1.000)	(1.000)	(0.581)	(1.000)	(1.000)	(1.000)	(0.290)	(1.000)
	LR_{CC}	0.413	0.061	0.413	0.061	0.061	0.413	0.061	0.061	0.061	1.292	0.061
	p-value	(0.813)	(0.970)	(0.813)	(0.970)	(0.970)	(0.813)	(0.970)	(0.970)	(0.970)	(0.524)	(0.970)
2.5%	LR_{UC}	0.775	0.033	2.350	2.350	0.033	0.775	0.035	0.033	0.330	4.622	0.289
	p-value	(0.379)	(0.855)	(0.125)	(0.125)	(0.855)	(0.379)	(0.852)	(0.855)	(0.566)	(0.032)	(0.591)
	LR_{CC}	1.467	0.473	3.353	3.353	0.473	1.467	0.371	0.473	0.576	4.796	0.848
	p-value	(0.480)	(0.789)	(0.187)	(0.187)	(0.789)	(0.480)	(0.831)	(0.789)	(0.750)	(0.091)	(0.654)
5%	LR_{UC}	4.847	1.596	3.025	3.889	1.039	4.847	1.596	3.025	1.039	3.025	2.259
	p-value	(0.028)	(0.207)	(0.082)	(0.049)	(0.308)	(0.028)	(0.207)	(0.082)	(0.308)	(0.082)	(0.133)
	LR_{CC}	5.583	1.702	3.1169	3.951	1.082	5.583	1.666	3.169	3.479	3.169	2.406
	p-value	(0.061)	(0.427)	(0.205)	(0.139)	(0.582)	(0.061)	(0.435)	(0.205)	(0.176)	(0.205)	(0.300)
Ethereum												
1	LR_{UC}	1.122	1.122	1.122	1.122	0.305	1.122	0.305	1.122	1.122	1.122	1.122
	p-value	(0.290)	(0.290)	(0.290)	(0.290)	(0.581)	(0.290)	(0.581)	(0.290)	(0.290)	(0.290)	(0.290)
	LR_{CC}	1.292	1.292	1.292	1.292	0.413	1.292	0.413	1.292	1.292	1.292	1.292
	p-value	(0.524)	(0.524)	(0.524)	(0.524)	(0.813)	(0.524)	(0.813)	(0.524)	(0.524)	(0.524)	(0.524)
2.5%	LR_{UC}	2.350	1.468	2.350	3.405	1.468	2.350	0.775	1.468	0.775	2.350	0.775
	p-value	(0.125)	(0.226)	(0.125)	(0.065)	(0.226)	(0.125)	(0.379)	(0.226)	(0.379)	(0.125)	(0.379)
	LR_{CC}	3.353	2.308	3.358	4.588	2.308	3.353	1.467	2.308	1.467	3.358	1.467
	p-value	(0.187)	(0.315)	(0.187)	(0.101)	(0.315)	(0.187)	(0.480)	(0.315)	(0.480)	(0.187)	(0.480)
5%	LR_{UC}	3.889	2.259	3.015	7.033	2.259	3.025	2.259	3.889	3.025	3.025	4.847
	p-value	(0.049)	(0.133)	(0.082)	(0.008)	(0.133)	(0.082)	(0.133)	(0.049)	(0.082)	(0.082)	(0.028)
	LR_{CC}	4.747	3.723	4.163	7.300	3.723	4.163	3.723	4.884	4.163	4.163	5.583
	p-value	(0.093)	(0.155)	(0.125)	(0.026)	(0.155)	(0.125)	(0.155)	(0.087)	(0.125)	(0.125)	(0.061)

Continued

Monero												
1%	LR_{UC}	0.305	0.000	0.305	0.305	0.305	0.305	0.382	2.348	0.305	1.122	-
	p-value	(0.581)	(1.000)	(0.581)	(0.581)	(0.581)	(0.581)	(0.537)	(0.125)	(0.581)	(0.290)	-
	LR_{CC}	0.413	0.061	0.413	0.413	0.413	0.413	0.408	2.594	0.413	1.292	-
	p-value	(-813)	(0.970)	(0.813)	(0.813)	(0.813)	(0.813)	(0.815)	(0.273)	(0.813)	(0.524)	-
2.5%	LR_{UC}	7.495	7.495	7.495	7.495	9.134	7.495	5.988	10.898	7.495	5.988	-
	p-value	(0.006)	(0.006)	(0.006)	(0.006)	(0.003)	(0.006)	(0.014)	(0.001)	(0.006)	(0.014)	-
	LR_{CC}	7.521	7.521	7.521	7.521	9.136	7.521	7.524	10.906	7.521	6.071	-
	p-value	(0.023)	(0.023)	(0.023)	(0.023)	(0.010)	(0.023)	(0.023)	(0.004)	(0.023)	(0.048)	-
5%	LR_{UC}	8.253	9.555	8.253	9.555	9.555	8.253	8.253	8.253	8.253	7.033	-
	p-value	(0.004)	(0.002)	(0.004)	(0.002)	(0.002)	(0.004)	(0.004)	(0.004)	(0.004)	(0.008)	
	LR_{CC}	9.397	10.949	9.397	10.949	10.949	9.397	9.397	9.397	9.397	7.949	-
	p-value	(0.009)	(0.004)	(0.009)	(0.004)	(0.004)	(0.009)	(0.009)	(0.009)	(0.009)	(0.019)	-
Litecoin												
1%	LR_{UC}	0.000	0.382	1.122	0.000	0.000	0.000	0.000	0.000	1.816	5.778	1.122
	p-value	(1.000)	(0.537)	(0.290)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(0.178)	(0.016)	(0.290)
	LR_{CC}	0.061	0.408	1.292	0.061	0.061	0.061	0.061	0.061	0.1.823	7.456	4.660
	p-value	(0.970)	(0.815)	(0.524)	(0.970)	(0.970)	(0.970)	(0.970)	(0.970)	(0.402)	(0.024)	(0.099)
2.5%	LR_{UC}	1.468	0.033	4.622	1.468	0.967	1.468	0.289	0.035	5.816	14.775	0.289
	p-value	(.226)	(0.855)	(0.032)	(0.226)	(0.566)	(0.315)	(0.591)	(0.852)	(0.016)	(0.000)	(0.591)
5%	LR_{UC}	13.924	3.025	13.393	8.253	3.889	13.924	4.847	2.259	0.072	12.393	2.259
	p-value	(0.000)	(0.082)	(0.000)	(0.004)	(0.049)	(0.000)	(0.028)	(0.000)	(0.789)	(0.000)	(0.133)
	LR_{CC}	14.565	4.163	12.856	8.353	4.747	14.565	5.469	3.723	0.246	12.765	5.898
	p-value	(0.001)	(0.125)	(0.002)	(0.015)	(0.093)	(0.001)	(0.065)	(0.155)	(0.884)	(0.002)	(0.052)
Dash												
1%	LR_{UC}	1.122	2.348	1.122	0.305	5.778	1.122	2.348	5.778	10.246	-	15.547
	p-value	(0.290)	(0.125)	(0.290)	(0.581)	(0.016)	(0.290)	(0.125)	(0.016)	(0.001)	-	(0.000)
	LR_{CC}	1.292	2.594	1.292	0.386	7.456	1.292	2.594	6.218	11.201	-	16.019
	p-value	(0.525)	(0,273)	(0.524)	(0.824)	(0.024)	(0.524)	(0.273)	(0.045)	(0.004)	-	(0.000)
2.5%	LR_{UC}	7.495	3.405	7.495	5.988	9.134	7.495	4.622	10.88	9.134	-	12.781
	p-value	(0.006)	(0.065)	(0.006)	(0.014)	(0.003)	(0.006)	(0.032)	(0.001)	(0.003)	-	(0.000)
	LR_{CC}	7.546	3.490	7.546	6.113	9.144	7.546	4.857	10.899	9.44	-	12.802
	p-value	(0.023)	(0.150)	(0.023)	(0.047)	(0.010)	(0.023)	(0.088)	(0.004)	(0.010)	-	(0.002)
5%	LR_{UC}	4.847	4.847	3.025	2.259	3.025	4.847	4.847	4.847	3.889	-	5.896
	p-value	(0.028)	(0.028)	(0.082)	(0.133)	(0.082)	(0.028)	(0.028)	(0.028)	(0.049)	-	(0.015)
	LRCC	4.862	4.862	3.169	2.406	3.276	4.862	4.862	4.862	3.951	-	5.896
	p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	-	(0.052)

Continued

Ripple												
1%	LR_{UC}	0.000	0.382	0.000	0.382	0.382	1.816	238.14	238.14	0.382	0.000	0.305
	p-value	(1.000)	(0.537)	(1.000)	(0.537)	(0.537)	(0.178)	(0.000)	(0.000)	(0.537)	(1.000)	(0.581)
	LR_{CC}	0.061	0.408	0.061	0.408	0.408	1.823	238.16	238.16	0.408	0.061	0.413
	p-value	(0.970)	(0.815)	(0.970)	(0.815)	(0.402)	(0.000)	(0.000)	(0.815)	(0.970)	(0.813)	
2.5%	LR_{UC}	0.775	0.775	0.775	0.775	0.289	0.035	217.14	222.21	0.033	0.289	2.350
	p-value	(0.379)	(0.379)	(0.379)	(0.379)	(0.591)	(0.852)	(0.000)	(0.000)	(0.855)	(0.591)	(0.125)
	LR_{CC}	1.467	1.467	1.467	1.467	0.848	0.371	217.71	223.03	0.473	0.848	2.822
	p-value	(0.480)	(0.480)	(0.480)	(0.480)	(0.654)	(0.831)	(0.000)	(0.000)	(0.789)	(0.654)	(0.244)
5	LR_{UC}	2.259	0.595	3.025	1.596	1.596	0.069	169.67	169.67	0.595	1.596	5.896
	p-value	(0.133)	(0.440)	(0.082)	(0.207)	(0.207)	(0.793)	(0.000)	(0.000)	(0.440)	(0.207)	(0.015)
	LR_{CC}	2.455	2.903	3.338	4.465	4.465	1.879	169.71	169.71	2.903	4.465	6.709
	p-value	(0.293)	(0.234)	(0.188)	(0.107)	(0.107)	(0.391)	(0.000)	(0.000)	(0.234)	(0.107)	(0.035)
Stellar												
1%	LR_{UC}	2.348	0.305	2.348	-	0.305	2.348	1.122	0.305	1.122	3.916	0.305
	p-value	(0.125)	(0.581)	(0.125)	-	(0.581)	(0.125)	(0.290)	(0.581)	(0.290)	(0.048)	(0.581)
	LR_{CC}	2.594	0.413	2.594	-	0.413	2.594	1.292	0.413	1.292	4.252	0.413
	p-value	(0.273)	(0.813)	(0.273)	-	(0.813)	(0.273)	(0.524)	(0.813)	(0.524)	(0.119)	(0.813)
2.5%	LR_{UC}	0.775	0.289	0.775	-	0.033	0.775	0.033	0.035	0.033	3.405	0.035
	p-value	(0.379)	(0.591)	(0.379)	-	(0.855)	(0.379)	(0.855)	(0.852)	(0.855)	(0.065)	(0.852)
	LR_{CC}	1.467	0.848	1.467	-	0.473	1.467	0.473	0.371	0.473	4.588	0.371
	p-value	(0.480)	(0.654)	(0.480)	-	(0.789)	(0.480)	(0.789)	(0.831)	(0.789)	(0.101)	(0.831)
5%	LR_{UC}	2.259	2.259	3.205	-	0.595	2.259	1.596	1.596	0.270	3.025	0.595
	p-value	(0.133)	(0.133)	(0.082)	-	(0.440)	(0.133)	(0.207)	(0.207)	(0.604)	(0.082)	(0.440)
	LR_{CC}	2.455	2.455	3.338	-	2.903	2.455	4.465	1.702	2.321	3.123	0.603
	p-value	(0.293)	(0.293)	(0.188)	-	(0.234)	(0.293)	(0.107)	(0.427)	(0.313)	(0.210)	(0.740)
NEM												
1%	LR_{UC}	0.000	1.122	0.000	0.305	1.122	0.382	1.122	1.122	1.122	1.122	2.348
	p-value	(1.000)	(0.290)	(1.000)	(0.581)	(0.290)	(0.537)	(0.290)	(0.290)	(0.290)	(0.290)	(0.125)
	LR_{CC}	0.06	1.292	0.061	0.413	1.292	0.408	1.292	1.292	1.292	1.292	2.594
	p-value	(0.970)	(0.524)	(0.970)	(0.813)	(0.524)	(0.815)	(0.524)	(0.524)	(0.524)	(0.524)	(0.273)
2.5%	LR_{UC}	3.405	1.468	3.405	0.775	1.468	0.775	1.468	2.350	0.289	3.405	4.622
	p-value	(0.065)	(0.226)	(0.065)	(0.379)	(0.226)	(0.379)	(0.226)	(0.125)	(0.591)	(0.065)	(0.032)
	LR_{CC}	4.588	2.308	4.588	1.467	2.308	1.467	2.308	3.353	0.848	4.588	5.998
	p-value	(0.101)	(0.315)	(0.101)	(0.480)	(0.315)	(0.480)	(0.315)	(0.187)	(0.654)	(0.101)	(0.050)
5%	LR_{UC}	8.253	8.253	7.033	4.847	9.555	7.033	9.555	8.253	8.253	8.253	12.393
	p-value	(0.004)	(0.004)	(0.008)	(0.028)	(0.002)	(0.008)	(0.002)	(0.004)	(0.004)	(0.004)	(0.000)
	LR_{CC}	8.353	8.353	7.070	5.382	9.748	7.070	9.748	8.353	8.353	8.353	12.293
	p-value	(0.015)	(0.015)	(0.029)	(0.068)	(0.008)	(0.029)	(0.008)	(0.015)	(0.015)	(0.015)	(0.002)

of significance. For example, regarding the first cryptocurrency (Bitcoin) in Table 7, the APARCH (1, 1) model pass both the LR_{uc} and LR_{cc} test at 95%, 97.5% and 99% confidence levels. While the EGARCH (1, 1) and AVGARCH (1, 1) pass the LR_{uc} test at only 97.5% and 99% confidence levels. Hence, the APARCH (1, 1) model has the highest number of passes and is therefore considered to be the most appropriate model in forecasting VaR for the Bitcoin. In relation to the other seven cryptocurrencies, the results are summarized as follows; the APARCH (1, 1) and TGARCH (1, 1) models have the highest number of passes for Ethereum; NAGARCH (1, 1) model for Moreno; NGARCH (1, 1) for Litecoin; CSGARCH (1, 1) for Dash, Ripple, and NEM. Finally, APARCH (1, 1) for Stellar. Both the LR_{uc} and LR_{cc} coverage tests recommend the same GARCH-type models in most of the cases. These results demonstrate that the asymmetric GARCH-type models mostly have better VaR forecast performance for all cryptocurrencies especially at 99% level of significance and are also consistent with those found in the failure rate performance. Moreover, the fact that more GARCH-type models pass the LR_{cc} test for 99% VaR than for 95% VaR can be explained by the independence test where a smaller number of exceedances makes it easier not to occur after each other.

Finally, the VaR forecast performance of GARCH-type models is greatly dependent on the GARCH-type specification, with most GARCH models performing fairly better at the 95% level of significance. The p-values for both conditional and unconditional coverage tests are relatively low for most of the GARCH models, with the TGARCH and AVGARCH models showing among the lowest probability values. Generally, the conditional variance component of the GARCH-type specification plays a significant role since it provides models with a long memory and a more flexible lag structure.

6. Conclusions

Cryptocurrencies are relatively new and innovative investment assets that are characterized by high volatility and are uncorrelated with traditional financial assets such as stocks, currencies and bonds. In this paper, the focus is on modelling the volatility dynamics and out-of-sample forecasting performance of several GARCH-type models for cryptocurrency returns. Specifically, we have considered twelve symmetric and asymmetric GARCH processes, to evaluate the out-of-sample VaR forecasting performance of the eight major cryptocurrencies by market capitalization. This is implemented under the assumption that the innovations distributions of cryptocurrencies returns are skewed, heavy-tailed and leptokurtic. The out-of-sample VaR forecast performance of the GARCH-type specification is evaluated using by means of backtesting using conditional and unconditional coverage tests.

The empirical results of the study can be summarized as follows. Firstly, innovations distributions that capture skewness, kurtosis and heavy tails constitute excellent tools in modelling distribution of cryptocurrencies returns. The skewed versions of Student-t, GED and hyperbolic distributions for return innovations confirm their predominance over the alternatives in terms of better predictive ability. Secondly, the GARCH-type volatility models combined with a skewed distribution of return innovations, like the skewed *t*-Student or the Skewed-GED, provide acceptable VaR forecasts. While the results do not guarantee a straightforward preference between GARCH-type models, the asymmetric GARCH models with long memory property with skewed and heavy-tailed innovations distributions demonstrate better overall performance for all cryptocurrencies. Finally, regarding the accuracy tests, the VaR forecast performance comparison results vary with the cryptocurrencies. Given the high volatility dynamics present in all the cryptocurrencies, investors need to be cautious about their investments decisions in any cryptocurrency while investment managers should select asymmetric GARCH-type models with a long memory to forecast the VaR of cryptocurrencies.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Nakamoto, S. (2008) Bitcoin: A Peer-to-Peer Electronic Cash System.

[2] ElBahrawy, A., Alessandretti, L., Kandler, A., Pastor-Satorras, R. and Baronchelli, A. (2017) Evolutionary Dynamics of the Cryptocurrency Market. *Royal Society Open Science*, **4**, Article ID: 170623. https://doi.org/10.1098/rsos.170623

[3] CoinMarketCap (2018) Cryptocurrency Market Capitalizations.

[4] Osterrieder, J. and Lorenz, J. (2017) A Statistical Risk Assessment of Bitcoin and Its Extreme Tail Behavior. *Annals of Financial Economics*, **12**, Article ID: 1750003. https://doi.org/10.1142/S2010495217500038

[5] Dwyer, G.P. (2015) The Economics of Bitcoin and Similar Private Digital Currencies. *Journal of Financial Stability*, **17**, 81-91. https://doi.org/10.1016/j.jfs.2014.11.006

[6] Baek, C. and Elbeck, M. (2015) Bitcoins as an Investment or Speculative Vehicle? A First Look. *Applied Economics Letters*, **22**, 30-34. https://doi.org/10.1080/13504851.2014.916379

[7] Urquhart, A. (2016) The Ineffciency of Bitcoin. *Economics Letters*, **148**, 80-82. https://doi.org/10.1016/j.econlet.2016.09.019

[8] Fry, J. and Cheah, E.T. (2016) Negative Bubbles and Shocks in Cryptocurrency Markets. *International Review of Financial Analysis*, **47**, 343-352. https://doi.org/10.1016/j.irfa.2016.02.008

[9] Balcilar, M., Bouri, E., Gupta, R. and Roubaud, D. (2017) Can Volume Predict Bitcoin Returns and Volatility? A Quantiles-Based Approach. *Economic Modelling*, **64**, 74-81. https://doi.org/10.1016/j.econmod.2017.03.019

[10] Nadarajah, S. and Chu, J. (2017) On the Ineffciency of Bitcoin. *Economics Letters*, **150**, 6-9. https://doi.org/10.1016/j.econlet.2016.10.033

[11] Bouri, E., Molnar, P., Azzi, G., Roubaud, D. and Hagfors, L.I. (2017) On the Hedge

and Safe Haven Properties of Bitcoin: Is It Really More than a Diversifier? *Finance Research Letters*, **20**, 192-198. https://doi.org/10.1016/j.frl.2016.09.025

[12] Urquhart, A. (2017) Price Clustering in Bitcoin. *Economics Letters*, **159**, 145-148. https://doi.org/10.1016/j.econlet.2017.07.035

[13] Baur, D.G., Hong, K. and Lee, A.D. (2018) Bitcoin: Medium of Exchange or Speculative Assets? *Journal of International Financial Markets, Institutions and Money*, **54**, 177-189. https://doi.org/10.1016/j.intfin.2017.12.004

[14] Jiang, Y., Nie, H. and Ruan, W. (2018) Time-Varying Long-Term Memory in Bitcoin Market. *Finance Research Letters*, **25**, 280-284. https://doi.org/10.1016/j.frl.2017.12.009

[15] Dyhrberg, A.H. (2016) Bitcoin, Gold and the Dollar: A Garch Volatility Analysis. *Finance Research Letters*, **16**, 85-92. https://doi.org/10.1016/j.frl.2015.10.008

[16] Katsiampa, P. (2017) Volatility Estimation for Bitcoin: A Comparison of Garch Models. *Economics Letters*, **158**, 3-6. https://doi.org/10.1016/j.econlet.2017.06.023

[17] Charles, A. and Darne, O. (2018) Volatility Estimation for Bitcoin: Replication and Robustness. *International Economics*, **157**, 23-32. https://doi.org/10.1016/j.inteco.2018.06.004

[18] Pichl, L. and Kaizoji, T. (2017) Volatility Analysis of Bitcoin. *Quantitative Finance and Economics*, **1**, 474-485. https://doi.org/10.3934/QFE.2017.4.474

[19] Bariviera, A.F. (2017) The Ineffciency of Bitcoin Revisited: A Dynamic Approach. *Economics Letters*, **161**, 1-4. https://doi.org/10.1016/j.econlet.2017.09.013

[20] Urquhart, A. and Zhang, H. (2018) Is Bitcoin a Hedge or Safe-Haven for Currencies. An Intraday Analysis. 1-25. https://doi.org/10.2139/ssrn.3114108

[21] Liu, Y. and Tsyvinski, A. (2018) Risks and Returns of Cryptocurrency. Technical Report, National Bureau of Economic Research, Cambridge. https://doi.org/10.3386/w24877

[22] Chu, J., Chan, S., Nadarajah, S. and Osterrieder, J. (2017) Garch Modelling of Cryptocurrencies. *Journal of Risk and Financial Management*, **10**, 17. https://doi.org/10.3390/jrfm10040017

[23] Trucios, C. (2018) Forecasting Bitcoin Risk Measures: A Robust Approach. *International Journal of Forecasting*, **35**, 836-847. https://doi.org/10.1016/j.ijforecast.2019.01.003

[24] Naimy, V.Y. and Hayek, M.R. (2018) Modelling and Predicting the Bitcoin Volatility Using Garch Models. *International Journal of Mathematical Modelling and Numerical Optimisation*, **8**, 197-215. https://doi.org/10.1504/IJMMNO.2018.088994

[25] Peng, Y., Albuquerque, P.H.M., de Sa, J.M.C., Padula, A.J.A. and Montenegro, M.R. (2018) The Best of Two Worlds: Forecasting High Frequency Volatility for Cryptocurrencies and Traditional Currencies with Support Vector Regression. *Expert Systems with Applications*, **97**, 177-192. https://doi.org/10.1016/j.eswa.2017.12.004

[26] Bollerslev, T. (1986) Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, **31**, 307-327. https://doi.org/10.1016/0304-4076(86)90063-1

[27] Hansen, P.R. and Lunde, A. (2005) A Forecast Comparison of Volatility Models: Does Anything Beat a Garch (1, 1)? *Journal of Applied Econometrics*, **20**, 873-889. https://doi.org/10.1002/jae.800

[28] Ghalanos, A. (2018) Introduction to the Rugarch Package. Version 1.3-1, Technical Report V. http://cran.r-project.org/web/packages

[29] Bollerslev, T. and Wooldridge, J.M. (1992) Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances. *Econometric Reviews*, **11**, 143-172. https://doi.org/10.1080/07474939208800229

[30] Engle, R.F. and Bollerslev, T. (1986) Modelling the Persistence of Conditional Variances. *Econometric Reviews*, **5**, 1-50. https://doi.org/10.1080/07474938608800095

[31] Nelson, D.B. (1991) Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, *Journal of the Econometric Society*, **59**, 347-370. https://doi.org/10.2307/2938260

[32] Glosten, L.R., Jagannathan, R. and Runkle, D.E. (1993) On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance*, **48**, 1779-1801. https://doi.org/10.1111/j.1540-6261.1993.tb05128.x

[33] Ding, Z., Granger, C.W. and Engle, R.F. (1993) A Long Memory Property of Stock Market Returns and a New Model. *Journal of Empirical Finance*, **1**, 83-106. https://doi.org/10.1016/0927-5398(93)90006-D

[34] Engle, R.F. and Lee, G. (1999) A Long-Run and Short-Run Component Model of Stock Return Volatility. In: Engle, R.F. and White, H., Eds., *Cointegration*, *Causality*, *and Forecasting*: *A Festschrift in Honor of Clive W.J. Granger*, Oxford University Press, Oxford, 475-497.

[35] Zakoian, J.-M. (1994) Threshold Heteroskedastic Models. *Journal of Economic Dynamics and Control*, **18**, 931-955. https://doi.org/10.1016/0165-1889(94)90039-6

[36] Schwert, G.W. and Seguin, P.J. (1990) Heteroskedasticity in Stock Returns. *The Journal of Finance*, **45**, 1129-1155. https://doi.org/10.1111/j.1540-6261.1990.tb02430.x

[37] Higgins, M.L. and Bera, A.K. (1992) A Class of Nonlinear Arch Models. *International Economic Review*, **33**, 137-158. https://doi.org/10.2307/2526988

[38] Engle, R.F. and Ng, V.K. (1993) Measuring and Testing the Impact of News on Volatility. *The Journal of Finance*, **48**, 1749-1778. https://doi.org/10.1111/j.1540-6261.1993.tb05127.x

[39] Hentschel, L., *et al.* (1995) All in the Family: Nesting Symmetric and Asymmetric Garch Models. *Journal of Financial Economics*, **39**, 71-104. https://doi.org/10.1016/0304-405X(94)00821-H

[40] Baillie, R.T., Bollerslev, T. and Mikkelsen, H.O. (1996) Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, **74**, 3-30. https://doi.org/10.1016/S0304-4076(95)01749-6

[41] Kupiec, P. (1995) Techniques for Verifying the Accuracy of Risk Measurement Models. *The Journal of Derivatives Winter*, **3**, 73-84. https://doi.org/10.3905/jod.1995.407942

[42] Christoffersen, P.F. (1998) Evaluating Interval Forecasts. *International Economic Review*, **39**, 841-862. https://doi.org/10.2307/2527341