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From Dynamic Linear Evaluation Rule to Dynamic CAPM in a Fractional Brownian Motion Environment*

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ABSTRACT

In this paper, we present the fundamental framework of the evaluation problem under which the evaluation operator satisfying some axioms is linear. Based on the dynamic linear evaluation mechanism of contingent claims, studying this evaluation rule in the market driven by fractional Brownian motions has led to a dynamic capital asset pricing model. It is deduced here mainly with the fractional Girsanov theorem and the Clark-Haussmann-Ocone theorem.

Keywords: Fractional Brownian Motion; Clark-Haussmann-Ocone Theorem; Fractional Girsanov Theorem; Evaluation Operator; Capital Asset Pricing Model

1. Introduction

In the 1960s, Sharpe (1964), Lintner (1965) and Mossin (1966) established the famous Capital Asset Pricing Model (CAPM for short). The CAPM has been used and cited in the literature over the past several decades. Some efforts have been made to extend this model. Dybvig and Ingersoll [1] investigated the relationship between the linear evaluation rule and the CAPM. They proved that the standard mean-variance separation theorem obtained in a complete market only if all investors had quadratic utility. In addition, the familiar CAPM pricing relation could hold for all assets in a complete market only if arbitrage opportunities existed. A description of the relationship between the linear evaluation rule and the theory of Markowitz portfolio choice can be found in [2], which they derived a general representation for asset prices that displayed the role of conditioning information. This representation was then used to examine restrictions implied by asset pricing models on the unconditional moments of asset payoffs and prices. An exhaustive discussion of the equivalence of these three theories (the linear evaluation rule, the CAPM and the theory of Markowitz portfolio choice) was presented in [3]. Shi [4] gave a fundamental probability model in the two-period security market. Under some conditions, if the linear evaluation rule holds, then there would be a stochastic discount factor. If this is true, all three theories (CAPM, linear evaluation rule and Markowitz portfolio choice) are equivalent. They are mainly deduced by the method of Hilbert space and stochastic discount factor. Particularly, CAPM could be deduced from the linear evaluation rule in the intertemporal market.

Since nowadays the market fluctuates promptly and dealings in securities require extremely high speed, no discrete-time model could adapt to the market well. However, the continuous-time model is regarded as a good approximation to real scenarios. If we assume that the model is continuous, then it facilitates the use of stochastic differential equations, stochastic analysis, and so on, to obtain some profound and concise conclusions. The famous Black-Scholes option pricing model is a classic issue of continuous-time finance. The fundamental theorem of asset pricing, the portfolio choice of securities and the CAPM all have their continuous-time version. Zhou and Wu [5] deduced the dynamic CAPM from the dynamic linear evaluation rule in the market driven by the Levy processes. They mainly used the predictable representation property in weak form and the Girsanov theorem of the Levy processes to obtain the results.

Ever since the pioneering work of Hurst [6,7] and Mandelbrot [8], the fractional Brownian motion has played an increasingly important role in various fields such as hydrology, economics, and telecommunications [9-12]. In this paper, we study the dynamic CAPM in the fractional Brownian motion environment, which represents a new perspective.

The remaining sections of this article are organized as

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follows: Some preliminaries of fractional Brownian motion are presented in Section 2. Section 3 presents the fundamental framework of the evaluation problem under which the evaluation operator satisfying some axioms is linear. In Section 4, we investigate the relationship between the dynamic linear evaluation rule and the dynamic CAPM in the market driven by fractional Brownian motions. Section 5 provides the conclusions.

2. Preliminaries of Fractional Brownian Motion

As preparation, collecting some important results concerning fractional Brownian motion is essential in this section. Also, it is necessary to introduce notation for further use.

Recall that if $0 < H < 1$, then the fractional Brownian motion with Hurst parameter H is a Gaussian process $\{B_t^H, t \ge 0\}$ with mean $E\left[B_t^H\right] = 0$ and covariance

$$E\left[B_t^H B_s^H\right] = \frac{1}{2}\left\{|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right\}$$

where $s,t \ge 0$ and $E = E_{\mu_H}$ denotes the expectation with respect to the probability law for $B^H = B^H(t,\omega)$. Assume that μ_H is defined on the σ-algebra $\mathscr{F}^{(H)}$ of subsets of Ω generated by the random variables $\{B_t^H, t \ge 0\}$. For simplicity we assume $B_H(0) = 0$.

If $H = 1/2$, then B_t^H coincides with the standard Brownian motion W_t, which has independent increments. If $H > \frac{1}{2}$, then B_t^H has a long-range dependence, in the sense that if we put:
$r(n) = \mathrm{cov}\left(B_1^H, \left(B_{n+1}^H - B_n^H\right)\right)$, then $\sum_{n=1}^{\infty} r(n) = \infty$.
For any $H \in (0,1)$ the process B_t^H is self-similar in the sense that $B_{\alpha t}^H$ has the same law as $\alpha^H B_t^H$ for any $\alpha > 0$. See [8,12] for more information about fractional Brownian motion.

Due to these properties, B_t^H with Hurst parameter $H \in \left(\frac{1}{2}, 1\right)$ has been suggested as a useful tool in many applications [11], including finance.

Fix a Hurst constant H, $\frac{1}{2} < H < 1$. Since H is fixed, the probability measure is denoted by P and the filtration is denoted by $\mathscr{F}$. In this case we have the integral representation [13] and the references therein):

$$B_t^H = \int_0^t K_H(t,s)\,\mathrm{d}W_s,$$

where $\{W_s, s \ge 0\}$ is a standard Brownian motion (Wiener process) and

$$K_H(t,s) = C_H\left[\left(\frac{t}{s}\right)^{H-\frac{1}{2}}(t-s)^{H-\frac{1}{2}} - \left(H - \frac{1}{2}\right)s^{\frac{1}{2}-H}\int_0^t u^{H-\frac{3}{2}}(u-s)^{H-\frac{1}{2}}\,\mathrm{d}u\right],$$

with C_H being a constant such that

$$\int_0^1 K_H^2(1,s)\,\mathrm{d}s = 1.$$

With this $K_H(t,s)$ we associate an operator

$$(K_H h)(t) = \int_0^t K_H(t,s)h(s)\,\mathrm{d}s,\ 0 \le t < \infty.$$

Recently, stochastic calculus for fractional Brownian motion has been developed by many researchers [13,14].

2.1. Quasi-Conditional Expectation and Fractional Girsanov Theorem

The quasi-conditional expectation is important to obtain the main results. It was initially introduced to find the hedging strategy in an application to finance [9]. Let f and g be two continuous functions on [0, T], where $T \in [0,\infty]$ is a fixed time horizon. Define

$$\langle f, g\rangle_t = \int_0^t\int_0^t \phi(u-v) f_u g_v \,\mathrm{d}u\mathrm{d}v,$$

where $\phi(x) = 2H(2H-1)|x|^{2H-2}$.

When $f = g$, denote: $\|f\|_t^2 = \langle f, f\rangle_t$.

Apparently, for any $t \in [0,T], \langle f,g\rangle_t$ is a Hilbert scalar product. Let Θ_t be the completion of the continuous functions under this Hilbert norm. The elements in Θ_t may be distributions [15].

For any $t \in (0,T]$, let $\mathscr{H}_t^{\otimes n}$ denote the set of all real symmetric functions f_n of n variables on $[0,t]^n$ such that

$$\sum_{n=0}^{\infty} n!\int_{[0,t]^{2n}} \prod_{i=1}^{n}\phi(s_i - r_i)\left|f_n(s_1,\cdots,s_n)\right| \left|f_n(r_1,\cdots,r_n)\right|\mathrm{d}s_1\cdots\mathrm{d}s_n\mathrm{d}r_1\cdots\mathrm{d}r_n < \infty$$

It is known [15] that $\mathscr{H}_t$ is a subspace of Θ_t and they are not identical. Let $\hat{L}^2(\Omega, \mathscr{F}, P)$ denote the set of $F \in L^2(\Omega, \mathscr{F}, P)$ such that F has the following chaos expansion: $F = \sum_{n=0}^{\infty} I_n(f_n)$,
where f_n when restricted to $[0,t]^n$ is in $H_t^{\otimes n}$ for all $0 < t \le T$ and

$$I_n(f_n) = \int_{0\le t_1,\cdots,t_n \le T} f_n(t_1,\cdots,t_n)\,\mathrm{d}B_{t_1}^H \cdots \mathrm{d}B_{t_n}^H$$

is the multiple stochastic integral (for the multiple integrals and the chaos expansion, [14,16].

Definition 2.1 If $F \in \hat{L}^2(\Omega, \mathscr{F}, P)$, then the quasi-conditional expectation [7] is defined as

$$\hat{E}\left[F\middle|\mathscr{F}_t\right]=\sum_{n=0}^{\infty}I_n\left(f_nI_{[0,t]}^{\otimes n}\right),$$

where

$$I_{[0,t]}^{\otimes n}\left(t_1,\cdots,t_n\right)=I_{[0,t]}\left(t_1\right)\cdots I_{[0,t]}t_n.$$

The following Lemma 2.1 (resp. Lemma 2.2) is from [17] Theorem 3.9 (resp. Theorem 3.11).

Lemma 2.1 Let σ be continuous such that $\|\sigma\|_t$ is an increasing function. Denote $\xi=g\left(\eta_T\right)$, where g is a measurable real valued function of polynomial growth and $\eta_T=\int_0^T\sigma_s\mathrm{d}B_s^H$. Then $E\left|\hat{E}\left[\xi\middle|\mathscr{F}\right]\right|^2\le E|\xi|^2$.

The following lemma is an analogue of the Striebel-Kallianpur formula. It is called a form of fractional Girsanov theorem.

Lemma 2.2 Let f_s and σ_s be continuous functions of s in [0, T] and $\eta_T=\int_0^T\sigma_s\mathrm{d}B_s^H$.

Consider the translation of B_t^H :

$$\hat{B}_t^H=B_t^H+\int_0^t f_s\mathrm{d}s,\ 0\le t\le T.$$

Let Q denote the probability measure given by

$$\frac{\mathrm{d}Q}{\mathrm{d}P}=\exp\left\{-\int_0^T\left(K_H^{-1}\int_0 f_r\mathrm{d}r\right)(s)\mathrm{d}W_s-\frac{1}{2}\int_0^T\left(K_H^{-1}\int_0 f_r\mathrm{d}r\right)^2(s)\mathrm{d}s\right\}$$

Then $\left\{\hat{B}_t^H,0\le t\le T\right\}$ is a fractional Brownian motion under Q. If h_t satisfies the integral equation

$$\sigma_t\int_0^t\phi(t,v)h_v\mathrm{d}v+h_t\int_0^t\phi(t,v)\sigma_v\mathrm{d}v+f_t\sigma_t=0,\quad 0\le t\le T,$$

then for any measurable function g of exponential growth,

$$\hat{E}^Q\left[g\left(\eta_T\right)\middle|\mathscr{F}_t\right]=\frac{\hat{E}\left[\rho_T g\left(\eta_T\right)\middle|\mathscr{F}_t\right]}{\hat{E}\left[\rho_T\middle|\mathscr{F}_t\right]},$$

where $\rho_T=\exp\left(\int_0^T h_s\mathrm{d}B_s^H-\frac{1}{2}\|h\|_T^2\right)$.

The above two lemmas are indispensable to the main results.

2.2. A fractional Clark-Haussmann-Ocone (CHO) Theorem

Finally let us review a fractional version of the Clark-Haussmann-Ocone (CHO) representation obtained in Theorem 4.5 in [9].

Lemma 2.3 Let $G(\omega)\in L^2(\Omega,\mathscr{F},P)$ be $\mathscr{F}_T$ -measurable, then $\hat{E}\left[D_tG\middle|\mathscr{F}_t\right]$ exists. Define $\psi(t,\omega)=\hat{E}\left[D_tG\middle|\mathscr{F}_t\right]$

Here $D_tG=\frac{\mathrm{d}G}{\mathrm{d}\omega}(t,\omega)$ is the stochastic gradient (Malliavin derivative) of G at t. Then,

$$G(\omega)=E[G]+\int_0^T\psi(t,\omega)\mathrm{d}B_t^H$$

Refer to Section 4 in [9] for details.

3. Mechanism of Evaluation of Contingent Claims

The mathematical formulation to the evaluation problem is provided below.

Let $(\Omega,\mathscr{F},P)$ be a complete probability space defined in Section 2, and $\left(\mathscr{F}_t\right)_{0\le t\le T}=\left(\mathscr{F}_t^{(H)}\right)_{0\le t\le T}$, a filtration satisfying the usual conditions, $\mathscr{F}_T=\mathscr{F},\mathscr{F}_0=\sigma\{\varnothing,\Omega\}$. Fix a time interval $[0,\mathrm{T}]$ and set

$$L^2(P)=\left\{\xi:\xi\ is\ \mathscr{F}_T\text{-measurable random variable},\ \text{and } E\left[|\xi|\right]<\infty\right\}$$

For all $\xi_1,\xi_2\in L^2(P)$, define the inner product of these two random variables $E\left[\xi_1\xi_2\right]$. Then $L^2(P)$ is a Hilbert space, which denotes the subspace of all contingent claims. $L^2\left(\Omega,\mathscr{F}_t,P\right)$ is the space of $\mathscr{F}_T$ -measurable and square-integrable random variables.

For all contingent claims $\xi\in L^2(P)$, denote the evaluated value by $\pi_t(\xi),t\in[0,T]$.

At each time t,

$$\pi_t:L^2(P)\to L^2\left(\Omega,\mathscr{F}_t,P\right)$$

is an evaluation operator. We will present the following axiomatic hypotheses of the evaluation operator:

(H1) $\pi_0(0)=0$.

(H2) If $\xi_i\in L^2(P),i=1,2,\cdots,\sum_{i=1}^{\infty}\xi_i=\xi\in L^2(P)$, then $\pi_0(\xi)=\sum_{i=1}^{\infty}\pi_0\left(\xi_i\right)$.

The following lemma is from [5] Lemma 2.1.

Lemma 3.1 For $\pi_0(\cdot)$, hypotheses (H1) and (H2) hold if and only if it is a continuous linear function defined on $L^2(P)$

(H3) For each $\xi\in L^2(P)$, if $\xi\ge 0$ $a.s.$, then $\pi_0(\xi)\ge 0$, if in addition $\xi\ge 0$ $a.s.$, $P(\xi>0)>0$, then $\pi_0(\xi)>0$.

Remark 3.1 The financial meaning of hypothesis (H1) is self evident. Hypothesis (H3) is similar to that there is no arbitrage in the market. Hypotheses (H1)-(H3) are the static properties of the linear evaluation operator.

Remark 3.2 From Lemma 3.1, we know that π_0 is a continuous linear function defined on the Hilbert space $L^2(P)$. It then follows from (H3) and the Riesz representative theorem that: there exists $\rho\in L^2(P),\rho>0$

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a.s., such that

$$\pi_0(\xi)=E[\rho\xi],\ \forall\xi\in L^2(P).$$

Since $\rho\in L^2(P)$, by [16] Theorem 3.1 and 3.2, without loss of generality, we may assume that there exists a Borel measurable (deterministic) function h_s such that

$$\rho=\exp\left(\int_0^T h_s\mathrm{d}B_s^H-\frac{1}{2}\|h\|_T^2\right).$$

(H4) For each $\xi\in L^2(P)$,

$$\frac{\pi_t(\xi)}{\pi_t(1)}=\frac{\hat{E}[\rho\xi|\mathscr{F}_t]}{\hat{E}[\rho|\mathscr{F}]_t},t\in[0,T]. \tag{3.1}$$

Remark 3.3 Hypothesis (H4) is the dynamic characteristic of the linear evaluation rules, which is shown uniquely in this paper. For the financial meaning of equation 3.1, you may see [5] Remark 2.2.

Now comes the explicit form of this evaluation operator in the market driven by fractional Brownian motion.

4. Deduce the Dynamic CAPM from the Dynamic Linear Evaluation Rule

4.1. The Explicit Form of the Evaluation Operator

Theorem 4.1 Let σ be continuous such that $\|\sigma\|_t$ is an increasing function. Denote $\xi=g(\eta_T)$, where g is measurable real valued function of polynomial growth such that $\xi\in L^2(P)$ and $\eta_T=\int_0^T\sigma_s\mathrm{d}B_s^H$. If there exists a continuous function f_s satisfying the integral equation

$$\sigma_t\int_0^t\phi(t,v)h_v\mathrm{d}v+h_t\int_0^t\phi(t,v)\sigma_v\mathrm{d}v+f_t\sigma_t=0,$$
$$0\le t\le T,$$

then let Q denote the probability measure given by

$$\frac{\mathrm{d}Q}{\mathrm{d}P}=\exp\left\{-\int_0^T\left(K_H^{-1}\int_0 f_r\mathrm{d}r\right)(s)\mathrm{d}W_s-\frac{1}{2}\int_0^T\left(K_H^{-1}\int_0 f_r\mathrm{d}r\right)^2(s)\mathrm{d}s\right\}$$

We have

$$\frac{\pi_t(\xi)}{\pi_t(1)}=E^Q\left[\hat{E}^Q[\xi|\mathscr{F}_t]\right]+\int_0^t\psi(s,\omega)\mathrm{d}B_s^H+\int_0^t\psi(s,\omega)f_s\mathrm{d}s,\ 0\le t\le T, \tag{4.1}$$

where $\psi(s,\omega)=\hat{E}^Q\left[D_s\left(\hat{E}^Q[\xi|\mathscr{F}_t]\right)\middle|\mathscr{F}_s\right]$.

Here $D_sG=\frac{\mathrm{d}G}{\mathrm{d}\omega}(s,\omega)$ is the stochastic gradient (Malliavin derivative) of G at s.

Proof. Let σ be continuous function such that $\|\sigma\|_t$ is an increasing function. Denote $\xi=g(\eta_T)$, where g is measurable real valued function of polynomial growth such that $\xi\in L^2(P)$ and $\eta_T=\int_0^T\sigma_s\mathrm{d}B_s^H$. By Lemma 2.2, we know that if there exists a continuous function f_s satisfying the integral equation

$$\sigma_t\int_0^t\phi(t,v)h_v\mathrm{d}v+h_t\int_0^t\phi(t,v)\sigma_v\mathrm{d}v+f_t\sigma_t=0,$$
$$0\le t\le T,$$

then consider the translation of B_t^H:

$$\hat{B}_t^H=B_t^H+\int_0^t f_s\mathrm{d}s,\ 0\le t\le T.$$

Let Q denote the probability measure given by

$$\frac{\mathrm{d}Q}{\mathrm{d}P}=\exp\left\{-\int_0^T\left(K_H^{-1}\int_0 f_r\mathrm{d}r\right)(s)\mathrm{d}W_s-\frac{1}{2}\int_0^T\left(K_H^{-1}\int_0 f_r\mathrm{d}r\right)^2(s)\mathrm{d}s\right\}$$

Then $\{\hat{B}^H,0\le t\le T\}$ is a fractional Brownian motion under Q. It follows from (3.1) and Lemma 2.2 that

$$\frac{\pi_t(\xi)}{\pi_t(1)}=\frac{\hat{E}[\rho\xi|\mathscr{F}]_t}{\hat{E}[\rho|\mathscr{F}_t]}=\hat{E}^Q[\xi|\mathscr{F}_t].$$

Let $G=\hat{E}^Q[\xi|\mathscr{F}_t]$. Define

$$\psi(s,\omega)=\hat{E}^Q[D_sG|\mathscr{F}_s],0\le s\le t,$$

where $D_sG=\frac{\mathrm{d}G}{\mathrm{d}\omega}(s,\omega)$ is the stochastic gradient (Malliavin derivative) of G at s. By Lemma 2.1 and 2.3, we obtain that

$$G(\omega)=E^Q[G]+\int_0^t\psi(s,\omega)\mathrm{d}\hat{B}_s^H$$

Thus,

$$\begin{aligned}\frac{\pi_t(\xi)}{\pi_t(1)}&=E^Q\left[\hat{E}^Q[\xi|\mathscr{F}_t]\right]+\int_0^t\psi(s,\omega)\mathrm{d}\left(B_s^H+\int_0^s f_u\mathrm{d}u\right)\\&=E^Q\left[\hat{E}^Q[\xi|\mathscr{F}_t]\right]+\int_0^t\psi(s,\omega)\mathrm{d}B_s^H+\int_0^t\psi(s,\omega)f_s\mathrm{d}s.\end{aligned}$$

The theorem is proved.

Suppose that $\pi_t(1)=1$. Remark Equation (4.1) can be formally expressed as

$$\mathrm{d}\pi_t(\xi)=\psi(t,\omega)\mathrm{d}B_t^H+\psi(t,\omega)f_t\mathrm{d}t. \tag{4.2}$$

4.2. Deduce the Dynamic CAPM from the Dynamic Linear Evaluation Rule

In fact, the CAPM attempts to relate R_i the one-period rate of return of a specified security *i*, to R_m the one-period rate of return of the entire market (as measured, say, by the Standard and Poor's index of 500 stocks). If r_f is the risk-free interest rate (usually taken to be the current rate of a US Treasury bill) then the model assumes that, for some constant β_i,

$$R_i = r_f + \beta_i\left(R_m - r_f\right) + \varepsilon_i,$$

where ε_i is a normal random variable with mean 0 that s values of R_i and R_m be r_i and r_m (resp.), the CAPM model (which treats r_f as a constant) implies that

$$r_i = r_f + \beta_i\left(r_m - r_f\right)$$

or, equivalently, that $r_i - r_f = \beta_i\left(r_m - r_f\right)$, where $\beta_i = \mathrm{Cov}\left[R_i, R_m\right] / \mathrm{Var}\left[R_M\right]$. That is, the difference between the expected rate of return of the security and the risk-free interest rate is assumed to equal β_i times the difference between the expected rate of return of the market and the risk-free interest rate.

From the above formula, we know that the rate of return of a single security is determined by the relationship between this single security and the market portfolio.

Assume that the non-risk interest rate is 0. Equation (4.2) indicates that the instantaneous return of the contingent claim can be decomposed into two parts. Containing fractional Brownian motion, the first term to the right side denotes the stochastic volatility and $\psi(t,\omega)$ denotes the volatility rate. The volatility rate changes in compliance with the change of the contingent claim ξ. In the other term, f is determined by the evaluation operator π_t itself and it reflects the mechanism of the market. $\psi(t,\omega)$ reflects the extent to which the instantaneous return of the contingent claim and the return of the market portfolio are related. Therefore, the instantaneous return of the contingent claim is mainly determined by this dependence. Equation (4.2) can be regarded as another version of CAPM. Accordingly, Equation (4.2) indicates that the instantaneous return of the contingent claim is mainly determined by the extent to which the instantaneous return of the contingent claim and the return of the market portfolio are related.

From the discussion above, we deduce the dynamic CAPM from the dynamic linear evaluation rule in the market driven by fractional Brownian motion.

5. Conclusion

In this paper, we first give some preliminaries of fractional Brownian motion. Then, we present the fundamental framework of the evaluation problem under which the evaluation operator satisfying some axioms is linear. Based on the dynamic linear evaluation mechanism of contingent claims, studying this evaluation rule in the market driven by fractional Brownian motions has led to a dynamic capital asset pricing model. It is deduced here mainly with the fractional Girsanov theorem and the Clark-Haussmann-Ocone theorem.

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