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Regime-Switching Model on Hourly Electricity Spot Price Dynamics

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Abstract

A robust time-varying regime-switching model for price dynamics of hourly spot price of electricity on the electricity market is developed. We propose a two-state Markov Regime Switching (MRS) model that gives weight to the existence of different variance for each regime. Our model is tractable as it integrates the main features exhibited in the hourly spot price dynamics on the electricity market. The parameters of our hourly spot price of electricity market model are estimated using the Expectation Maximization algorithm. Based on this model, an efficient and tractable pricing technique can be developed to price the dynamics of the hourly spot price of electricity.

Keywords

Regime-Switching Model, Time-Varying Volatility, Hourly Spot Price of Electricity, Expectation-Maximization (EM)-Algorithm

1. Introduction

Electricity, among other commodities, is one of the most important blessings science has given to the world. It is an essential commodity for social and economic development of developing countries. Most small household and manufacturing industries depend on electricity for their activities. According to the World Banks Global Tracking Framework (GTF), released in April 2017, 1.06 billion people live without electricity—a negligible improvement since 2012 (http://www.worldbank.org/en/topic/energy/overview, accessed on 02/09/2017) and this impedes the growth of countries economy due to the over-reliance of most activities on electricity. Crousillat, Hamilton, and Antmann (2010) stated that "eventhough electricity alone is not sufficient to spur economic growth, it is certainly necessary for human development" [1].

The electricity market facilitates the purchase of electricity through bids to buy; sales, through offers to sell; and short-term trades. In the early 1990s, the deregulation of the energy market (electricity market in our case) started in some countries (among others were the United Kingdom, Australia, and Norway) and gradually spread out to the European Union and the United States. This has created competitive markets that boost wholesale trading in most countries. This deregulation instigated substantial elements of risk such as uncertain demand, price risk, and volumetric risk; the principal of them being electricity price volatility. In result of this, there is the need to understand and model the spot price dynamics of the electricity market accurately to aid in an efficient pricing of electricity spots.

2. Electricity Spot Prices: Markets and Models

The spot price dynamics of electricity show signs of strong seasonality, high volatility, and generally unexpected extreme changes known as "spike" or "jumps" [2] [3]. Electricity spot prices (the underlying) show forms of nonlinear dynamics. Among the nonlinearities of the series of the price dynamics are the clustering of large shocks, and non-constant variance. These distinctive characteristics make it difficult for practitioners and researchers to model accurately the spot price of electricity on the electricity market. Most researchers have modelled electricity spot price dynamics using single regime stochastic models where it is assumed that there is no changes in state of the underlying spot price dynamics. But a single stochastic model may not be able to incorporate the dynamics of these electricity spot price dynamics accurately [4] [5] [6]. In reality, the underlying can go through different unobservable (latent) states in a particular period of interval. The underlying exhibits switching mechanism that needs different stochastic model for each switching state. Hence, the need to formulate appropriate models that can capture efficiently the electricity price dynamics to help in the proper pricing of spot and futures contract. From **Figure 1**, it is clear that mean-reversion is the optimal choice for electricity spot price dynamics. To build up the efficiency of Markov Regime Switching (MRS) models to electricity spot price dynamics, we proposed a model that gives weight to the existence of different variance for two Markov Regime Switching (MRS) models. The dynamics of electricity are more complex than normal spot price models allow and it can be noted that in the deregulated market, the dynamics of electricity prices are characterized with a combination of low price behavior and sharp price spikes as illustrated in **Figure 1**.

Ethier and Mount (1998) presented MRS models to electricty prices [7]. Huisman and Mahieu (2003) presented a three regime-switching model that separates price spikes from normal price [8]. They indicated that power spikes are short-lived and that stochastic jumps process cannot adequately model the electricity price behaviour. To substantiate their proposed model, they stated

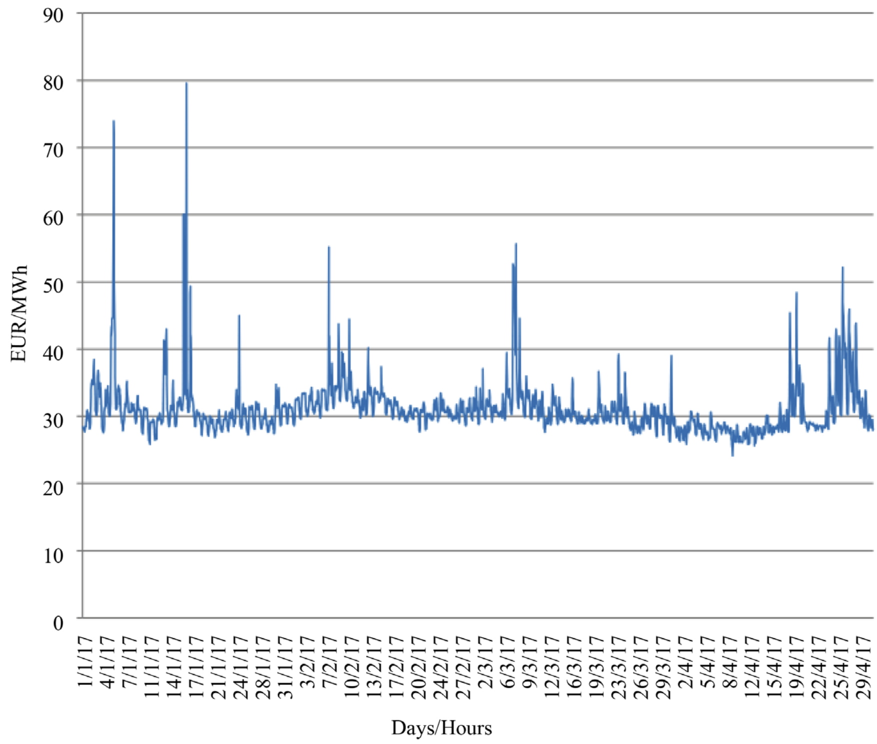

Figure 1. Hourly spot price of the NordPool Electricity market from 01/01/2017-31/04/2017.

that the mean-reverting rate of the long-run price level and the normal periods are not the same. Weron, Bierbrauer, and Trück (2004) modelled spot electricity prices by reviewing different electricity spot models [3]. Bierbrauer, Trück, and Weron (2010) addressed the problem of modeling spot electricity prices from the Nordic power exchange with a regime switching model [9]. The performance of the regime switching model was evaluated by comparing simulated and market prices. De Jong (2006) presented a two-regime MRS model by considering an autoregressive, spike regime dynamics driven by a Poisson distribution [10]. Weron (2008) also presented a model that decreases the computational time induced by independent regimes [11].

In this paper, we develop a robust two state regime-switching model with time-varying volatility for the price dynamics of the electricity spot price on the electricity market. The model is mathematically tractable to represent well the characteristics of the spot price dynamics of the electricity market.

3. Regime-Switching Brownian-"Jump" Model

Suppose in a two independent state regime switching, each state undergoes discrete shifts between states S_t of the process. Then S_t follows a first order Markov chain with the transition matrix:

$$\boldsymbol{P} = \begin{bmatrix} \mathbb{P}(S_t = 1 \mid S_{t-1} = 1) & \mathbb{P}(S_t = 1 \mid S_{t-1} = 2) \\ \mathbb{P}(S_t = 2 \mid S_{t-1} = 1) & \mathbb{P}(S_t = 2 \mid S_{t-1} = 2) \end{bmatrix}$$

$$\boldsymbol{P}=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}=\begin{bmatrix} 1-P_{12} & P_{12} \\ P_{21} & 1-P_{21} \end{bmatrix}$$

The transition matrix $\boldsymbol{P}$ contains the probabilities $P_{ij}\left(i, j=0,1\right)=P\left\{S_t=j \mid S_{t-1}=1\right\}$, $0 \le P_{ij} \le 1$, $\sum P_{ij}=1$ and satisfying $P_{i0}+P_{i1}=1, \forall i=0,1$

Keeping the stylized features of the spot price dynamics of electricity in mind, we propose a two-state Markov regime-Switching model with base regime driven by a mean-reverting process and a shifted regime driven by a Brownian-"Jump" process. In both regimes, we assume that the volatility of the current spot price is dependent on the current spot price level X_t. The "jump" behaviour is as a result of an "extreme" Brownian motion with a greater extreme drift and volatility than the standard mean-reverting regime. The "jump" regime is modelled with a simple Itô process. Given a time interval $\left[0,T\right]$ at a finite time horizon $\left[T<\infty\right]$, assume there is trading activities in the electricty market. Suppose, given a probability space $\left(\Omega,\mathcal{F},\mathbb{P}\right)$

$$X\left(t\right)=\begin{cases} X_{t,1}: \mathrm{d}X_{t,1}=\left(\alpha_1-\lambda X_{t,1}\right)\mathrm{d}t+\sigma_1 X_t^{\gamma}\mathrm{d}W_t^M, & \text{if } X\left(t\right) \text{ is in regime 1} \\ X_{t,2}: \mathrm{d}X_{t,2}=\mu_2\mathrm{d}t+\sigma_2 X_t^{\gamma}\mathrm{d}W_t^J, & \text{if } X\left(t\right) \text{ is in regime 2} \end{cases} \tag{1}$$

where λ is the mean-reversion rate of the base regime, $\frac{\alpha_1}{\lambda}$ is the long-term mean for the spot price reverting to, $\sigma_1 X_t^{\gamma}$ and σ_2 are the daily price volatility of the base and shifted regimes respectively, W_t^M and W_t^J are the standard Brownian motion and a Brownian-"Jump" process respectively, T_t is the temperature at time *t*. Let p_1 and $1-p_1$ be the probabilities that the process is in regime 1 and 2, respectively. The base regime model is based on the mean-reverting constant elasticity variance developed by [12]. There is a strong positive correlation between the price level and the price change for a psoitive γ. Assume $\gamma=1$, then model (1) can be reformulated as

$$X\left(t\right)=\begin{cases} X_{t,1}: \mathrm{d}X_{t,1}=\left(\alpha_1-\lambda X_{t,1}\right)dt+\sigma_1 X_{t,1}\mathrm{d}W_t^M, & \text{if } X\left(t\right) \text{ is in regime 1} \\ X_{t,2}: \mathrm{d}X_{t,2}=\alpha_2\mathrm{d}t+\sigma_2 X_{t,2}\mathrm{d}W_t^J, & \text{if } X\left(t\right) \text{ is in regime 2} \end{cases} \tag{2}$$

By the application of Itô's lemma to model (2), the integral form to the base regime and shifted regime is explicitly given in integral form as

$$X_{t,1}=\frac{\alpha_1}{\lambda}+\mathrm{e}^{-\lambda t}\left[X_{0,1}-\frac{\alpha_1}{\lambda}\right]+\sigma_1\int_t^0 X_{s,1}\mathrm{e}^{-\lambda(t-s)}\mathrm{d}W_s^M \tag{3}$$

$$X_{t,2}=X_{0,2}+\mu_2 t+\int_0^t \sigma_2\mathrm{d}W_s^J \tag{4}$$

4. Parameter Estimation

The EM algorithm was first introduced by Dempster, Laird, and Rubin (1977) [13]. Estimating the parameters in a MRS models is not trivial since the regime is not directly observable(latent). Hamilton (1990) was the first to apply Expectation-Maximization (EM)-algorithm [14]. The EM-algorithm is a comprehensive procedure for finding the maximum-likelihood estimate of the parameters of a distribution from a specified data given that the data is either incomplete or have missing (or hidden) values. The optimal set of unknown parameters to be estimated in the base and shifted regime are $\theta_1 = \{\alpha_1, \lambda, \sigma_1, P_1\}$ and $\theta_2 = \{\alpha_2, \sigma_2, P_2\}$ respectively. The optimal set of the unknown parameters in the model to estimate is $\Theta = \{\theta_1, \theta_2\}$.

4.1. Discretization

The discretized version of model 1 in the base and shifted regime is respectively given as

$$X_{t+1,1} = \alpha_1 + X_{t,1}(1-\lambda) + \sigma_1 X_{t,1}\epsilon_{t+1,1} \tag{5}$$

$$X_{t+1,2} = \alpha_2 + X_{t,2} + \sigma_2 X_{t,2}\epsilon_{t+1,2} \tag{6}$$

where $\epsilon_{t+1} \sim \mathcal{N}(0,1)$. Let $\mathcal{F}_{t_k}^X$ be the vector of past $k+1$ last values of (5) and (6), *i.e.* $\mathcal{F}_{t_k}^X = (X_{t_0}, X_{t_1}, X_{t_2}, \cdots, X_{t_k})$. Also, let $H+1$ be the size of the past data and Ψ be the equivalent increasing pattern of time at which the data is recorded, *i.e.* $\Psi = \{t_j; 0 = t_0 \le t_1 \le t_2 \le \cdots \le t_{H-1} \le t_H = T\}$.

4.2. E-Step

As stated earlier, the regime switching model is latent, hence the inference of the regimes are given by the equations below:

$\forall\, i = \{1,2\}$, $k = 1,2,3,\cdots,H$, and n = number of iterations.

$$A_{i,t_k}^n = \mathbb{P}\left(S_{t_k} = i \mid \mathcal{F}_{t_k}^X; \hat{\Theta}^{(n)}\right) = \frac{\mathbb{P}\left(S_{t_k}, X_{t_k} \mid \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right)}{f\left(X_{t_k} \mid \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right)} \tag{7}$$

$$= \frac{\mathbb{P}\left(S_{t_k} = i \mid \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right) f\left(X_{t_k} \mid S_{t_k} = i; \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right)}{\sum_{i=1}^{2} \mathbb{P}\left(S_{t_k} = i \mid \mathcal{F}_{t_{k-1}}^X\right) f\left(X_{t_k} \mid S_{t_k} = i; \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right)} \tag{8}$$

with

$$\mathbb{P}\left(S_{t_k} = i \mid \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right) = \sum_{i=1}^{2} \mathbb{P}\left(S_{t_k} = i, S_{t_{k-1}} = j \mid \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right) \tag{9}$$

$$= \sum_{i=1}^{2} \prod_{ji}^{(n)} \mathbb{P}\left(S_{t_k} = j \mid \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right) \tag{10}$$

$f\left(X_{t_k} \mid S_{t_k} = i; \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right)$ is the density process at time t_k, conditional on the process in regime *i*. From (2) and (5), the base regime has a conditional Gaussian distribution with mean $\alpha_1 + X_{t_k,1}(1-\lambda)$ and standard deviation $\sigma_1 X_{t_k,1}$. The shifted regime has a conditional Gaussian distribution with mean $\alpha_2 + X_{t_k,2}$ and standard deviation $\sigma_2 X_{t_k,2}$.

The probability density functions (pdf) of the base and shifted regimes are respectively given as

$$f\left(X_{t_k} \mid S_{t_k} = i; \mathcal{F}_{t_{k-1}}^X; \hat{\theta}_1^{(n)}\right) = \frac{1}{\sqrt{2\pi}\sigma_1^{(n)} X_{t_{k-1}}} \exp\left(-\frac{\left(X_{t_k} - \left(1-\lambda^{(n)}\right) X_{t_{k-1}} - \alpha_1^{(n)}\right)^2}{2\left(\sigma_1^{(n)}\right)^2 X_{t_{k-1}}^2}\right) \tag{11}$$

$$f\left(X_{t_k} \mid S_{t_k} = i; \mathcal{F}_{t_{k-1}}^X; \hat{\theta}_2^{(n)}\right) = \frac{1}{\sqrt{2\pi}\sigma_2^{(n)} X_{t_{k-1}}} \exp\left(-\frac{\left(X_{t_k} - X_{t_{k-1}} - \alpha_2^{(n)}\right)^2}{2\left(\sigma_2^{(n)}\right)^2 X_{t_{k-1}}^2}\right) \tag{12}$$

4.3. M-Step

We compute the maximum likelihood estimates $\Theta^{(n+1)}$ for the unknown parameters in (5) and (6).

The transition probabilities P_{1j} and P_{2j} $\left(\sum P_{ij} = 1\right)$ are estimated based on the formula below

$$P_{1j}^{(n+1)} = \frac{\sum_{k=2}^{N}\left[\mathbb{P}\left(S_{t_k} = 1 \mid \mathcal{F}_{t_k}^X; \hat{\Theta}^{(n)}\right) \frac{P_{1j}^{(n)} \mathbb{P}\left(S_{t_{k-1}} = 1 \mid \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right)}{\sum_{k=2}^{N} \mathbb{P}\left(S_{t_k} = 1 \mid \mathcal{F}_{t_{k-1}}^X; \hat{\Theta}^{(n)}\right)}\right]}{\sum_{k=2}^{N}\left[\mathbb{P}\left(S_{t_{k-1}} = 1 \mid \mathcal{F}_{t_N}^X; \hat{\Theta}^{(n)}\right)\right]} \tag{13}$$

$$P_2 j = 1 - P_1 j \tag{14}$$

From (11) and (12), the log-likelihood functions of the base and shifted regimes are given respectively as

$$\log\left[L\left(\hat{\theta}_1^{(n)}; \mathcal{F}_{t_k}^X, S_{t_k}\right)\right] = \sum_{k=2}^{H} A_{1,t_k}^{(n)}\left[\log P_{1j} - \log\left(\sqrt{2\pi}\sigma_1 X_{t_{k-1}}\right) - \frac{1}{2\sigma_1^2 X_{t_{k-1}}^2}\left(X_{t_k} - (1-\lambda) X_{t_{k-1}} - \alpha_1\right)^2\right] \tag{15}$$

$$\log\left[L\left(\hat{\theta}_2^{(n)}; \mathcal{F}_{t_k}^X, S_{t_k}\right)\right] = \sum_{k=2}^{H} A_{2,t_k}^{(n)}\left[\log P_{2j} - \log\left(\sqrt{2\pi}\sigma_2 X_{t_{k-1}}\right) - \frac{1}{2\sigma_2^2 X_{t_{k-1}}^2}\left(X_{t_k} - X_{t_{k-1}} - \alpha_2\right)^2\right] \tag{16}$$

From (15), each of the parameter in the base regime can be estimated by differentiating the log-likelihood with respect to that parameter.

$$\left(\sigma_1^{(n+1)}\right)^2 = \frac{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-2}\left(X_{t_k} - \left(1-\lambda^{n+1}\right) X_{t_{k-1}} - \alpha_1^{(n+1)}\right)^2}{\sum_{k=2}^{H} A_{1,t_k}^{(n)}} \tag{17}$$

$$\lambda^{(n+1)} = \frac{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-1}\left[C_1\right]}{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-2}\left[C_2\right]} \tag{18}$$

where

$$C_1 = X_{t_k} - X_{t_{k-1}} \frac{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-2}\left(X_t - X_{t-1}\right)}{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-2}}$$

$$C_2 = X_{t_{k-1}}^2 - \frac{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-2}}{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-2}}$$

$$\alpha_1^{(n+1)} = \frac{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-2} \left(X_{t_k} - X_{t-1} \left(1 - \lambda^{(n+1)} \right) \right)}{\sum_{k=2}^{H} A_{1,t_k}^{(n)} X_{t_{k-1}}^{-2}} \quad (19)$$

From (16), each of the parameter in the shifted regime can be estimated by differentiating the log-likelihood with respect to that parameter.

$$\left(\sigma_2^{(n+1)} \right)^2 = \frac{\sum_{k=2}^{H} A_{2,t_k} \left(X_{t_k} - X_{t_{k-1}} - \alpha_2^{(n+1)} \right)^2}{\sum_{k=2}^{H} A_{2,t_k}} \quad (20)$$

$$\alpha_2^{(n+1)} = \frac{\sum_{k=2}^{H} A_{2,t_k} \left(X_{t_{k-1}} - X_{t_k} \right)}{\sum_{k=2}^{H} A_{2,t_k}} \quad (21)$$

5. Data Description, Results, and Discussion

Historical electricity hourly spot price on the NordPool market is used, specifically we took an hourly data of Oslo. The data set consist of 764 hours spanning from 01/03/2017-31/03/2017. From Table 1, the kurtosis of the data was found to be 24.6572 which is by far greater than the kurtosis of a gaussian distributed data, hence the existence of extreme data points in hourly spot price of the NordPool Electricity. These extreme data points can be described as "jumps". The presence of extreme data points in the hourly electricity data are clearly illustrated in Figure 2, as the normal curve was not able to fit well on the histograph. This shows that the consumption of electricity depends largely on the peak hours and normal hours in a day, hence the need to model electricity spot price dynamics hourly. Also from Table 1, with a skewness of 3.9056, the data is skewed to the right. This also shows that the hourly spot price of the NordPool Electricity is not normally distributed. The parameter estimates for both regimes depends on the hourly spot price of the NordPool Electricity market from 01/03/2017-31/03/2017.

The estimated results of the model is found in Table 2. The probability of hourly price remaining in the base regime is very high, 0.8471. With a lower but

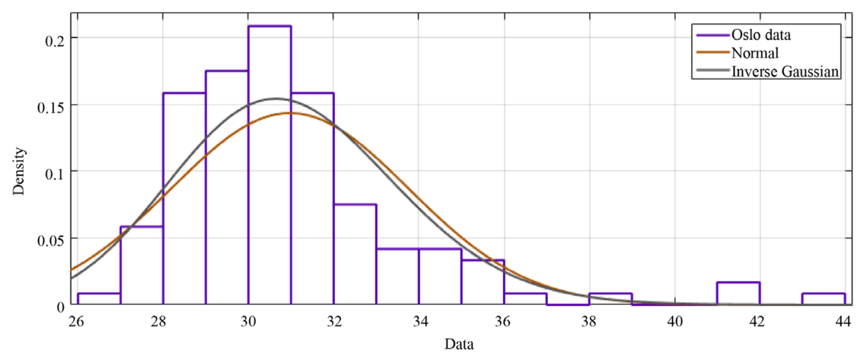

Figure 2. The distribution fit of the hourly spot price of the NordPool Electricity market from 01/03/2017-31/03/2017.

significant probability of 0.1529, the hourly price will remain in the shifted regime.

6. Conclusion

In this paper, a two-state Markov Regime switching model for the dynamics of the hourly spot price of electricity is developed. It is clear from the illustrated Figure 3 and Figure 1 that electricity hourly spot price exhibits mean reversion, heteroscedastic volatility in both regimes, price spikes and jumps. Our model is tractable as it integrates the main features exhibited in the hourly spot price on the electricity market. The parameters of our hourly spot price electricity market model are estimated using the EM algorithm. Based on this model, an efficient and tractable pricing technique can be developed to price the dynamics of the hourly spot price of electricity. To the best of our knowledge, our proposed model is the first to consider hourly spot price of electricity. From Figure 2, it is evident that the distribution of hourly spot price of the NordPool Electricity is not normal; hence it will be appropriate to use other distributions like the Normal Inverse Gaussian (NIG) or the Gamma distribution to capture this effect.

Table 1. The distribution (skewness and kurtosis) of hourly spot price of the NordPool Electricity market from 01/03/2017-31/03/2017.

Skewness	Kurtosis
3.9056	24.6572

Table 2. Parameter estimates for the two state MRS model. The parameters are estimated using the EM algorithm based on the hourly spot price of the NordPool Electricity market for 01/03/2017-31/03/2017.

Parameter	σ_1	λ	α_1	P_1	P_2	σ_2	α_2
Estimate	10.9876	1.7671	6.8814	0.8471	0.1529	7.0913	4.0936

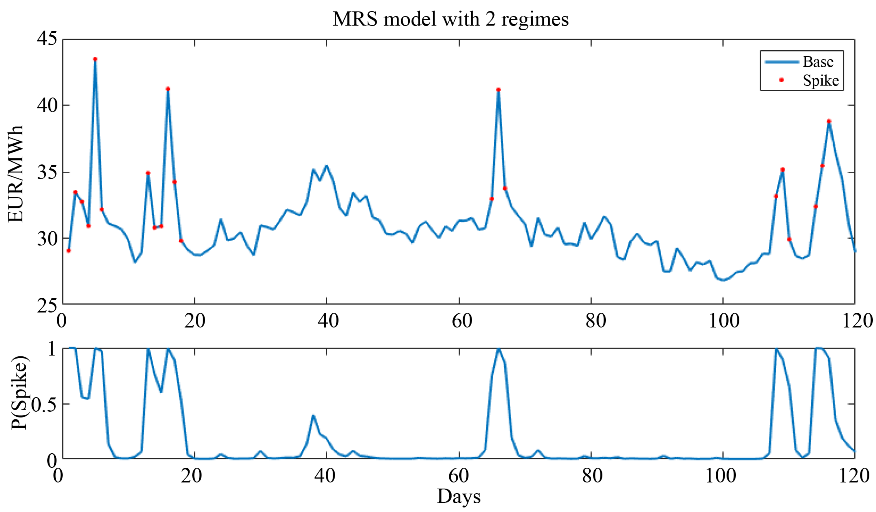

Figure 3. Calibration of two state MRS model with independent regimes fitted to deseasonalized hourly electricity spot price from the NordPool Electricity market.

NIG and Gamma distributions can capture the extreme and skewed features of the hourly spot price of the NordPool Electricity data.

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