

Retraction Notice

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Comment:

The paper is withdrawn from "Journal of Intelligent Learning Systems and Applications" due to data errors in this paper which has been confirmed by EIC of this journal.

This article has been retracted to straighten the academic record. In making this decision the Editorial Board follows COPE's [Retraction Guidelines](#). The aim is to promote the circulation of scientific research by offering an ideal research publication platform with due consideration of internationally accepted standards on publication ethics. The Editorial Board would like to extend its sincere apologies for any inconvenience this retraction may have caused.

Editor guiding this retraction: Anita LIU (Editorial Assistant of JILSA)

Local Correlated Noise Improvement of Signal-to-Noise Ratio Gain in an Ensemble of Noisy Neuron

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Abstract

We theoretically investigate the collective response of an ensemble of leaky integrate-and-fire neuron units to a noisy periodic signal by including local spatially correlated noise. By using the linear response theory, we obtained the analytic expression of signal-to-noise ratio (*SNR*). Numerical simulation results show that the rms amplitude of internal noise can be increased up to an optimal value where the output *SNR* reaches a maximum value. Due to the existence of the local spatially correlated noise in the units of the ensemble, the *SNR* gain of the collective ensemble response can exceed unity and can be optimized when the nearest-neighborhood correlation is negative. This non-linear collective phenomenon of *SNR* gain amplification in an ensemble of leaky integrate-and-fire neuron units can be related to the array stochastic resonance (SR) phenomenon. Furthermore, we also show that the *SNR* gain can also be optimized by tuning the number of neuron units, frequency and amplitude of the weak periodic signal. The present study illustrates the potential to utilize the local spatially correlation noise and the number of ensemble units for optimizing the collective response of the neuron to inputs, as well as a guidance in the design of information processing devices to weak signal detection.

Keywords

Array Stochastic Resonance, Signal-to-Noise Ratio, Local Correlation, Noisy Neurons

1. Introduction

The concept of stochastic resonance (SR) was invented in 1981-82 in the rather

exotic context of the evolution of the earth's climate [1]. SR is a phenomenon that occurs in a threshold measurement system when an appropriate measure of information transfer is maximized in the presence of a non-zero level of stochastic input noise thereby lowering the response threshold. The system resonates at a particular noise level. For a nonlinear system with periodic input, the signal-to-noise ratio (*SNR*) provides a measure of the relative importance of the noisy and the systematic parts of the response. The SR has been observed in different systems [2] [3] [4], ranging from lasers, to sets of neurons and to solid-state devices.

Sensory neurons transform a signal from the environment into trains of spikes that propagate to other structures in the nervous system. Since internal and external noise are ubiquitous and unavoidable, many studies involving peripheral sensory systems that have exhibited SR have been carried out. The SR phenomena have received considerable attention because of the surprisingly beneficial effect of noise in the community of neuroscience. In this regard, we should note that SR has been found in various neuron models and related experimental observations have also been reported [5]-[10]. A large proportion of the published work has concentrated on the phenomenon of SR in a single neuron unit. Because of the complexity in the structure of neural networks and the variability in the origin of noise, one should examine the collective response and possibility of *SNR* gains in an ensemble of neuron units.

The importance of the study of the *SNR* gains on signal transduction across ensembles was stressed in [5]. The results in [5] demonstrated that the ensembles of neuron units can reliably detect subthreshold pulses by including appropriate noise. Recently, the enhancement of *SNR* gain (larger than unity) is reliably observed in parallel arrays of nonlinear subsystems assisted by the independent internal noises [11] [12] [13] [14]. This regular model of uncoupled parallel arrays of nonlinear subsystems elicits many important mechanisms of non-conventional SR effects, such as, SR without tuning [7], suprathreshold SR [9] and array SR [11]. In such an ensemble, all units have a common input, and their outputs are summed as the collective response [11] [12] [13] [14]. Motivated by the fact of large numbers of neurons in the nervous systems of animals and humans with variations in structure, function, and size, the potential exploitation of SR in neuroscience becomes an interesting open question [15], especially in a summing parallel threshold-based sensory neuron model. In spite of efforts devoted to examining the constructive roles of external or internal noise, the existing investigations have not completely clarified the mechanism of variability in the origin of noise for the complex neural networks, such as, the optimize of the noise level and the optimized correlation of the internal noise that benefit signal transmission through nonlinear elements.

As far as an ensemble of neuron units is concerned, the existing investigations were mainly focused on independent internal noise of each unit [11] [12] [13] [14]. However, as for the correlation of the internal noise, few related investigation has been carried out in an ensemble of neuron units, which motivates

the current work. Based on the fact that negative correlated background noise are less noisy as a whole, we infer that the advantage of local spatial negative correlation over its statistical independent or positive correlated counterpart should be universal in an ensemble of neuron units, and consequently the local spatial negative correlation should be more helpful for the ensemble to detect weak coherent signal. In this paper, we examine the effect of local negative spatial correlation on SR in parallel arrays of leaky integrate-and-fire neurons. We should note that the considering the correlation of internal noise is a more realistic assessment of SR in the brain neuroscience [16]. For example, the neurotransmitter trains which are released in quantum from inner hair cells and the auditory signals are random point interfering by the correlation of internal noise indirectly by simulating sound.

It is worth noting that we consider an ensemble of N leaky integrate-and-fire neuron units in the present paper. As a matter of fact, the leaky integrate-and-fire neuron model has become widely accepted as one of the canonical models for the study of neural systems. The model provides a good description of the subthreshold integration of synaptic inputs. Assembling the leaky integrate-and-fire neuron units into arrays, we will show that the collective response of a parallel array to a given noisy signal can be enhanced by the internal array noise. For a noisy signal, the *SNR* gain is employed and numerically analyzed. The regions of the *SNR* gain exceeding unity, testify the efficiency of the parallel array assembled by this kind of leaky integrate-and-fire neuron units.

2. The Ensemble Model and Method Description

Let us consider an ensemble of N leaky integrate-and-fire (LIF) neuron units receiving the same periodic stimulation $S(t)$, as shown in **Figure 1**. Each LIF neuron is characterized by its membrane potential, denoted by V_i . The membrane potential of the single neuron is governed by the following Langevin Equation [13] [14].

$$\tau \frac{dV_i(t)}{dt} = -V_i(t) + \mu + \sqrt{2D}\xi_i(t) + S(t) \quad (1)$$

with $1 \leq i \leq N$. Here the parameter D stands for the intensity of the white Gaussian noise (*i.e.*, $\xi_i(t)$) while μ is a dc component in the noisy synaptic input and τ is the membrane time constant for the subthreshold dynamics. $S(t)$ is a common component in the input and the Gaussian white noise $\xi_i(t)$ models the internal stochastic component for the i th neuron. The dynamics Equation (1) is complemented by the well-known fire and reset rule: whenever the membrane potential reaches a prescribed constant V_T , the neuron fires and the potential is kept fixed for an absolute refractory period τ_R and then reset to a value V_R . In the following we set $V_T = 1$ and $V_R = 0$. In Equation (1), the single neuron properties are described by $-V_i(t)$, μ , and $\xi_i(t)$ standing for a leakage term, a constant base dc component, and an internal Gaussian white

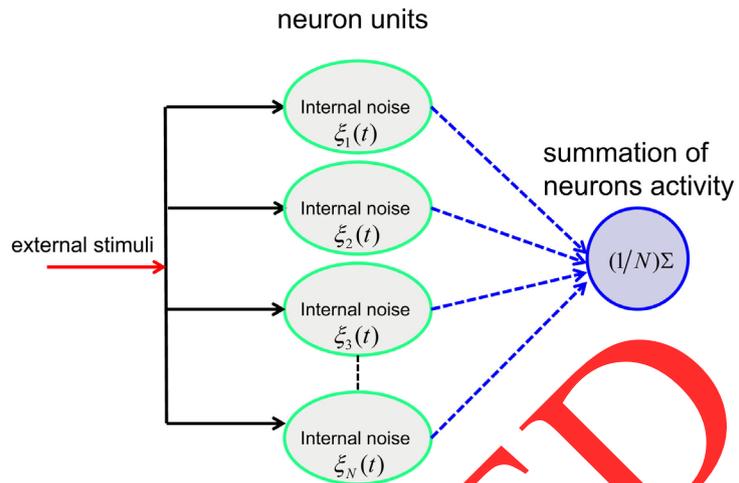


Figure 1. Illustration of a parallel array of N leaky integrate-and-fire neuron units. Each element is subject to the same noisy input $S(t) + \eta(t)$. The Gaussian white noise $\xi_i(t)$ ($1 < i \leq N$) models the internal stochastic component for the i th neuron. The average spike train $y(t) = \frac{1}{N} \sum_i x_i(t)$ is taken as the output for summing parallel array.

noise of intensity D , respectively. This intrinsic noise leads to spontaneous activity even in the absence of the external stimuli. The output of the i th LIF neuron is a δ spike train determined by the j th instants of threshold crossing of the i th neuron $t_{i,j}$

$$x_i(t) = \sum_j \delta(t - t_{i,j}) \quad (2)$$

The ensemble response $y(t)$ is taken as the average of output $x_i(t)$ as,

$$y(t) = \frac{1}{N} \sum_{i=1}^N x_i(t) \quad (3)$$

By using the leaky integrate-and-fire model, some authors [13] [14] have investigated the transmission of noise coded versus additive signals through a neuronal ensemble, in which the ensemble of neuron units is driven by the noise coded parts of a periodic signal and independent internal noise. These investigations showed that the both noise coded parts of the periodic signal can induce SR and efficient high frequency transmission. However, a number of recent developments have promoted interest in the analysis of the ensemble of neurons or other systems with correlation of the internal noise [17]-[23]. In the following, we will focus on the theoretical analysis and numerical simulations that are used to study SR and SNR gain in an ensemble of leaky integrate-and-fire neuron units by including the correlation of the internal noise. The present study represent significant extensions to the existing investigations [13] [14]. We now consider the response of the ensemble of neuron units when submitted to a superposition of a weak periodic signal $S(t)$ and the internal noise $\xi_i(t)$ with a tunable nearest-neighborhood correlation coefficient λ . Assuming that the internal noise $\xi_i(t)$ satisfies

$$\langle \xi_i(t) \xi_j(t + \tau) \rangle = [\delta_{i,j=i} + \lambda \delta_{i,j=i+1}] \delta(\tau) \quad (4)$$

with $1 \leq i, j \leq N$, where the first term describes the self-correlation of each neuron unit, and the second term describes the correlation between two nearest-neighborhood neuron units with λ being a tunable nearest-neighborhood correlation coefficient. The function $\delta(\tau)$ means the correlation must be instantaneous. Because response of each neuron is not affected by the local correlation between the nearest-neighborhood neuron units, it is necessary to calculate the spectral statistics $R_{yy}(\omega) = \langle Y(\omega) Y^*(\omega) \rangle$ for the ensemble output spike train, where

$$Y(\omega) = \frac{1}{\sqrt{T}} \int_0^T dt \exp(i\omega t) (y(t) - r_0(D)) \quad (5)$$

is the Fourier transform of the zero average output spike train with $r_0(D)$ being the stationary firing rate at the noise level D . For weak signals, we can adopt the linear response theory. Based on the linear response theory [5], we assume that the output equals that of the unperturbed system (denoted by $Y_{i,0}(\omega, D)$) plus a correlation term consisting of the perturbed terms multiplied with the transfer function (i.e., the susceptibility $B(\omega, D)$). Thus the frequency domain linear response for each neuron unit can be written as,

$$Y_i(\omega) = Y_{i,0}(\omega, D) + B(\omega, D) S(\omega) \quad (6)$$

Equation (6) provides a system of equations relating spike train power spectrum $\langle Y_i(\omega) Y_i^*(\omega) \rangle$, cross spectrum between distinct spike trains $\langle Y_i(\omega) Y_j^*(\omega) \rangle (j \neq i)$, and cross spectrum between the stimulus and a spike train $\langle Y_i(\omega) S^*(\omega) \rangle$ with * denotes complex conjugation. We further assume that $\langle Y_{0,i}(\omega) Y_{0,j}^*(\omega) \rangle = \langle Y_{0,i}(\omega) S^*(\omega) \rangle = \langle Y_i(\omega) \xi_j^*(\omega) \rangle = 0 (i \neq j)$. We assume the external stimuli ($\eta(t)$) to be Gaussian white noise of intensity D_η . For such a stimulus a linear correction of the spectral quantities is not valid anymore, because the variance of the white noise is not small but in fact infinite. Then we replace the spectrum of the transmitted stimulus as

$$S_{st}(\omega) = S_0(\omega, D) + |B(\omega, D)|^2 R_{ss}(\omega) \quad (7)$$

with $R_{ss}(\omega)$ standing for the power spectrum of the input coherent signal. Here we explicitly show the parametric dependence of the power spectrum and the susceptibility on the internal noise level. This is a linear approximation of $S_{0,Q} = S_0(\omega, Q)$ with $Q = D + D_\eta$ the intensity of the summed internal and external noise sources. If both internal and external noises are white and Gaussian, the single neuron cannot distinguish between both kinds of noise, thus to replace the power spectrum of unperturbed system $S_0(\omega, D)$ by $S_0(\omega, Q)$ seems to be plausible. This also leads to the firing rate and the susceptibility functions that should be taken at noise intensity Q and not at D anymore. As a matter of fact, an external stimulus treated by the linear response in Equation (7) will never affect the stationary firing rate of the neuron. In contrast to this we expect an increase in firing rate for a neuron that experiences a white noise of

total intensity $Q = D + D_\eta$ compared to the unperturbed case ($D_\eta = 0$). Then the susceptibility $B(\omega, D)$ at full noise level Q can be replaced by $B(\omega, Q)$. For the self-consistent determination of the firing rate we also use the full noise intensity Q instead of D . For the LIF model we can calculate the firing rate by the following expression [14].

$$r_0(\mu, Q) = \left(\tau_R + \sqrt{\pi} \int_{(\mu-V_T)/\sqrt{2Q}}^{(\mu-V_R)/\sqrt{2Q}} dz e^{-z^2} \operatorname{erfc}(z) \right)^{-1} \quad (8)$$

The power spectrum of the unperturbed system $S_0(\omega, Q)$ and the susceptibility $B(\omega, Q)$ [13] [14] can be respectively obtained as

$$S_0(\omega, Q) = r_0(\mu, Q) \frac{\left| \mathcal{D}_{i\omega} \left(\frac{\mu - V_T}{\sqrt{Q}} \right) \right|^2 - e^{2\beta} \left| \mathcal{D}_{i\omega} \left(\frac{\mu - V_R}{\sqrt{Q}} \right) \right|^2}{\left| \mathcal{D}_{i\omega} \left(\frac{\mu - V_T}{\sqrt{Q}} \right) \right|^2 e^{i\omega\tau_R} - \mathcal{D}_{i\omega} \left(\frac{\mu - V_R}{\sqrt{Q}} \right) \right|^2} \quad (9)$$

$$B(\omega, Q) = \frac{r_0(\mu, Q) i\omega}{\sqrt{Q}(i\omega - 1)} \frac{\mathcal{D}_{i\omega-1} \left(\frac{\mu - V_T}{\sqrt{Q}} \right) - e^\beta \mathcal{D}_{i\omega-1} \left(\frac{\mu - V_R}{\sqrt{Q}} \right)}{\mathcal{D}_{i\omega} \left(\frac{\mu - V_T}{\sqrt{Q}} \right) - e^{i\omega\tau_R} e^\beta \mathcal{D}_{i\omega} \left(\frac{\mu - V_R}{\sqrt{Q}} \right)} \quad (10)$$

where $\beta = [V_R^2 - V_T^2 + 2\mu(V_T - V_R)]/4Q$, and $\mathcal{D}_a(z)$ denotes the parabolic cylinder function [14] that can be obtained by means of computer programs like MATHEMATICA or MAPLE. The cross spectrum in the unperturbed case corresponds to that for the system of neuron ensemble driven by the dc component μ , a common external noise $\eta(t)$, and internal noise $\xi_i(t)$. In this case the response of the i th neuron by including the perturbed part is written as,

$$\frac{dV_i(t)}{dt} = -V_i(t) + \mu + \sqrt{2Q(1-|\lambda|)}\xi_i(t) + \delta \left[\sqrt{2Q|\lambda|}\eta(t) + S(t) \right] \quad (11)$$

where $\xi_i(t)$ and $\eta(t)$ in the above Equation (11) are mutually independent Gaussian white noises. The terms $\delta \left[\sqrt{2Q|\lambda|}\eta(t) + S(t) \right]$ are perturbation parts. The response of the $(I + 1)$ th neuron is governed by

$$\frac{dV_{i+1}(t)}{dt} = -V_{i+1}(t) + \mu + \sqrt{2Q(1-|\lambda|)}\xi_{i+1}(t) + \delta \left[\sqrt{2Q} \operatorname{sgn}(t) \sqrt{|\lambda|}\eta(t) + S(t) \right] \quad (12)$$

where $\operatorname{sgn}(x)$ being sign function. As far as we know there is not any stochastic neuron model for which an exact expression for this cross spectrum is given. If we know an expression or we measure it from a simulation of or experiment on two uncoupled neurons (*i.e.*, the i th and $(I + 1)$ th neurons), we can use the following relations for a better approximation of the cross spectrum,

$$S_{cross}(\omega) = \left| B(\omega, Q(1-|\lambda|)) \right|^2 \left[2\lambda Q + R_{ss}(\omega) \right] \quad (13)$$

As mentioned above, $R_{ss}(\omega)$ is the power spectrum of the input coherent

signal. Based on Equations (7) and (13), we separate $S_{st}(\omega)$ and $S_{cross}(\omega)$ into coherent (i.e., $S_{st}^{co}(\omega) = |B(\omega, Q)|^2 R_{ss}(\omega)$, $S_{cross}^{co}(\omega) = |B(\omega, Q(1-|\lambda|))|^2 R_{ss}(\omega)$) and incoherent terms i.e., $S_{st}^{in}(\omega) = S_0(\omega, Q)$, $S_{cross}^{in}(\omega) = |B(\omega, Q(1-|\lambda|))|^2 2\lambda Q$). We can also calculate a better approximation for the spectrum for the ensemble of the neuron units

$$R_{yy}(\omega) = \left[1 - \frac{2(N-1)}{N^2} \right] S_{st}^{co}(\omega) + \frac{1}{N} S_{st}^{in}(\omega) + \frac{2(N-1)}{N^2} S_{cross}^{co}(\omega) + \frac{2(N-1)}{N^2} S_{cross}^{in}(\omega) \tag{14}$$

In order to study the transmission of the weak periodic signal induced by the noise we can use the definition of the output signal-to-noise ratio (SNR) in [11], where the transmission of periodic additive and noise coded signals were studied. For a weak periodic signal $S(t) = A \cos(\omega t)$ ($A < 1$), the output signal-to-noise ratio (SNR) can be obtained as

$$R_{out} = \frac{\pi^2 A^2 \left[(N^2 - 2N + 2) |B(\omega, Q)|^2 + 2(N-1) |B(\omega, Q(1-|\lambda|))|^2 \right]}{NS_0(\omega, Q) + 4\lambda Q(N-1) |B(\omega, Q(1-|\lambda|))|^2} \tag{15}$$

In the same way, the collective input SNR R_{in} is defined by replacing the numerator and denominator in Equation (15) with the corresponding quantities for the collective inputs. Based on the definitions of output and input SNR, the SNR gain of the ensemble of neuron units can be defined as the ratio of the output SNR of the ensemble to the input SNR for the coherent component, follows as

$$G = \frac{R_{out}}{R_{in}} \tag{16}$$

The above Equations (15) and (16) can at best provide a generic theory of evaluating SNR of ensemble of neuron units. If the SNR gain G exceeds unity, the interactions of ensemble of neuron units and controllable internal noise provide a specific potentiality for neural signal processing.

The performance of a single leaky integrate-and-fire neuron model was analyzed in detail in many works [5]-[10]. Here we mainly focus on the collective dynamics of an ensemble of leaky integrate-and-fire neuron units. In this realization of Gaussian white noise $\xi_i(t)$ and $\eta(t)$, we have $2D = \sigma_\xi^2 \Delta t$ and $2D_\eta = \sigma_\eta^2 \Delta t$. In the present paper, we represent σ_η as the amplitude of input noise $\eta(t)$, and σ_ξ as the amplitude of internal noise $\xi_i(t)$ of each unit. The quantities introduced in Equations (15) and (16) can be also calculating by computer simulations. In order to verify the analytical results, we have used a simple Euler-Maruyama scheme with a time step between 10^{-3} and 10^{-5} (latter at high noise intensity) for the integration of Equations (11) and (12).

3. Numerical Results of SR and SNR Gain

As mentioned in the introduction, SR phenomenon will be exhibited by studying

the variations of the output SNR as a function of the additive noise power density D . Based on the definitions of SNR (R_{out}) and SNR gain (G) in Equations (15) and (16), we will analyze numerically the evolution of output R_{out} and G with different parameters of system. If the SNR gain satisfies $G > 1$ at the optimal internal noise amplitude, then we say that the input signal can be improved or enhanced by the internal noise of the ensemble of neuron units.

Based on the definitions of SNR (R_{out}) and SNR gain (G) in Equations (15) and (16), we will analyze numerically the evolution of output R_{out} and G with different parameters of system. The numerical method for calculating SNR was introduced in [12], in detail. We firstly analyze the dependence of R_{out} on the rms amplitude σ of internal noise $\xi_i(t)$ with nearest-neighborhood correlation λ for different numbers of neuron units N in the present ensemble of leaky integrate-and-fire neuron units. Here, we choose the values of parameters as $V_T = 1$, $V_R = 0$, $\tau_R = 0.1$, $\mu = 0.8$, $\omega = 0.1$ and $A = 0.3$. If the number of neuron units is $N = 1$ and the response $y(t) = \frac{1}{N} \sum_i x_i(t) = x_1(t)$, this is the case of a single leaky integrate-and-fire neuron model displaying the conventional SR or residual SR phenomena [5] [10]. If the number of neuron units is $N = 100$, we plot in **Figure 2(a)** the variation of the R_{out} as a function of the rms amplitude σ of internal noise $\xi_i(t)$ for different values of λ , i.e., $\lambda = -0.3$ (blue “o” line), $\lambda = 0.0$ (blue “□” line), and $\lambda = 0.3$ (red “*” line). The curves of **Figure 2(a)** display nonmonotonic evolution of the R_{out} with the rms amplitude of internal noise, and exhibit typical SR characteristic. These results present that the internal noise inputs allows collective response of neuron ensemble for a weak signal input. Furthermore, the rms amplitude σ of internal noise can be increased up to an optimal value (marked with dashed line) where R_{out} reaches a maximum. In addition, one can also find from **Figure 2(a)** that the maximal value of R_{out} is increased as the value of nearest-neighborhood

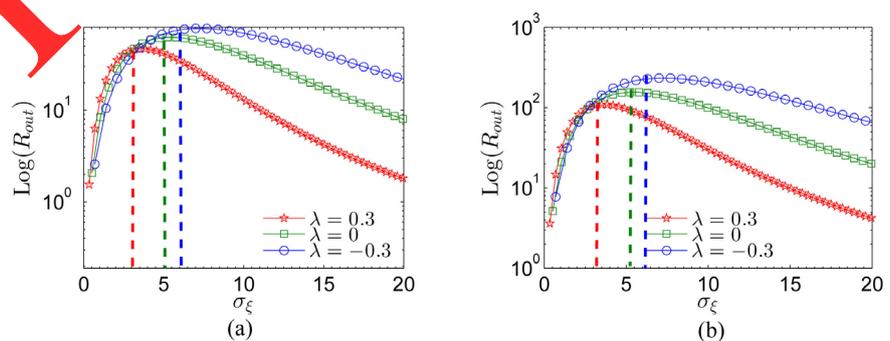


Figure 2. Plots of the output SNR R_{out} as a function of the rms amplitude σ of internal noise $\xi_i(t)$ for an ensemble of leaky integrate-and-fire neuron units with different nearest-neighborhood correlation λ , i.e., $\lambda = -0.3$ (blue “o” line), $\lambda = 0.0$ (green “□” line), and $\lambda = 0.3$ (red “*” line). The number of neuron units is (a) $N = 100$; (b) $N = 10000$. The other parameters are chosen as $V_T = 1$, $V_R = 0$, $\tau_R = 0.1$, $\mu = 0.8$, $\sigma_\eta = 1.6$, $\omega = 0.1$ and $A = 0.3$.

correlation λ varies from positive correlation ($\lambda = 0.3$) to negative correlation $\lambda = -0.3$, which implies that the negative correlation is optimal for triggering the neuron units and transmission of the weak signal among the three cases of statistically independence ($\lambda = 0$), positive correlation ($\lambda = 0.3$) and negative correlation ($\lambda = -0.3$). In **Figure 2(b)**, we show the variation of the output SNR R_{out} as a function of the rms amplitude σ of internal noise $\xi_i(t)$ for $N = 10000$ with different nearest-neighborhood correlation λ . It can be found that the maximal output SNR R_{out} is enhanced as the numbers of the neuron units N . It is also noted that the region of the internal noise level that improves the output SNR R_{out} is gradually expanded as N increases.

In **Figure 3**, we show the SNR gain G as a function of the rms amplitude of internal noise $\xi_i(t)$ with a fixed noisy periodic signal $S(t) + \eta(t)$ for an ensemble of leaky integrate-and-fire neuron units with different nearest-neighborhood correlation λ , i.e., $\lambda = -0.3$ (blue “o” line), $\lambda = 0.0$ (green “□” line), and $\lambda = 0.3$ (red “*” line). From **Figure 3**, one can find that the SNR gain G also displays nonmonotonic behavior with the increasing of rms amplitude σ of internal noise for both cases of statistically independence ($\lambda = 0$) and negative correlation ($\lambda = -0.3$). More importantly, in these cases, the SNR gain G can be larger than unity at the optimal rms amplitude σ of internal noise. However, the SNR gain G at the optimal rms amplitude σ of internal noise can not reach unity for the case of positive correlation. Direct comparison in **Figure 3(a)** and **Figure 3(b)** implies that the negative correlation assists the collective response of the ensemble of the neuron units for signal transmission, and its positive role is much more manifest at a large number of the neuron units. In addition, the regions of the SNR gain larger than unity are obvious for the input signal plus Gaussian white noise when the nearest-neighborhood correlation coefficient λ is negative, as indicated in **Figure 3**.

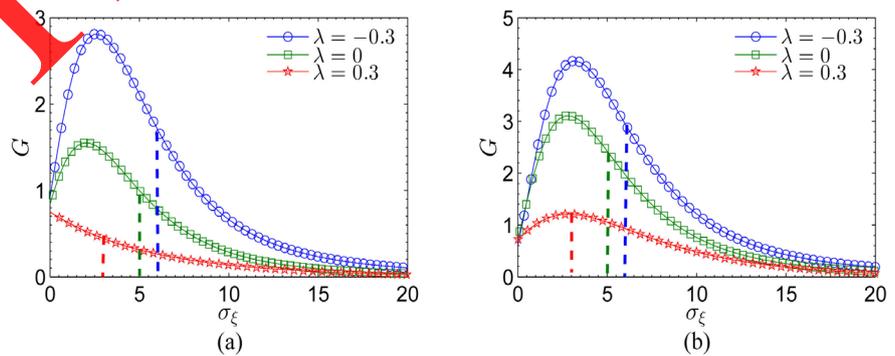


Figure 3. Plots of the SNR gain G as a function of the rms amplitude σ of internal noise $\xi_i(t)$ for an ensemble of leaky integrate-and-fire neuron units with different nearest-neighborhood correlation λ , i.e., $\lambda = -0.3$ (blue “o” line), $\lambda = 0.0$ (green “□” line), and $\lambda = 0.3$ (red “*” line). The number of neuron units is (a) $N = 100$; (b) $N = 10000$. The values of other parameters are chosen as $V_T = 1$, $V_R = 0$, $\tau_R = 0.1$, $\mu = 0.8$, $\sigma_\eta = 1.6$, $\omega = 0.1$ and $A = 0.3$.

As shown in **Figure 2**, for a fixed frequency $\omega = 0.1$, we have demonstrated that the negative nearest-neighborhood correlation can lead to the larger output *SNR* at the optimal rms amplitude of internal noise $\xi_i(t)$. We have also demonstrated in **Figure 3** that the negative nearest-neighborhood correlation will also lead to the larger *SNR* gain for a fixed frequency $\omega = 0.1$ at corresponding optimal rms amplitude of internal noise $\xi_i(t)$. However, a key question is whether the above results can also be observed for different frequency ω . In the following, we study numerically the evolution of the output *SNR* R_{out} and *SNR* gain G as a function of the rms amplitude of internal noise $\xi_i(t)$ with a fixed negative correlation $\lambda = -0.3$ for different frequency ω . We plot in **Figure 4(a)** and **Figure 4(b)**, respectively, the variations of the output *SNR* R_{out} and *SNR* gain G , as a function of the rms amplitude of internal noise $\xi_i(t)$ with different value of frequencies ω , *i.e.*, $\omega = 0.05$ (red “*” line), $\omega = 0.1$ (green “□” line), and $\omega = 0.2$ (blue “O” line). As shown in **Figure 4(a)**, the maximum of R_{out} corresponding to the optimal noise power density increases as the frequency ω increases from 0.05 to 0.2. The corresponding optimal values of rms amplitude of internal noise $\xi_i(t)$ have been marked with dashed lines. The value of the rms amplitude of internal noise $\xi_i(t)$ for frequency 0.05 is larger than the one for frequency 0.2. **Figure 4(b)** shows that the *SNR* gain G at the corresponding optimal rms amplitude of internal noise $\xi_i(t)$ increases as the frequency ω increases from 0.05 to 0.2. Obviously, one can find from **Figure 4(b)** that the *SNR* gain G corresponding to the optimal rms amplitude of internal noise $\xi_i(t)$ is larger than unity for all the frequencies (*i.e.*, $\omega = 0.05, 0.1, 0.2$). In other words, we demonstrate that $G > 1$ (*i.e.*, *SNR* gain is larger than unity) is a general characteristics for the case of negative correlation in an ensemble of the leaky integrate-and-fire neuron units.

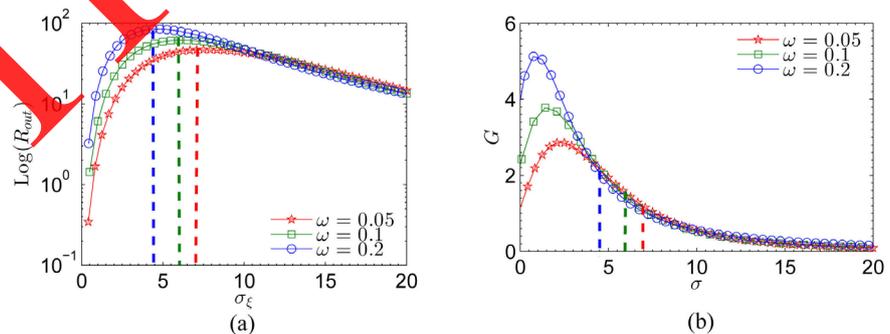


Figure 4. (a) Plots of the output *SNR* SNR_{out} as a function of the rms amplitude of internal noise $\xi_i(t)$ with a fixed noisy periodic signal $S(t) + \eta(t)$ for different frequencies, *i.e.*, $\omega = 0.05$ (red “*” line), $\omega = 0.1$ (green “□” line), and $\omega = 0.2$ (blue “O” line); (b) Plots of the *SNR* gain G as a function of the rms amplitude of internal noise $\xi_i(t)$ with a fixed noisy periodic signal $S(t) + \eta(t)$ for different frequencies, *i.e.*, $\omega = 0.05$ (red “*” line), $\omega = 0.1$ (green “□” line), and $\omega = 0.2$ (blue “O” line). The values of other parameters are chosen as $V_T = 1$, $V_R = 0$, $\tau_R = 0.1$, $\mu = 0.8$, $\sigma_{\eta} = 1.6$, $\lambda = -0.3$, $N = 100$ and $A = 0.3$.

In addition, one can also find in **Figure 4(b)** that the *SNR* gain G at the optimal rms amplitude of internal noise $\xi_i(t)$ becomes larger as the input signal becomes slower. Therefore, we can conclude that the slower signal input (*i.e.*, $\omega = 0.05$) in noisy environment is more possible to induce the collective response of the ensemble of neurons. The present results agree with the observations in many nonlinear physical and biological systems [5] [24].

Up to now, we have demonstrated that *SNR* gain can be optimized by choosing appropriate nearest-neighborhood correlation λ , input signal frequency ω , as well as the rms amplitude of internal noise $\xi_i(t)$. However, these results showed in **Figures 2-4** focused on the weak noisy periodic signal with fixed amplitude $A = 0.3$ in an ensemble of leaky integrate-and-fire neuron units by including the internal noise. When one studies the sensory nervous system, it is inevitable to consider the influences of the complex signals. For examining the external stimulus on the sensory nervous system, the external signal can be coded by means of the amplitude of calcium ion oscillation, and the periodical calcium ion signal further enters into the nucleus to change the responsible activities [25] [26] [27] [28]. Some experiments and theories indicated that calcium ion signal in response to external stimulus can encode information via amplitude modulation [26] [27].

One of the most interesting characters of the input periodic signal with amplitude modulation is that the output *SNR* R_{out} and *SNR* gain G can be optimized by varying the amplitude A of the input signal. Whether there is an optimal amplitude that produces the largest output *SNR* and *SNR* gain. In the following, we will study the influence of the amplitude modulation on the output *SNR* and *SNR* gain. We plot in **Figure 5(a)** and **Figure 5(b)** the variation of the R_{out} as a function of the rms amplitude of internal noise $\xi_i(t)$ for different values of amplitude A , *i.e.*, $A = 0.6$ (blue "O" line), $A = 0.4$ (green "□" line), and $A = 0.3$ (red "×" line). The **Figure 5(a)** and **Figure 5(b)**, respectively, correspond to the two cases of $N = 100$ and $N = 10000$. No matter when the number of the neuron units N is 100 or 1000, the curves of **Figure 5(a)** and **Figure 5(b)** displays a monotonic evolutions of the maximum *SNR* as the amplitude A of signal increases. The maximum *SNR* correspond to the case of $A = 0.6$. In other words, for a weak signal ($A < 1$), the larger amplitude of input signal induces more pronounced collective response of the ensemble of neurons. In addition, one can readily find from **Figure 5(a)** and **Figure 5(b)** that the maximum values of the output *SNR* for different signal amplitude A located at the identical optimal noise level. Furthermore, the directly comparison between **Figure 5(a)** and **Figure 5(b)** shows that the maximum value of output *SNR* corresponding to the amplitude $A = 0.6$ for large number of neuron units $N = 10000$ is larger than the one for the number of neuron units $N = 100$. These results agree with the ones shown in **Figure 2**. We also plot in **Figure 5(c)** and **Figure 5(d)** the variation of the *SNR* gain G as a function of the rms amplitude of internal noise $\xi_i(t)$ for different values of signal amplitude A , *i.e.*,

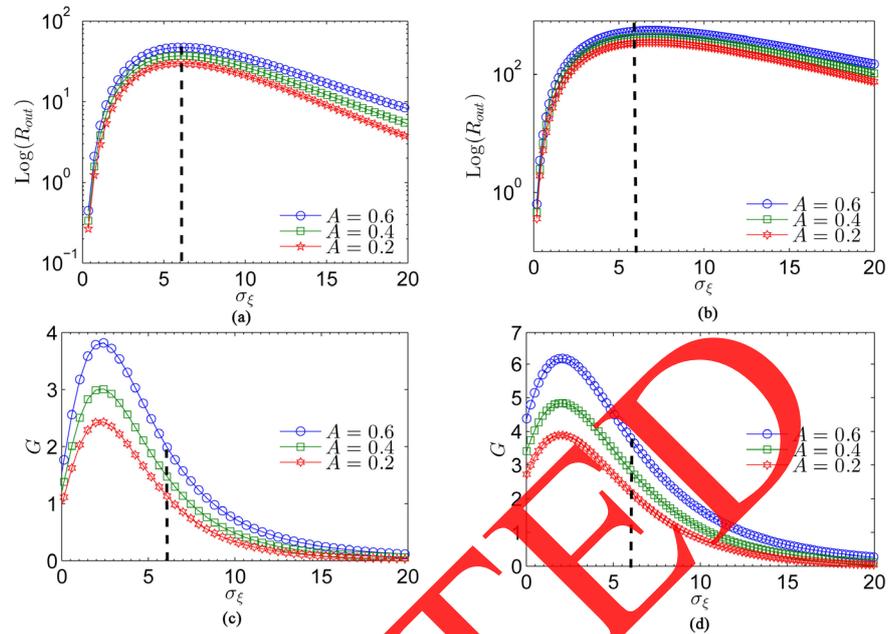


Figure 5. (a), (b) Plots of the output SNR R_{out} as a function of the rms amplitude of internal noise $\xi_i(t)$ with a noisy periodic signal $S(t) + \eta(t)$ for different numbers of neuron units is (a) $N = 100$; (b) $N = 10000$, in which the values of signal amplitudes vary as, *i.e.*, $A = 0.6$ (blue “O” line), $A = 0.4$ (green “□” line), and $A = 0.2$ (red “*” line); (c), (d) Plots of the SNR gain G as a function of the rms amplitude of internal noise $\xi_i(t)$ with a noisy periodic signal $S(t) + \eta(t)$ for different numbers of neuron units is (a) $N = 100$; (b) $N = 10000$, in which the values of signal amplitudes vary as, *i.e.*, $A = 0.6$ (blue “O” line), $A = 0.4$ (green “□” line), and $A = 0.2$ (red “*” line). The values of other parameters are chosen as $V_T = 1$, $V_R = 0$, $\tau_R = 0.1$, $\mu = 0.8$, $\sigma_\eta = 1.6$, $\lambda = -0.3$, and $\omega = 0.1$.

$A = 0.6$ (blue “O” line), $A = 0.4$ (green “□” line), and $A = 0.3$ (red “*” line). Figure 5(c) and Figure 5(d), respectively, correspond to the two cases of $N = 100$ and $N = 10000$. These two figures demonstrate that the SNR gain G can be larger than unity even if the signal amplitude is $A = 0.2$ when the nearest-neighborhood correlation is negative, *i.e.*, $\lambda = -0.3$. In addition, the SNR gain G becomes more pronounced as the number of the neuron units increases with the same signal amplitude. Thus, we can conclude that both the output SNR and SNR gain can be improved by varying the amplitude of a weak periodic signal when the negative nearest-neighborhood correlation exists. In this way, the present investigation may provide another factor for improving the transmission of signal in an ensemble of leaky integrate-and-fire neuron units.

For a better insight into the effects of the amplitude A of the weak periodic signal $S(t)$ and the rms amplitude σ_ξ of internal noise $\xi_i(t)$ on global behavior of the output SNR, the contour map of the output SNR as the function of both the amplitude and rms amplitude σ_ξ of internal noise is shown in Figure 6 for different numbers of neuron units N . The Figure 6(a) and Figure 6(b), respectively, correspond to the two cases of $N = 100$ and $N = 10000$.

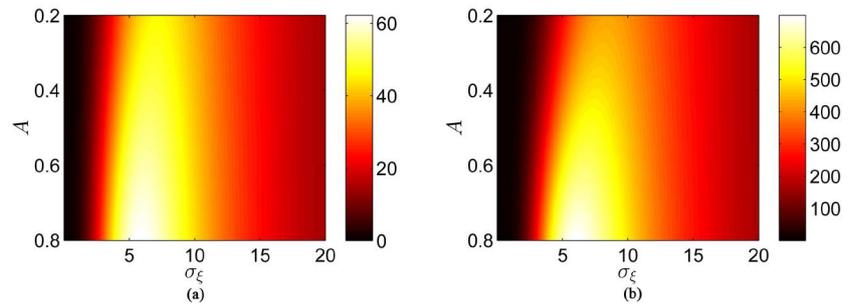


Figure 6. The contour plots of the $SNR R_{out}$ as the function of the rms amplitude σ_{ξ} of internal noise $\xi_i(t)$ and the amplitude A of the weak noisy periodic signal $S(t) + \eta(t)$ for different numbers of neuron units is (a) $N = 100$; (b) $N = 10000$. The values of other parameters are chosen as $V_T = 1$, $V_R = 0$, $\tau_R = 0.1$, $\mu = 0.8$, $\sigma_{\eta} = 1.6$, $\lambda = -0.3$, and $\omega = 0.1$.

Obviously, the curves of both **Figure 6(a)** and **Figure 6(b)** display the nonmonotonic evolutions of the output SNR with the increasing of the rms amplitude σ_{ξ} of internal noise $\xi_i(t)$. However, one can find from **Figure 6** that the output SNR displays a monotonic evolutions of the SNR as the amplitude A of weak periodic signal ($A < 1$) increases. It can be also found from **Figure 6** that there is a certain region of A and σ_{ξ} where the output SNR reaches maximum values, that is the signature of typical SR. From the direct comparison between **Figure 6(a)** and **Figure 6(b)**, one can find that the output SNR depends sensitively on the value of the number of the neuron units N . The maximum value of output SNR corresponding to the optimal rms amplitude σ_{ξ} of internal noise $\xi_i(t)$ and amplitude A of weak periodic signal for $N = 10000$ is larger than the one for $N = 100$. In other words, the maximum value of R_{out} increases as the number of neuron units N increases. It is worth noting that the rms amplitude σ_{ξ} of internal noise $\xi_i(t)$ and the amplitude A of weak periodic signal can vary in a wide range which can be modulated in realistic signal processing [16]. Especially, we should note that signaling through amplitude modulation is conceptually straightforward because it is governed, at least in part, by the calcium ion affinity of the decoder or sensor.

4 Conclusions

In conclusion, we have investigated the collective response of an ensemble of leaky integrate-and-fire neuron units to a noisy periodic signal by including the internal spatially correlated noise. By using the linear response theory, we obtained the analytic expression of signal-to-noise ratio (SNR). The present investigation shows that the rms amplitude of internal noise can be increased up to an optimal value where the output SNR reaches a maximum value. This property of noise-enhanced transmission of weak periodic signal can be related to the stochastic resonance phenomenon. The results also show that the local spatial correlation of the internal noise in an ensemble of leaky integrate-and-fire neuron units can influence the collective response. Furthermore, the curve of

output *SNR* versus the RMS amplitude of internal noise has more pronounced peak when the internal noise correlation is negative, which implies that the negative correlation has advantage over positive correlation and zero correlation in transmission of a weak periodic signal in the ensemble of leaky integrate-and-fire neuron units. The improvement of the *SNR* gain results from the internal noise can be related to the so called array SR phenomenon.

More interestingly, we find that the *SNR* gain can exceed unity and can be optimized by tuning the number of the neuron units and input signal frequency. For a weak periodic signal, our investigation shows that the larger number of the neuron units can induce more optimal *SNR* gain whether the nearest-neighborhood correlation is statistically independence (zero correlation), negative or positive. Thus we can conclude that the larger number of the neuron units might be more useful for assisting the ensemble of neurons to process the weak periodic signal. It is worth noting that our results have also demonstrated that the slower periodic signal induce larger *SNR* gain in comparison to the fast periodic signal, which is similar with previous investigation in *SNR* gain of a single noisy LIF neuron that transmits subthreshold periodic spike trains [8].

Furthermore, we found that the *SNR* gain can exceed unity and can also be optimized by tuning the amplitude of the weak periodic signal. The present study proved the possibility of *SNR* gain in an ensemble of LIF neuron units with more realistic signal inputs involving modulated amplitude. These results might provide the theoretical mechanism for the investigations in the spontaneously released neurotransmitter trains of inner hair cells and spontaneous auditory signals. The present results fit into the main features of SR of hair cells. While refinements of the model and further experiments may be needed to optimize the model parameters, the present correspondence between theoretical and experimental works [24] strengthens our confidence in the biophysics of the phenomenon and in the suggestion that the internal noise in the inner ear can be sometimes helpful rather than always harmful. In summary, the present study illustrate the potential to utilize the local spatially correlation noise and the number of ensemble units for optimizing the collective response of the neuron to inputs, as well as a guidance in the design of information processing devices to weak signal detection.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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