Journal of Mathematical Finance, 2016, 6, 711-727
http://www.scirp.org/journal/jmf
ISSN Online: 2162-2442
ISSN Print: 2162-2434

# A New Fama-French 5-Factor Model Based on SSAEPD Error and GARCH-Type Volatility

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**How to cite this paper:** Zhou, W.T. and Li, L.L. (2016) A New Fama-French 5-Factor Model Based on SSAEPD Error and GARCH-Type Volatility. *Journal of Mathematical Finance*, **6**, 711-727.
http://dx.doi.org/10.4236/jmf.2016.65050

**Received:** July 30, 2016
**Accepted:** November 13, 2016
**Published:** November 16, 2016



## Abstract

In this paper, we extend the 5-factor model in Fama and French (2015) with the non-Normal errors distribution of SSAEPD (Standardized Standard Asymmetric Exponential Power Distribution) in Zhu and Zinde-Walsh (2009) and the GARCH-type volatility. The focus is on finding out whether our new model can outperform the original Fama-French 5-factor model. We use Fama-French 25 value-weighted portfolios to conduct our research. The MLE is used to estimate the parameters. The LR test and KS test are used for model diagnostics. Models are compared by AIC. Empirical results show that with GARCH-type volatilities and non-normal errors, the Fama-French 5 factors are still alive. Our new model can successfully capture the skewness, fat-tailness and asymmetric kurtosis in the data and has better in-sample fit than the 5-factors model in Fama and French (2015). Our study provides an update to existing asset pricing literature and reference for investors.

## Keywords

Fama-French 5-Factor Model (FF5), Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD), GARCH, Asset Pricing

## 1. Introduction

The capital asset pricing model of Sharpe and Lintner (1965) marks the birth of asset pricing theory [1], which discovers that there exists a positive linear relation between expected returns and their market betas. Three decades later, Fama and French (1993) proposed a three-factor model relating to market premium, Size, B/M and confirmed that the 3-factor model outperformed the single-factor CAPM. [2]

However, recent studies have discovered that many other important patterns in average returns are left unexplained by the 3-factor model. Panel A of Table 1 documents

**Table 1.** Researches about factor model for stock market.

| Author (Year) | Research Purpose | Model | Estimation Method | Data Country | Model factors | Frequency & Period |
|---|---|---|---|---|---|---|
| Panel A: Extension of Factor Model | | | | | | |
| Fama *et.al.* (1993) | CAPM Extension | FF3 | - | USA | Mkt, SMB, HML, WML | M1963:7 - 1991:12 |
| Carhart (1997) | FF3 Extention | CAPM, FF3, C4 | OLS | USA | Mkt, SMB, HML, WML | M1962:1 - 1993:12 |
| Griffin (2002) | FF3 Extention | World, Domestic or International FF3 | - | Global | Mkt, SMB, HML | M1981:1995:12 |
| Bali *et.al.* (2004) | FF3 Extention | FF3 with VAR | OLS | USA | Mkt, SMB, HML, VAR | M1963:1 - 2001:12 |
| Chan *et.al.* (2005) | FF3 Extension | FF3 with IML | GMM | Australia | Mkt, SMB, HML, IML | M1990:1 - 1998:12 |
| Chan *et.al.* (2007) | FF3 Extension | FF3 with Default factor | GMM | Australia | Mkt, SMB, HML, DEF | M1996 - 2004 |
| He (2008) | FF3 Extention | FF3, FF3 with State Switch | OLS | China | Mkt, SMB, HML, State Switch | M1995:6 - 2005:12 |
| Xiao *et.al.* (2007) | FF3 Extention | FF3 with Sustainability Factor | GMM | Global | Mkt, SMB, HML, SUS | M1999 - 2007 |
| Fama *et.al.* (2013) | FF3 Extention | FF4 | - | USA | Mkt, SMB, HML, RMW | M1963:7 - 2012:12 |
| Yang (2013) | FF3 Extention | FF3 with SSAEPD, EGARCH | MLE | USA | Mkt, SMB, HML | M1926 - 2011 |
| Fama *et.al.* (2015) | FF4 Extention | FF5 | - | USA | Mkt, SMB, HML, RMW, CMA | M1963:7 - 2013:12 |
| Mu (2015) | C Extension | C4 with SSAEPD, EGARCH | MLE | USA | Mkt, SMB, HML, WML | M1927:1 - 2014:12 |
| Panel B: Fama-French 5-Factor Model comparison | | | | | | |
| Fama *et.al.* (2014) | Model Comparison | CAPM, FF3, FF4, FF5, FF5 with WML | - | USA | Mkt, SMB, HML, RMW, CMA, WML | M1963:7 - 2014:12 |
| Hou *et.al.* (2015) | Model Comparison | FF5, C, q-factor | - | USA | Mkt, SMB, HML, RMW, CMA, WML | M1967:1 - 2013:12 |
| Harshita *et.al.* (2015) | Model Comparison | CAPM, FF3, FF5 | - | India | Mkt, SMB, HML, RMW, CMA | M1999:10 - 2014:9 |
| Chiah *et.al.* (2015) | Model Comparison | FF3, FF5 | HAC-adjusted OLS | Australia | Mkt, SMB, HML, RMW, CMA | M1982:12 - 2012:12 |

the development of the factor model in stock market. For example, Carhart (1997) incorporates momentum factor into the Fama-French 3-factor (FF3) model and establishes a Carhart 4-factor (C4) model which documents that stocks performing the best in the short run tend to continue this trend [3]. Chan and Faff (2005) construct a liquidity-augmented FF3 model [4]. Connor, Hagmann and Linton (2012) consider a five-factor extension of the C4 model which suggests an own-volatility factor [5]. Xiao, Faff, Gharghori and Min (2012) incorporate a sustainability factor into 3-factor model which explains the sustainability of the world price better [6].

In 2015, Fama and French proposed a 5-factor model directed at capturing the size, value, profitability and investment patterns in average stock returns and found it performed better than their 3-factor model [7]. Since then, many studies focusing on Fama-French 5-factor (FF5) model have been done. Panel B of **Table 1** presents the researches for the FF5 model. These researches are focused on empirical analysis of FF5 model in different stock markets and comparison between the FF5 model and other models. For example, Hou, Xue and Zhang (2015) find that the 4-factor q-model performs better than the FF5 model in US market [8]. Harshita, Singh, S. and Yadav, S. S. (2015) discover that the FF5 model works better in India than CAPM and FF3 model [9].

Different from previous researches, our research tries to extend the 5-factor model in Fama and French (2015). Many asset pricing models in the existing literature just assume that financial time series follow the normal distribution, but more and more researches and studies have observed the unique distributional properties of financial data—more kurtosis and higher peak—contradicting the assumption of normality [10]. Thus, instead of adding new factors, we incorporate the GARCH-type volatilities of Bollerslev (1986) into FF5 model and employ non-normal errors of SSAEPD proposed by Zhu and Zinde-Walsh (2009) for the error term. SSAEPD is capable of capturing many stylized facts in financial time series such as skewness, fat tails and asymmetric kurtosis [11]. We denote our new model as FF5-SSAEPD-GARCH. Based on our new model, we try to figure out the following two questions:

1) With GARCH-type volatilities and SSAEPD errors, are the Fama-French 5 factors still alive?

2) Can our new model beat the 5 factor model in Fama and French (2015)?

To answer these questions, we first run simulation to test whether the MatLab program we write can be used in our analysis. Then, Fama-French 25 value-weighted portfolios are analyzed. Data are downloaded from the French's Data Library, and the sample period is from Jul. 1963 to Dec. 2013. Method of Maximum Likelihood Estimation (MLE) is used to estimate the parameters. Likelihood Ratio test (LR) and Kolmogorov-Smirnov test (KS) are exploited for model diagnostics. Akaike Information Criterion (AIC) is employed for model comparison.

Simulation results show our MatLab program can be employed for our empirical analysis. According to the empirical results, we find out the 5 factors in Fama and French (2015) are still alive! The new model fits the data well and has better in-sample fit than the 5-factor model in Fama and French (2015).

The paper proceeds as follows. The model and methodology are discussed in Section 2. Simulation analysis is reported in Section 3. Empirical results and the model comparisons are presented in Section 4. Section 5 provides the conclusions and future extensions.

## 2. Model and Methodology

### 2.1. Fama-French 5-Factor Model (FF5-Normal)

Fama and French (2015) propose a 5-factor model (denoted as FF5) to capture the size, value, profitability, and investment patterns in expected stock returns, and show this model empirically outperforms their 3 factor model. The 5-factor model is:

$$R_t - R_{ft} = \beta_0 + \beta_1 * \left(R_{mt} - R_{ft}\right) + \beta_2 * SMB_t + \beta_3 * HMLO_t + \beta_4 * RMW_t + \beta_5 * CMA_t + u_t, u_t \sim Normal\left(\mu, \sigma^2\right). \quad (1)$$

where $\theta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \mu, \sigma)$ are parameters to be estimated in this model. $R_t$ is the return on stock portfolio for period $t$. $R_{ft}$ is the risk-free return. $R_{mt}$ is the value-weighted market return. $SMB_t$ is the return on small stock portfolio minus the return on big stock portfolio. $HMLO_t$ is the high book-to-market ratio minus low book-to-market ratio orthogonalized1. $RMW_t$ stands for robust operating profitability portfolios minus weak operating profitability portfolios. $CMA_t$ stands for conservative investment portfolios minus aggressive investment portfolios. The error term $u_t$ is distributed as the Normal.

## 2.2. The FF5-SSAEPD-GARCH Model

Based on the GARCH-type volatility in Bollerslev (1986) and non-Normal error distribution of SSAEPD in Zhu and Zinde-Walsh (2009), we extend Fama-French (2015) five-factor model in this section. The new model is denoted as FF5-SSAEPD-GARCH and its math formula is:

$$R_t - R_{ft} = \beta_0 + \beta_1\left(R_{mt} - R_{ft}\right) + \beta_2 SMB_t + \beta_3 HMLO_t + \beta_4 RMW_t + \beta_5 CMA_t + u_t, t = 1, 2, \cdots, T, \quad (2)$$

$$u_t = \sigma_t z_t, z_t \sim SSAEPD\left(\alpha, p_1, p_2\right), \quad (3)$$

$$\sigma_t^2 = a_0 + \sum_{i=1}^{r} a_i u_{t-i}^2 + \sum_{i=1}^{s} b_i \sigma_{t-i}^2. \quad (4)$$

where $a_0 > 0$, $a_i \ge 0$, $b_i \ge 0$, $\sum_{i=1}^{\max(r,s)}\left(a_i + b_i\right) < 1$.

$\boldsymbol{\theta} = \left(\beta_1, \beta_2, a_0, \{a_i\}_{i=1}^{r}, \{b_i\}_{i=1}^{s}, \alpha, p_1, p_2\right)$ are the parameter vectors to be estimated. $T$ is the sample size. The error term $z_t$ is distributed as the Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD) proposed by Zhu and Zinde-Walsh (2009). $\sigma_t$ is the conditional standard deviation, *i.e.*, volatility. With $\{a_0 = 0\}_{i=1}^{r}$, $\{b_i = 0\}_{i=1}^{s}$, $\alpha = 0.5$, $p_1 = p_2 = 2$, the FF5-SSAEPD-GARCH model reduces to the FF5-Normal model.

- Standardized Standard AEPD (SSAEPD)

The probability density function (PDF) of the SSAEPD proposed by Zhu and Zinde-Walsh (2009) is

$$f\left(z_t \mid \beta\right) = \begin{cases} \delta\left(\dfrac{\alpha}{\alpha^*}\right) K\left(p_1\right) \exp\left(-\dfrac{1}{p_1}\left|\dfrac{w + z_t\delta}{2\alpha^*}\right|^{p_1}\right), & \text{if } z_t \le -\dfrac{w}{\delta}, \\ \delta\left(\dfrac{1-\alpha}{1-\alpha^*}\right) K\left(p_2\right) \exp\left(-\dfrac{1}{p_2}\left|\dfrac{w + z_t\delta}{2\left(1-\alpha^*\right)}\right|^{p_2}\right), & \text{if } z_t > -\dfrac{w}{\delta}. \end{cases} \quad (5)$$

where

$$\alpha^* = \frac{\alpha K\left(p_1\right)}{\alpha K\left(p_1\right) + \left(1-\alpha\right) K\left(p_2\right)}, \quad (6)$$

$$K(p)=\frac{1}{2p^{1/p}\Gamma(1+1/p)}, \tag{7}$$

$$\Gamma(x)=\int_0^\infty y^{x-1}\mathrm{e}^{-y}\mathrm{d}y, \tag{8}$$

$$w=\frac{1}{B}\left[(1-\alpha)^2\frac{p_2\Gamma(2/p_2)}{\Gamma^2(1/p_2)}-\alpha^2\frac{p_1\Gamma(2/p_1)}{\Gamma^2(1/p_1)}\right], \tag{9}$$

$$\begin{aligned}\delta^2=\frac{1}{B^2}\Bigg\{&\left[(1-\alpha)^3\frac{p_2^2\Gamma(3/p_2)}{\Gamma^3(1/p_2)}+\alpha^3\frac{p_1^2\Gamma(3/p_1)}{\Gamma^3(1/p_1)}\right]\\&-\left[(1-\alpha)^2\frac{p_2\Gamma(2/p_2)}{\Gamma^2(1/p_2)}-\alpha^2\frac{p_1\Gamma(2/p_1)}{\Gamma^2(1/p_1)}\right]^2\Bigg\},\end{aligned} \tag{10}$$

$$B=\alpha K(p_1)+(1-\alpha)K(p_2). \tag{11}$$

And $\mu\in R,\sigma>0,p_1>0,p_2>0,\alpha\in(0,1)$. $p_1$ and $p_2$ are the parameters controlling the left tail and right tail, respectively. Parameter $\alpha$ controls the skewness of SSAEPD. The smaller $p_1$ and $p_2$ are, the fatter the tails are. If $\alpha$ is smaller than 0.5, it indicates a left-skewed distribution. If $\alpha$ is larger than 0.5, it indicates a right-skewed distribution When $\alpha$ = 0.5, $p_1$ = $p_2$ = 2, SSAEPD can be reduced to Normal (0, 1). The mean of $z_t$ is zero and its variance is 1.

### 2.3. Maximum Likelihood Estimation

In this paper, we estimate the FF5-SSAEPD-GARCH model with Maximum Likelihood Estimation (MLE). The likelihood function is

$$\begin{aligned}&L\left(\left\{R_t-R_{ft},R_{mt}-R_{ft},SMB_t,HMLO_t,RMW_t,CMA_t\right\}_{t=1}^T;\theta\right)\\&=\prod_{t=1}^T f\left(R_t-R_{ft}\right)\\&=\prod_{i=1}^n\begin{cases}\frac{\delta}{\eta}\left(\frac{\alpha}{\alpha^*}\right)K(p_1)\exp\left(-\frac{1}{p_1}\left|\frac{\omega+\delta z_t}{2\alpha^*}\right|^{p_1}\right), & z_t\le-\frac{\omega}{\delta},\\ \frac{\delta}{\eta}\left(\frac{1-\alpha}{1-\alpha^*}\right)K(p_2)\exp\left(-\frac{1}{p_2}\left|\frac{\omega+\delta z_t}{2(1-\alpha^*)}\right|^{p_2}\right), & z_t>-\frac{\omega}{\delta}.\end{cases}\end{aligned} \tag{12}$$

where

$$z_t=\frac{R_t-R_{ft}-\beta_0-\beta_1(R_{mt}-R_{ft})-\beta_2SMB_t-\beta_3HMLO_t-\beta_4RMW_t-\beta_5CMA_t}{\sigma_t}, \tag{13}$$

$$\sigma_t^2=a_0+\sum_{i=1}^r a_iu_{t-i}^2+\sum_{i=1}^s b_i\sigma_{t-i}^2. \tag{14}$$

$\boldsymbol{\theta}=\left(\beta_1,\beta_2,a_0,\{a_i\}_{i=1}^r,\{b_i\}_{i=1}^s,\alpha,p_1,p_2\right)$ is the parameter vector to be estimated.

## 3. Simulation Analysis

In this section, we first generate random number series for $(R_{mt}-R_{ft})$, $SMB_t$, $HMLO_t$, $RMW_t$, $CMA_t$ and $\{Y_t\}_{t=1}^T$. Then we use them to run the simulations to test whether the program we write in MatLab can be applied to our empirical analysis. The FF5-SSAEPD-GARCH (1, 1) is simulated as follows:

$$R_t - R_{ft} = \beta_0 + \beta_1 \left( R_{mt} - R_{ft} \right) + \beta_2 SMB_t + \beta_3 HMLO_t + \beta_4 RMW_t + \beta_5 CMA_t + u_t, t = 1, 2, \cdots, T, \quad (15)$$

$$u_t = \sigma_t z_t, z_t \sim SSAEPD\left(\alpha, p_1, p_2\right), \quad (16)$$

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + b_1 \sigma_{t-1}^2. \quad (17)$$

We choose $\beta_0 = 0.2, \beta_1 = 1, \beta_2 = 0.5, \beta_3 = 0.5, \beta_4 = 0.5, \beta_5 = 0.5, \alpha = 0.5, p_1 = p_2 = 2,$ $a = 0.3, b = 0.5, c = 0.4$ as the true values of the parameters. The data is generated as follows:

1) Given $\alpha = 0.5$, $p_1 = p_2 = 2$, we can generate SSAEPD random number series

2) Set the initial value $\sigma_0^2 = 1$, $\varepsilon_0 = 0$, and given $a = 0.3$, $b = 0.5$, $c = 0.4$, we can generate $\left\{\sigma_t^2\right\}_{t=15}^{T}$ and $\left\{u_t\right\}_{t=1}^{T}$, with the following formula:

$$\sigma_1^2 = a_0 + a_1 \sigma_0^2 z_t^2 + b_1 \sigma_0^2,$$

$u_1 = z_1 \sigma_1$.

3) Generate random number series $\left(R_{mt} - R_{ft}\right)$, $SMB_t$, $HMLO_t$, $RMW_t$, $CMA_t$ from Uniform (0, 1).

4) Set $\beta_0 = 0.2$, $\beta_1 = 1$, $\beta_2 = 0.5$, $\beta_3 = 0.5$, $\beta_4 = 0.5$, $\beta_5 = 0.5$ and we can get $\left\{Y_t\right\}_{t=1}^{T}$, with the following formula:

After getting the simulated data $\left(R_{mt} - R_{ft}\right)$, $SMB_t$, $HMLO_t$, $RMW_t$, $CMA_t$ and $\left\{Y_t\right\}_{t=1}^{T}$, we can use them to estimate the parameters in the FF5-SSAEPD-GARCH model. Using above-mentioned true values of the parameters, we do the simulations. The simulation results are reported in Table 2, all the estimates are close to the true values of the parameters. For robustness exam, we then alter the true values of the parameters and re-run the simulation. All the simulation results show that most of estimates are very close to the true values of the parameters, since most of errors are equal to or less than 10%. Also, we get the same estimates as those in Fama and French (2015) (see. Appendix 1). Hence, we can draw the conclusion that this MatLab program can be applied to estimate and analyze empirical data for FF5-SSAEPD-GARCH.

## 4. Empirical Analysis

### 4.1. Data

The data we analyze are the monthly returns from the Fama-French 25 value-weighted portfolios for US stock market, which are the same as data used in Fama and French (2015). The descriptive statistics of sample data are calculated by MatLab and listed in Table 3. For each observation, the skewness estimates (except one case) is not 0 and all kurtosis estimates are more than 3, which suggests that the data follows a leptokurtic distribution with the high peak and fat tail. The P-value of Jarque-Bera test for each portfolio is 0, which is smaller than 5% significance level. Hence, we can reject the null hypothesis and conclude that the asset returns do not follow Normal distribution. Thus, non-Normal error of SSAEPD might be able to fit the data better.

**Table 2.** Simulation results.

| | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $\alpha$ | $p_1$ | $p_2$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | 0.2 | 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 2 | 2 | 0.3 | 0.5 | 0.4 |
| E | 0.1823 | 1.0260 | 0.5096 | 0.4801 | 0.5306 | 0.5096 | 0.4951 | 1.9497 | 2.0179 | 0.2998 | 0.4972 | 0.4028 |
| P | 8.84% | 2.60% | 1.92% | 3.97% | 6.12% | 1.93% | 0.97% | 2.52% | 0.89% | 0.07% | 0.57% | 0.70% |
| T | 0.2 | 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 2 | 2 | 0.3 | 0.5 | 0.4 |
| E | 0.1963 | 1.0303 | 0.5234 | 0.4957 | 0.5381 | 0.4601 | 0.5054 | 2.0098 | 2.0179 | 0.2917 | 0.5027 | 0.3943 |
| P | 1.85% | 3.03% | 4.69% | 0.85% | 7.62% | 7.98% | 1.09% | 0.49% | 0.90% | 2.78% | 1.17% | 1.43% |
| T | 0.3 | 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 2 | 2 | 0.3 | 0.5 | 0.4 |
| E | 0.3123 | 1.0108 | 0.5262 | 0.4729 | 0.4787 | 0.4955 | 0.4990 | 1.9998 | 1.9994 | 0.3019 | 0.4929 | 0.4009 |
| P | 4.10% | 1.08% | 5.24% | 5.41% | 4.25% | 0.89% | 0.19% | 0.01% | 0.03% | 0.65% | 1.42% | 0.24% |
| T | 0.2 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 2 | 2 | 0.3 | 0.5 | 0.4 |
| E | 0.1911 | 0.5242 | 0.4724 | 0.4775 | 0.5320 | 0.5356 | 0.5000 | 2.0000 | 2.0000 | 0.2918 | 0.5185 | 0.3926 |
| P | 4.43% | 4.85% | 5.52% | 4.51% | 6.40% | 7.12% | 0.00% | 0.00% | 0.00% | 2.73% | 3.71% | 2.05% |
| T | 0.2 | 1 | 1 | 0.5 | 0.5 | 0.5 | 0.5 | 2 | 2 | 0.3 | 0.5 | 0.4 |
| E | 0.1871 | 1.0070 | 0.9952 | 0.4981 | 0.5193 | 0.5135 | 0.5000 | 1.9998 | 2.0001 | 0.3281 | 0.5289 | 0.3634 |
| P | 6.43% | 0.70% | 0.48% | 0.38% | 3.87% | 2.69% | 0.00% | 0.01% | 0.00% | 9.38% | 5.78% | 9.15% |
| T | 0.2 | 1 | 0.5 | 1 | 0.5 | 0.5 | 0.5 | 2 | 2 | 0.3 | 0.5 | 0.4 |
| E | 0.1884 | 0.9851 | 0.5161 | 1.0033 | 0.5163 | 0.4963 | 0.5000 | 2.0000 | 2.0000 | 0.2745 | 0.4918 | 0.4212 |
| P | 5.78% | 1.49% | 3.22% | 0.33% | 3.27% | 0.073% | 0.00% | 0.00% | 0.00% | 8.51% | 1.64% | 5.31% |
| T | 0.2 | 1 | 0.5 | 0.5 | 1 | 0.5 | 0.5 | 2 | 2 | 0.3 | 0.5 | 0.4 |
| E | 0.1922 | 0.9619 | 0.4798 | 0.4775 | 1.0282 | 0.5010 | 0.5004 | 1.9959 | 1.9947 | 0.3059 | 0.4688 | 0.4174 |
| P | 3.90% | 3.81% | 4.05% | 4.49% | 2.82% | 0.20% | 0.07% | 0.21% | 0.26% | 1.95% | 6.25% | 4.34% |
| T | 0.2 | 1 | 0.5 | 0.5 | 0.5 | 1 | 0.5 | 2 | 2 | 0.3 | 0.5 | 0.4 |
| E | 0.1944 | 1.0429 | 0.5075 | 0.5375 | 0.4979 | 0.9589 | 0.5000 | 2.0000 | 2.0000 | 0.3088 | 0.5055 | 0.3911 |
| P | 2.81% | 4.29% | 1.51% | 7.50% | 0.42% | 4.11% | 0.00% | 0.00% | 0.00% | 2.94% | 1.09% | 2.22% |
| T | 0.2 | 1 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 2 | 2 | 0.3 | 0.4 | 0.5 |
| E | 0.2042 | 1.0001 | 0.4898 | 0.4987 | 0.4843 | 0.4620 | 0.5000 | 1.9996 | 1.9995 | 0.2905 | 0.3948 | 0.5013 |
| P | 2.10% | 0.01% | 2.05% | 0.25% | 3.13% | 7.59% | 0.01% | 0.02% | 0.03% | 3.17% | 1.30% | 0.25% |

Notes: T means the true value of parameters. E means the estimates. P means the error in percentage.

Table 3. Descriptive statistics (1963:7 - 2013:12).

| | Size | | | | | Book-to-market quintiles | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quintile | Low | 2 | 3 | 4 | High | Low | 2 | 3 | 4 | High |
| | Mean | | | | | Median | | | | |
| Small | 0.68 | 1.22 | 1.26 | 1.42 | 1.56 | 1.06 | 1.43 | 1.36 | 1.54 | 1.57 |
| 2 | 0.89 | 1.14 | 1.35 | 1.36 | 1.44 | 1.37 | 1.56 | 1.60 | 1.55 | 1.85 |
| 3 | 0.91 | 1.19 | 1.21 | 1.29 | 1.49 | 1.51 | 1.47 | 1.57 | 1.50 | 1.61 |
| 4 | 1.01 | 0.99 | 1.12 | 1.26 | 1.27 | 1.09 | 1.24 | 1.45 | 1.54 | 1.60 |
| Big | 0.87 | 0.93 | 0.89 | 0.97 | 1.03 | 0.97 | 1.04 | 1.15 | 1.10 | 1.19 |
| | Maximum | | | | | Minimum | | | | |
| Small | 39.85 | 38.56 | 28.13 | 27.78 | 33.27 | −34.24 | −30.94 | −28.69 | −28.88 | −28.88 |
| 2 | 27.45 | 26.12 | 26.34 | 27.34 | 30.04 | −32.71 | −31.56 | −27.80 | −26.04 | −28.84 |
| 3 | 24.69 | 25.03 | 21.94 | 23.40 | 29.20 | −29.79 | −28.99 | −24.26 | −23.03 | −26.17 |
| 4 | 25.91 | 20.44 | 24.01 | 24.32 | 27.90 | −25.94 | −28.83 | −26.33 | −21.02 | −23.84 |
| Big | 22.35 | 16.53 | 19.08 | 19.76 | 17.57 | −21.64 | −22.36 | −21.74 | −19.32 | −19.13 |
| | Standard Deviation | | | | | Skewness | | | | |
| Small | 8.01 | 6.90 | 6.02 | 5.68 | 6.11 | −0.02 | 0.00 | −0.17 | −0.22 | −0.26 |
| 2 | 7.23 | 6.00 | 5.46 | 5.32 | 6.03 | −0.34 | −0.46 | −0.48 | −0.50 | −0.46 |
| 3 | 6.67 | 5.51 | 5.03 | 4.94 | 5.52 | −0.36 | −0.52 | −0.53 | −0.33 | −0.41 |
| 4 | 5.94 | 5.21 | 5.10 | 4.84 | 5.53 | −0.22 | −0.60 | −0.51 | −0.28 | −0.30 |
| Big | 4.70 | 4.47 | 4.38 | 4.39 | 5.05 | −0.22 | −0.38 | −0.31 | −0.25 | −0.30 |
| | Kurtosis | | | | | P-value of Jarque-Bera Test | | | | |
| Small | 5.26 | 6.12 | 5.63 | 5.90 | 6.34 | 0 | 0 | 0 | 0 | 0 |
| 2 | 4.45 | 5.33 | 5.91 | 6.11 | 6.16 | 0 | 0 | 0 | 0 | 0 |
| 3 | 4.44 | 5.64 | 5.26 | 5.43 | 6.26 | 0 | 0 | 0 | 0 | 0 |
| 4 | 4.72 | 5.90 | 6.35 | 5.06 | 5.52 | 0 | 0 | 0 | 0 | 0 |
| Big | 4.67 | 4.76 | 5.24 | 4.96 | 4.07 | 0 | 0 | 0 | 0 | 0 |

## 4.2. Estimation Results

### Estimates for the FF5-SSAEPD-GARCH Model

The estimates for our new model are displayed in Table 4. We find out that our model can successfully capture the skewness, fat-tailness and asymmetric kurtosis of the data. To be specific, the skewness parameters $\alpha$ are all not equal to 0.5, which captures the skewness in the data. 44 out of 50 estimates for the tail parameters $pi$ ($i = 1, 2$) are smaller than 2, which suggests that portfolio returns are fat-tailed distributed. Besides, all the tail parameters p1 and p2 are not equal to each other, which document the asymmetric kurtosis. And 15 out of 25 portfolios have bigger estimates for the left tail parameter p1, which means that these returns have thinner left tails.

**Table 4.** Estimates on FF5-SSAEPD-GARCH (Monthly, 1963:07 - 2013:12).

| | Size | | | | | Book-to-market quintiles | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quintile | Low | 2 | 3 | 4 | High | Low | 2 | 3 | 4 | High |
| | | | $\beta_0$ | | | | | $\beta_1$ | | |
| Small | −0.30 | 0.10 | 0.00 | 0.69 | 0.00 | 1.03 | 0.95 | 0.94 | 0.87 | 0.97 |
| 2 | −0.09 | −0.07 | −0.13 | 0.03 | −0.09 | 1.07 | 1.02 | 0.99 | 0.98 | 1.08 |
| 3 | 0.04 | 0.07 | −0.15 | 0.01 | −0.51 | 1.05 | 1.01 | 1.00 | 1.00 | 1.11 |
| 4 | 0.16 | −0.08 | 0.04 | 0.06 | −0.07 | 1.05 | 1.07 | 1.05 | 1.02 | 1.13 |
| Big | −0.15 | 0.38 | −0.05 | −0.01 | −0.30 | 1.02 | 0.98 | 0.98 | 0.98 | 1.09 |
| | | | $\beta_2$ | | | | | $\beta_3$ | | |
| Small | 1.26 | 1.20 | 1.06 | 1.00 | 1.11 | −0.48 | −0.07 | 0.11 | 0.30 | 0.48 |
| 2 | 0.97 | 0.90 | 0.84 | 0.76 | 0.87 | −0.46 | −0.04 | 0.26 | 0.42 | 0.67 |
| 3 | 0.69 | 0.63 | 0.53 | 0.48 | 0.65 | −0.43 | 0.00 | 0.30 | 0.51 | 0.63 |
| 4 | 0.34 | 0.28 | 0.25 | 0.24 | 0.30 | −0.41 | 0.01 | 0.27 | 0.52 | 0.77 |
| Big | −0.19 | −0.19 | −0.20 | −0.15 | −0.03 | −0.33 | 0.01 | 0.26 | 0.53 | 0.85 |
| | | | $\beta_4$ | | | | | $\beta_5$ | | |
| Small | −0.64 | −0.24 | 0.01 | 0.11 | 0.13 | −0.56 | −0.14 | 0.20 | 0.28 | 0.70 |
| 2 | −0.21 | 0.04 | 0.22 | 0.23 | 0.23 | −0.58 | 0.03 | 0.37 | 0.53 | 0.74 |
| 3 | −0.21 | 0.10 | 0.16 | 0.19 | 0.24 | −0.68 | 0.02 | 0.42 | 0.58 | 0.98 |
| 4 | −0.13 | 0.08 | 0.06 | 0.07 | 0.22 | −0.47 | 0.15 | 0.33 | 0.53 | 0.74 |
| Big | 0.13 | 0.12 | 0.02 | 0.09 | 0.06 | −0.26 | −0.01 | 0.38 | 0.50 | 0.77 |
| | | | $\alpha$ | | | | | $p_1$ | | |
| Small | 0.44 | 0.79 | 0.66 | 0.45 | 0.35 | 1.60 | 3.21 | 2.35 | 1.73 | 1.18 |
| 2 | 0.54 | 0.63 | 0.45 | 0.45 | 0.30 | 1.43 | 1.84 | 1.41 | 1.39 | 1.01 |
| 3 | 0.38 | 0.72 | 0.72 | 0.80 | 0.62 | 1.27 | 1.68 | 2.07 | 2.03 | 1.80 |
| 4 | 0.70 | 0.35 | 0.66 | 0.47 | 0.45 | 2.04 | 1.05 | 1.67 | 1.46 | 1.57 |
| Big | 0.53 | 0.56 | 0.63 | 0.38 | 0.41 | 1.71 | 1.93 | 1.78 | 1.38 | 1.28 |
| | | | $p_2$ | | | | | a | | |
| Small | 1.57 | 0.80 | 1.08 | 0.93 | 1.70 | 2.73 | 1.72 | 1.27 | 0.42 | 0.60 |
| 2 | 1.27 | 1.07 | 1.70 | 1.70 | 1.98 | 1.88 | 1.16 | 0.13 | 1.40 | 0.37 |
| 3 | 2.13 | 0.88 | 1.12 | 0.71 | 1.19 | 1.74 | 1.76 | 0.17 | 1.73 | 0.24 |
| 4 | 0.91 | 1.69 | 0.97 | 1.56 | 1.68 | 1.94 | 1.89 | 1.21 | 2.15 | 3.28 |
| Big | 1.66 | 1.45 | 1.22 | 1.96 | 1.72 | 0.04 | 0.12 | 2.00 | 0.15 | 0.25 |
| | | | b | | | | | c | | |
| Small | 0.37 | 0.27 | 0.22 | 0.08 | 0.22 | 0.00 | 0.00 | 0.00 | 0.82 | 0.49 |
| 2 | 0.12 | 0.31 | 0.07 | 0.10 | 0.15 | 0.00 | 0.00 | 0.85 | 0.00 | 0.66 |
| 3 | 0.14 | 0.28 | 0.13 | 0.24 | 0.11 | 0.00 | 0.00 | 0.80 | 0.00 | 0.82 |
| 4 | 0.14 | 0.33 | 0.53 | 0.15 | 0.12 | 0.00 | 0.00 | 0.14 | 0.00 | 0.00 |
| Big | 0.12 | 0.11 | 0.25 | 0.12 | 0.17 | 0.85 | 0.84 | 0.00 | 0.81 | 0.78 |

### 4.3. Model Diagnostics

To test the significance of coefficients in our new model, Likelihood Ratio test (LR) is applied, LR formula is from Neyman and Pearson (1993), which is Equation (18).

$$\text{LR} = -2\ln(\text{likelihood for null}) + 2\ln(\text{likelihood for alternative}) \quad (18)$$

#### 4.3.1. Tests for Parameter Restrictions

- Tests for Parameters in the Mean Equation

The P-values of LR are listed in Table 5. The null hypothesis of the joint significance test is $H_0$: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$. The P-values of the joint significance test for all the 25 portfolios are 0, which means $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$ and $\beta_5$ are statistically jointly significant under 5% significance level. The individual significance tests show that under 5% significance level the coefficient $\beta_1$ in all 25 portfolios are statistically significant; 24/25 portfolios have a statistically coefficient $\beta_2$ and $\beta_5$; 23/25 and 19/25 portfolios have a statistically coefficient $\beta_3$ and $\beta_4$, respectively. As for coefficient $\beta_0$, 16 out of the 25 portfolios don't have a statistically significant coefficient $\beta_0$ under 5% significance level. Thus, we can conclude that with non-Normal errors such as SSAEPD and GARCH- type volatilities, the Fama-French 5-factor model is still alive.

- Tests for Parameters in the GARCH Equation

In this part, some restrictions on the parameters in the GARCH equation are tested with Likelihood Ratio test (LR). And the results are listed in Table 5. Results show the GARCH-type volatility should be included in Fama-French 5 factor model. For instance, we do the joint significance test for hypothesis $H_0$: $b = c = 0$. The P-values of the LR are all smaller than the significance level 5%, which means our GARCH-type volatility is quite necessary. As for individual hypotheses, we discover that most P-values of LR are smaller than the significance level 5%. And to be specific, ARCH term ($H_0$: $b = 0$) is significant in 20 out of 25 portfolios and GARCH term (H0: $c = 0$) is significant in 18 out of 25 portfolios.

- Tests for Parameters in SSAEPD

We also run significance tests for the parameters in the SSAEPD and the results of parameter restrictions show strong non-Normality. And the results are listed in Table 5. For example, for the Hypothesis $H_0$: $\alpha = 0.5$, $p_1 = p_2 = 2$, 21 out of 25 p-values are smaller than the significance level 5%, which means that Normal error assumption is not supported by most of our data. Besides, Asymmetry is documented ($H_0$: $\alpha = 0.5$ is rejected by 7 out of 25 portfolios). And non-normality is found ($H_0$: $p_1 = 2$ is rejected by 8 out of 25 portfolios and 12 out of 25 portfolios reject the null $H_0$: $p_1 = 2$.).

#### 4.3.2. Residual Check

In this subsection, the residuals for previous models are checked with both Kolmogorov-Smirnov test and graphs. Our results show that 20 out of the 25 portfolios have residuals which do follow SSAEPD. That means our new model is adequate for the Fama-French 25 portfolios. But the FF5-Normal model is not adequate for the data, since 21 portfolios have residuals which do not follow the Normal error distribution.

**Table 5.** P-values of likelihood ratio test (LR).

| | Size | | | | | Book-to-market quintiles | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quintile | Low | 2 | 3 | 4 | High | Low | 2 | 3 | 4 | High |
| | $H_0{:}\beta_1=\beta_2=\beta_3=\beta_4=\beta_5=0$ | | | | | $H_0{:}\beta_0=0$ | | | | |
| Small | 0* | 0* | 0* | 0* | 0* | 0* | 0.08 | 0.95 | 1.00 | 0.31 |
| 2 | 0* | 0* | 0* | 0* | 0* | 0.13 | 0.19 | 0* | 0.58 | 0* |
| 3 | 0* | 0* | 0* | 0* | 0* | 0.46 | 0.17 | 0* | 1.00 | 1.00 |
| 4 | 0* | 0* | 0* | 0* | 0* | 0.01* | 0.24 | 0.57 | 0.36 | 0* |
| Big | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0.44 | 0.65 | 0* |
| | $H_0{:}\beta_1=0$ | | | | | $H_0{:}\beta_2=0$ | | | | |
| Small | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| 2 | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| 3 | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| 4 | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| Big | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0.20 |
| | $H_0{:}\beta_3=0$ | | | | | $H_0{:}\beta_4=0$ | | | | |
| Small | 0* | 0.03* | 0* | 0* | 0* | 0* | 0* | 1.00 | 0* | 0* |
| 2 | 0* | 0.57 | 0* | 0* | 0* | 0* | 0.96 | 0* | 0* | 0* |
| 3 | 0* | 0* | 0* | 0* | 0* | 0* | 0.01* | 0* | 0.01* | 0* |
| 4 | 0* | 0* | 0* | 0* | 0* | 0* | 0.91 | 0* | 0.53 | 0* |
| Big | 0* | 1.00 | 0* | 0* | 0* | 0* | 0* | 0.64 | 0* | 1.00 |
| | $H_0{:}\beta_5=0$ | | | | | $H_0{:}b=c=0$ | | | | |
| Small | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| 2 | 0* | 0.08 | 0* | 0* | 0* | 0* | 0* | 0* | 0.04* | 0.01* |
| 3 | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| 4 | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| Big | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| | $H_0{:}a=0$ | | | | | $H_0{:}b=0$ | | | | |
| Small | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 1.00 | 0* |
| 2 | 0* | 0* | 0* | 0* | 0* | 0.02* | 0* | 0* | 0.05 | 0* |
| 3 | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 1.00 |
| 4 | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| Big | 0* | 0* | 0* | 0* | 0* | 1.00 | 1.00 | 0* | 0* | 0* |
| | $H_0{:}c=0$ | | | | | $H_0{:}a=0.5, p_1=p_2=2$ | | | | |
| Small | 0* | 1.00 | 0* | 0* | 0* | 0* | 0* | 0* | 0* | 0* |
| 2 | 0* | 1.00 | 0* | 1.00 | 0* | 0* | 0* | 0.05 | 0.01* | 0* |
| 3 | 0* | 0* | 0* | 1.00 | 0* | 0.02* | 0* | 0* | 0* | 0* |
| 4 | 0* | 1.00 | 1.00 | 0.95 | 0* | 0* | 0* | 0* | 0* | 0.04* |
| Big | 0* | 0* | 0* | 0* | 0* | 0.35 | 0.37 | 0* | 0.23 | 0* |
| | $H_0{:}a=0.5$ | | | | | $H_0{:}p_1=p_2=2$ | | | | |
| Small | 0.57 | 0.01* | 0.17 | 0.62 | 0.08 | 0.02* | 0* | 0.01* | 0* | 0* |
| 2 | 0.47 | 0.06 | 0.40 | 0.46 | 0.01* | 0* | 0* | 0.02* | 0* | 0* |
| 3 | 0.07 | 0.04* | 0* | 0* | 0.18 | 0.01* | 0* | 0* | 0* | 0* |
| 4 | 0.02* | 0.08 | 0.03* | 0.69 | 0.42 | 0* | 0* | 0* | 0* | 0.04* |
| Big | 1.00 | 0.52 | 0.14 | 0.46 | 0.33 | 0.20 | 0.21 | 0* | 0.13 | 0* |
| | $H_0{:}p_1=2$ | | | | | $H_0{:}p_2=2$ | | | | |
| Small | 0.35 | 0.01* | 0.40 | 1.00 | 0.01* | 0.48 | 0* | 0.02* | 0* | 0.16 |
| 2 | 0.01* | 0.52 | 0.15 | 0.06 | 0* | 0* | 0* | 0.13 | 0.28 | 1.00 |
| 3 | 0* | 0.07 | 0.80 | 1.00 | 0.52 | 0.68 | 0* | 0* | 0* | 0* |
| 4 | 0.86 | 0* | 0.18 | 0.04* | 0.14 | 0* | 0.12 | 0* | 0.37 | 0.35 |
| Big | 0.44 | 0.95 | 0.49 | 0.21 | 0.02* | 0.31 | 0.15 | 0.01* | 0.90 | 0.23 |

Note: * means the null is rejected under 5% significant level.

- Kolmogorov-Smirnov Test for Residuals

To check the residuals, the Kolmogorov-Smirov test (KS) is employed. The P-value of KS test is displayed in Table 6. The P-values of KS test show the residuals from the new model do follow SSAEPD. For example, the P-value of the portfolio with Small Size and Low Book-to-market is 0.71, greater than 5%, which means under 5% significance level, the null hypothesis is not rejected and the residuals from our model do follow the SSAEPD. Similarly, the null hypothesis cannot be rejected for other 19 portfolios.

Then, we apply the KS test for the residuals from the FF5-Normal model. The P-values of the KS test are also listed in Table 6. 21 out of 25 portfolios have smaller P-values than 0.05, which means these 21 portfolios reject the nulls. Hence, the error terms of the portfolios do not follow Normal distribution. And the FF-Normal model is not adequate for the data.

- PDFs of Residuals

By method of "eye-rolling", the PDF of residuals is compared with theoretical PDFs. Taking the portfolio with Small Size and Low Book-to-market for example, in Figure 1, the probability density function (PDF) for the estimated residuals $\hat{z}_t$ in FF5-SSAEPD-GARCH and that of $SSAEPD\left(\hat{\alpha}, \hat{p}_1, \hat{p}_2\right)$ are plotted. These curves are very close to each other, indicating that the residuals are distributed as SSAEPD. Hence, the FF5-SSAEPD-GARCH model fits the data well.

Similarly, the probability density function (PDF) for the estimated residuals $\hat{u}_t$ in FF5-Normal and that of $Normal\left(\hat{\mu}, \hat{\sigma}^2\right)$ are shown in Figure 2. And there is a big difference between these two curves, indicating the residuals do not follow Normal distribution.

### 4.4. Model Diagnostics

In this subsection, we compare our new model with the 5-factor model of Fama and French (2015). The Akaike Information Criterion (AIC) is used as the model selection criterion. Table 7 displays the AIC values. We find that 23 out of 25 AIC values of the

**Table 6.** P-values of KS test for residuals.

| | Size | | | | | Book-to-market quintiles | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quintile | Low | 2 | 3 | 4 | High | Low | 2 | 3 | 4 | High |
| | *FF5-SSAEPD-GARCH*[a] | | | | | *FF5-Normal*[b] | | | | |
| Small | 0.71 | 0.50 | 0.91 | 0* | 0.28 | 0* | 0* | 0* | 0.01* | 0* |
| 2 | 0.98 | 1.00 | 0* | 0.94 | 0.76 | 0* | 0.07 | 0.13 | 0.06 | 0.01* |
| 3 | 0.67 | 0.87 | 0.17 | 0.46 | 0* | 0* | 0* | 0* | 0* | 0* |
| 4 | 0.81 | 0.93 | 0.19 | 0.96 | 0.92 | 0* | 0* | 0* | 0* | 0* |
| Big | 0* | 0* | 0.82 | 0.74 | 0.17 | 0.79 | 0* | 0* | 0* | 0* |

a. The null hypothesis $H_0$ is: FF5-SSAEPD-GARCH residuals are distributed as $\mathrm{SSAEPD}\left(\hat{\alpha}, \hat{p}_1, \hat{p}_2\right)$. b. The null hypothesis $H_0$ is: FF5-Normal residuals are distributed as $\mathrm{Normal}\left(\hat{\mu}, \hat{\sigma}^2\right)$. * means the null is rejected under 5% significant level.

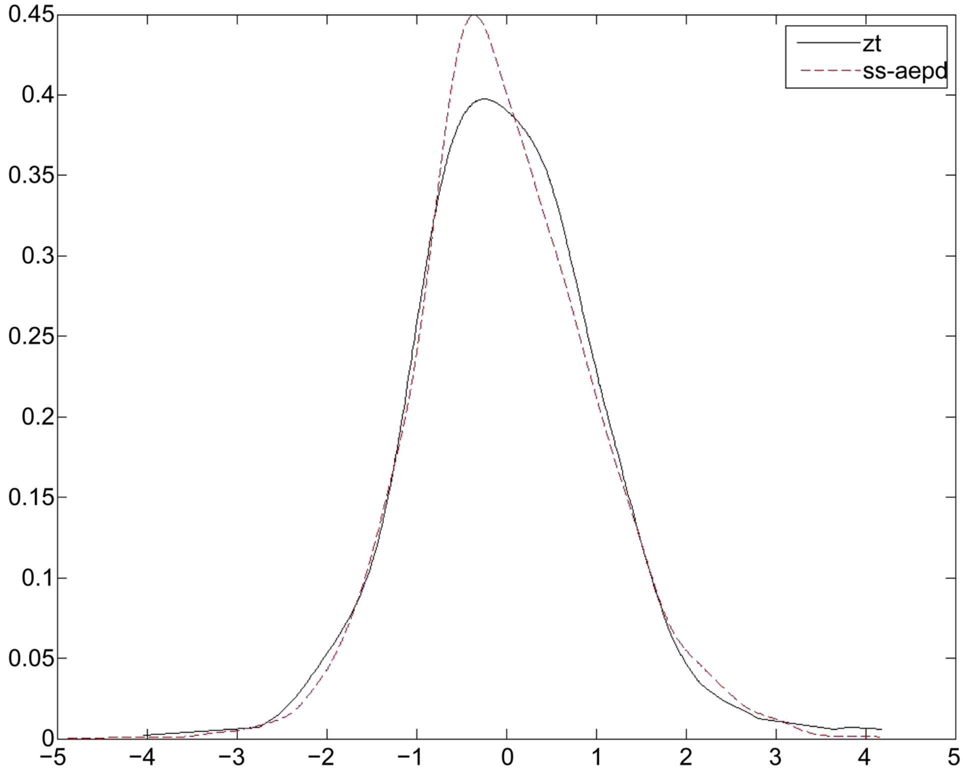


**Figure 1.** PDFs of the residuals (FF5-SSAEPD-GARCH) and $\mathrm{SSAEPD}\left(\hat{\alpha},\hat{p}_1,\hat{p}_2\right)$.

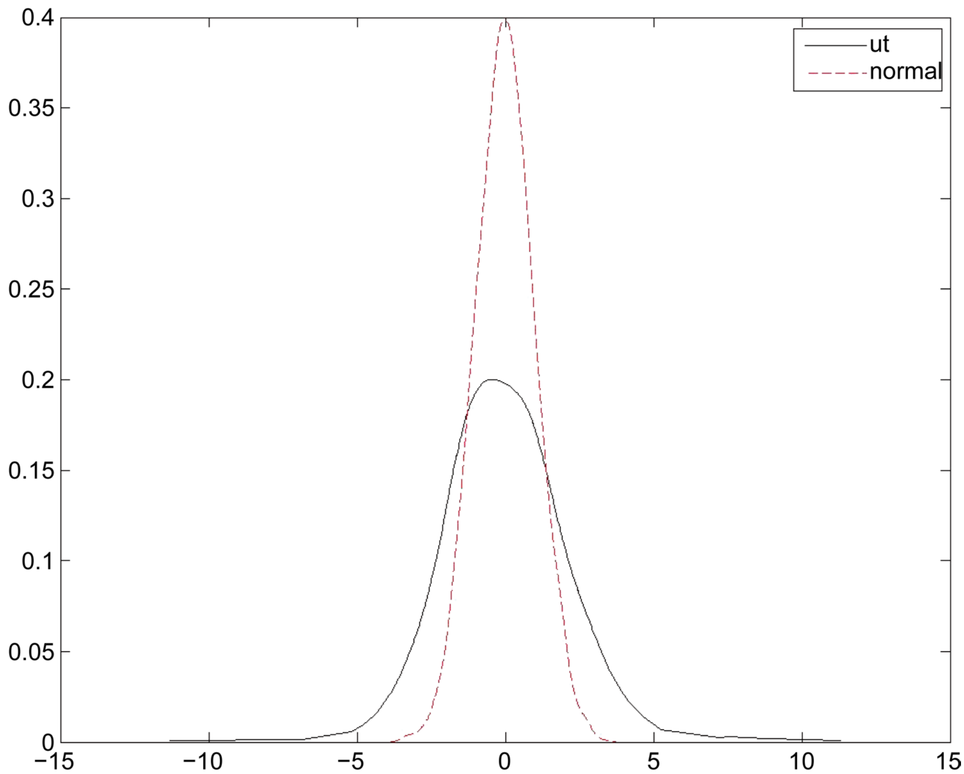


**Figure 2.** PDFs of the residuals (FF5-Normal) and $\mathrm{Normal}\left(\hat{\mu},\hat{\sigma}^2\right)$.

**Table 7.** AIC values (Monthly, 1963:07 - 2013:12).

| | Size | | | | | Book-to-market quintiles | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quintile | Low | 2 | 3 | 4 | High | Low | 2 | 3 | 4 | High |
| | *FF5-SSAEPD-GARCH* | | | | | *FF5-Normal* | | | | |
| Small | 4.19* | 3.60* | 3.29* | 3.45 | 3.49* | 4.32 | 3.71 | 3.36 | 3.43 | 3.60 |
| 2 | 3.58* | 3.26* | 3.24* | 3.29* | 3.44* | 3.63 | 3.38 | 3.27 | 3.32 | 3.54 |
| 3 | 3.54* | 3.61* | 3.57* | 3.54* | 3.96* | 3.59 | 3.74 | 3.68 | 3.68 | 4.00 |
| 4 | 3.60* | 3.76* | 3.80* | 3.77* | 4.16* | 3.67 | 3.85 | 3.93 | 3.82 | 4.18 |
| Big | 2.98* | 3.53 | 3.77* | 3.51* | 4.27* | 3.00 | 3.48 | 3.89 | 3.62 | 4.47 |

Note: Numbers with * are smaller AIC values.

FF5-SSAEPD-GARCH model are smaller than those of the FF5-Normal model. Hence, we can conclude that our new model (FF5-SSAEPD-GARCH) performs better than the 5-factor model in Fama and French (2015).

## 5. Conclusions

In this paper, we extend the 5-factor model in Fama and French (2015) by introducing a non-normal error term and time-varying volatilities. The non-normal error assumption we used is the SSAEPD in Zhu and Zinde-Walsh (2009). And the time-varying volatilities are the GARCH model in Bollerslev (1986). For comparison, monthly US stock returns in Fama and French (2015) (1963:07 - 2013:12) are analyzed. Method of Maximum Likelihood is used. Likelihood Ratio Test (LR) is used to test the hypotheses of parameter restrictions. Kolmogorov-Smirnov test (KS) is used to check residuals. Akaike Information Criterion (AIC) is used to compare models.

Simulation results show our MatLab program for the new model is valid. And empirical results show: 1) this new model can capture the skewness, fat tails and asymmetric kurtosis in the data; 2) With GARCH-type volatilities and non-normal errors, the Fama-French 5 factors are still alive, since the estimates are all significant; and 3) our new model can fit the data much better than 5-factor model in Fama and French (2015). Our study provides an update to existing asset pricing literature and reference for investors.

Future extensions will include but not limited to the following. First, we can exam our results with different data. Second, we can compare our results with those from other models such as ARIMA model. Last but not the least, other factors can be introduced into this model.

## Acknowledgements

We also want to thank participants in the 16th World Business Research Conference at San Francisco (28 - 29 July, 2016) and the seminars organized by Institute of Statistics and Econometrics, Nankai University. The support of Jiayi Zhu and Qingyu Zhu is gratefully acknowledged. The authors are responsible for all errors.

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## Appendix 1. Estimates for the FF5-Normal Model

To test our MatLab program, we also estimate the FF5-Normal model using the program by setting $\{a_i = 0\}_{i=1}^{r}$, $\{b_i = 0\}_{i=1}^{s}$, $\alpha = 0.5$, $p_1 = p_2 = 2$. The estimates are listed in Table 8. The estimates and their t - statistics are very close to those in Table 9, respectively. Thus, the MatLab program we write is valid.

Table 8. Estimates for FF5-normal model by our MatLab program (Monthly, 1963:07 - 2013:12).

| | Size | | | | | Book-to-market quintiles | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quintile | Low | 2 | 3 | 4 | High | Low | 2 | 3 | 4 | High |
| | | | *a* | | | | | *t*(*a*) | | |
| Small | −0.29 | 0.11 | 0.01 | 0.12 | 0.12 | −3.28 | 1.60 | 0.14 | 2.03 | 1.95 |
| 2 | −0.12 | −0.11 | 0.04 | 0.00 | −0.04 | −1.84 | −1.94 | 0.81 | 0.06 | −0.66 |
| 3 | 0.02 | −0.01 | −0.06 | −0.03 | 0.05 | 0.38 | −0.20 | −0.99 | −0.40 | 0.59 |
| 4 | 0.17 | −0.23 | −0.15 | 0.05 | −0.10 | 2.59 | −3.22 | −1.98 | 0.65 | −1.18 |
| Big | 0.11 | −0.10 | −0.11 | −0.15 | −0.10 | 2.49 | −1.66 | −1.55 | −2.42 | −0.99 |
| | | | *h* | | | | | *t*(*h*) | | |
| Small | −0.42 | −0.14 | 0.11 | 0.28 | 0.52 | −9.92 | −4.39 | 4.07 | 10.41 | 17.90 |
| 2 | −0.46 | −0.01 | 0.29 | 0.42 | 0.70 | −15.41 | −0.42 | 11.67 | 16.54 | 24.60 |
| 3 | −0.43 | 0.12 | 0.37 | 0.52 | 0.67 | −14.63 | 3.81 | 12.25 | 17.15 | 18.83 |
| 4 | −0.46 | 0.09 | 0.38 | 0.52 | 0.80 | −15.22 | 2.63 | 11.16 | 15.78 | 20.57 |
| Big | −0.31 | 0.03 | 0.26 | 0.62 | 0.84 | −14.29 | 1.13 | 7.68 | 20.86 | 18.56 |
| | | | *r* | | | | | *t*(*r*) | | |
| Small | −0.56 | −0.33 | 0.01 | 0.11 | 0.11 | −13.00 | −10.32 | 0.34 | 3.90 | 3.74 |
| 2 | −0.20 | 0.14 | 0.27 | 0.25 | 0.20 | −6.65 | 5.14 | 10.36 | 9.36 | 6.79 |
| 3 | −0.20 | 0.22 | 0.32 | 0.28 | 0.32 | −6.77 | 6.77 | 10.30 | 8.83 | 8.58 |
| 4 | −0.18 | 0.26 | 0.27 | 0.14 | 0.25 | −5.88 | 7.67 | 7.69 | 4.00 | 6.10 |
| Big | 0.13 | 0.24 | 0.08 | 0.22 | 0.01 | 5.86 | 8.45 | 2.29 | 7.31 | 0.23 |
| | | | *c* | | | | | *t*(*c*) | | |
| Small | −0.57 | −0.11 | 0.19 | 0.39 | 0.61 | −12.36 | −3.23 | 6.70 | 13.17 | 18.98 |
| 2 | −0.58 | 0.06 | 0.31 | 0.54 | 0.71 | −17.65 | 2.21 | 11.49 | 19.50 | 22.86 |
| 3 | −0.66 | 0.13 | 0.42 | 0.64 | 0.77 | −20.53 | 3.83 | 12.67 | 19.13 | 19.58 |
| 4 | −0.49 | 0.31 | 0.51 | 0.60 | 0.78 | −14.82 | 8.56 | 13.50 | 16.73 | 18.19 |
| Big | −0.38 | 0.25 | 0.41 | 0.65 | 0.72 | −16.04 | 8.31 | 11.15 | 20.17 | 14.50 |

**Table 9.** Estimates in Fama and French (2015) (Monthly, 1963:07 - 2013:12).

| | Size | | | | | Book-to-market quintiles | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Quintile | Low | 2 | 3 | 4 | High | Low | 2 | 3 | 4 | High |
| | | | *a* | | | | | *t*(*a*) | | |
| Small | −0.29 | 0.11 | 0.01 | 0.12 | 0.12 | −3.31 | 1.61 | 0.17 | 2.12 | 1.99 |
| 2 | −0.11 | −0.10 | 0.05 | −0.00 | −0.04 | −1.73 | −1.88 | 0.95 | −0.04 | −0.64 |
| 3 | 0.02 | −0.01 | −0.07 | −0.02 | 0.05 | 0.40 | −0.10 | −1.06 | −0.25 | 0.60 |
| 4 | 0.18 | −0.23 | −0.13 | 0.05 | −0.09 | 2.73 | −3.29 | −1.81 | 0.73 | −1.09 |
| Big | 0.12 | −0.11 | −0.10 | −0.15 | −0.09 | 2.50 | −1.82 | −1.39 | −2.33 | −0.93 |
| | | | *h* | | | | | *t*(*h*) | | |
| Small | −0.43 | −0.14 | 0.10 | 0.27 | 0.52 | −10.11 | −4.38 | 3.90 | 10.12 | 17.55 |
| 2 | −0.46 | −0.01 | 0.29 | 0.43 | 0.69 | −15.22 | −0.45 | 11.77 | 16.78 | 24.44 |
| 3 | −0.43 | 0.12 | 0.37 | 0.52 | 0.67 | −14.70 | 3.71 | 12.28 | 17.07 | 18.75 |
| 4 | −0.46 | 0.09 | 0.38 | 0.52 | 0.80 | −15.18 | 2.76 | 11.03 | 15.88 | 20.26 |
| Big | −0.31 | 0.03 | 0.26 | 0.62 | 0.85 | −14.12 | 1.09 | 7.54 | 21.05 | 18.74 |
| | | | *r* | | | | | *t*(*r*) | | |
| Small | −0.58 | −0.34 | 0.01 | 0.11 | 0.12 | −13.26 | −10.56 | 0.31 | 3.89 | 3.95 |
| 2 | −0.21 | 0.13 | 0.27 | 0.26 | 0.21 | −6.75 | 4.89 | 10.35 | 9.86 | 7.04 |
| 3 | −0.21 | 0.22 | 0.33 | 0.28 | 0.33 | −6.99 | 6.77 | 10.36 | 8.98 | 8.88 |
| 4 | −0.19 | 0.27 | 0.28 | 0.14 | 0.25 | −6.06 | 7.75 | 7.99 | 4.16 | 6.14 |
| Big | 0.13 | 0.25 | 0.07 | 0.23 | 0.02 | 5.64 | 8.79 | 2.07 | 7.62 | 0.49 |
| | | | *c* | | | | | *t*(*c*) | | |
| Small | −0.57 | −0.12 | 0.19 | 0.39 | 0.62 | −12.27 | −3.46 | 6.59 | 13.15 | 19.10 |
| 2 | −0.59 | 0.06 | 0.31 | 0.55 | 0.72 | −17.76 | 1.94 | 11.27 | 19.39 | 22.92 |
| 3 | −0.67 | 0.13 | 0.42 | 0.64 | 0.78 | −20.59 | 3.64 | 12.52 | 18.97 | 19.62 |
| 4 | −0.51 | 0.31 | 0.51 | 0.60 | 0.79 | −15.11 | 8.33 | 13.35 | 16.41 | 18.03 |
| Big | −0.39 | 0.26 | 0.41 | 0.66 | 0.73 | −16.08 | 8.38 | 10.80 | 19.88 | 14.54 |

Note: This table is quoted from the results in Table 7 on page 13 of Fama and French (2015).