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Resolving the Vacuum Catastrophe: A Generalized Holographic Approach*

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Abstract

We address the ~122 orders of magnitude discrepancy between the vacuum energy density at the cosmological scale and the vacuum density predicted by quantum field theory. This disagreement is known as the cosmological constant problem or the "vacuum catastrophe". Utilizing a generalized holographic model, we consider the total mass-energy density in the geometry of a spherical shell universe (as a first order approximation) and find an exact solution for the currently observed critical density of the universe. We discuss the validity of such an approach and consider its implications to cosmogenesis and universal evolution.

Keywords

Cosmological Constant, Critical Density, Dark Matter, Holographic Mass Solution, Vacuum Energy

1. Introduction

The vacuum energy density predicted by quantum field theory disagrees with cosmological observation by approximately 122 orders of magnitude. It is one of the biggest disagreements between theory, experiment and observation and is known as the vacuum catastrophe [1]. To resolve this discrepancy, we first review the fundamental nature of the vacuum energy density and its relationship to the cosmological constant.

The Einstein field equations of general relativity include a constant Λ known as the cosmological constant. Originally included to allow for static homogenous solutions to Einstein's equations, it was subsequently removed when

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the expansion of the universe was discovered [2]. However, since then the universe was found to be accelerating [3] and many cosmological models have been put forward with a nonzero Λ e.g. de Sitter, steady state and the Lemaitre models, where Λ acts as an additional expanding (dark energy) force.

With the inclusion of the cosmological constant, Einstein's field equations are:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, $g_{\mu\nu}$ is the metric tensor, r is the scalar curvature and $T_{\mu\nu}$ is the stress-energy tensor, which is modeled as a perfect fluid such that:

$$T_{\mu\nu} = \left(\rho + P/c^2\right) U_\mu U_\nu + P g_{\mu\nu} \tag{2}$$

The Robertson-Walker solution, which states that the rest frame of the fluid must be the same as the co-moving observer, reduces the Einstein equations to two Friedman equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{c^2 k}{a^2 R_o^2} \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P/c^2\right) \tag{4}$$

where a is the scaling factor, k is the curvature constant and R_o is the radius of the observable universe (*i.e.* $R_t = a_t r$, where r is the co-moving radius).

Based on astronomical observations the current cosmological model states that we live in a flat, Λ dominated, homogeneous and isotropic universe, composed of radiation, baryonic matter and non-baryonic dark matter [3]-[8].

The Friedman equation for a flat universe (*i.e.* $k = 0$) is thus given in the form:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} \tag{5}$$

If we then take the assumption that the universe is pervaded by a form of energy (*i.e.* dark energy), which is the current consensus in both cosmology and particle physics [9] [10] [11] then the cosmological constant can be interpreted as an energy density [12] [13] and given in terms of the dark energy density, $\Lambda = 8\pi G\rho_\Lambda$. Note, this result can also be found by assuming a static universe (*i.e.* $\dot{a} = 0$).

In either case the Friedman equation thus takes the form:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho + \rho_\Lambda\right) \tag{6}$$

Friedman's solutions suggest that there is a critical density at which the universe must be flat, where the ratio of the total mass-energy density to the critical density is known as the density parameter $\Omega = \frac{\rho}{\rho_{crit}}$ and is currently measured as $\Omega \sim 1$ [6] [8] [14].

The contributions to this density parameter come from: the vacuum density (dark energy), $\Omega_\Lambda = 0.683$; the dark matter, $\Omega_d = 0.268$; and the baryonic matter, $\Omega_b = 0.049$, totaling to $\Omega_T = 1$ [14].

The Friedman equation thus takes the form of an Einstein-de Sitter model in which the cosmological constant is coupled to the density:

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\left(\rho_b + \rho_d + \rho_\Lambda\right) \\ &= \frac{8\pi G}{3}\left(0.049\rho_{crit} + 0.268\rho_{crit} + 0.683\rho_{crit}\right) \\ &= \frac{8\pi G}{3}\rho_{crit}\end{aligned} \tag{7}$$

where ρ_b is the density due to baryonic matter; ρ_d is the density due to dark matter; ρ_Λ is the density due to dark energy; and $\rho_{crit} = \frac{3H_o^2}{8\pi G}$.

Using the current value of $H_o = 67.4 \pm 0.5\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ for Hubble's constant [14], gives the critical density at the present time as, $\rho_{crit} = 8.53\times10^{-30}\ \text{g/cm}^3$ and thus $\rho_b = 0.049\rho_{crit} = 4.18\times10^{-31}\ \text{g/cm}^3$, $\rho_d = 0.268\rho_{crit} = 2.29\times10^{-30}\ \text{g/cm}^3$ and $\rho_\Lambda = 0.683\rho_{crit} = 5.83\times10^{-30}\ \text{g/cm}^3$. The vacuum energy density at the cosmological scale is thus of the order $10^{-30}\ \text{g/cm}^3$.

However, quantum field theory determines the vacuum energy density by summing the energies $\hbar\omega/2$ over all oscillatory modes. See reference [1] for a more detailed overview. As quantum fluctuations predict infinite oscillatory modes [15] [16] this yields an infinite result unless renormalized at the Planck cutoff. In utilizing such a cutoff value, the vacuum energy density is found to be:

$$\rho_{vac} = \frac{c^5}{\hbar G^2} = \frac{m_\ell}{\ell^3} = 5.16\times10^{93}\ \text{g/cm}^3 \tag{8}$$

where $m_\ell = 2.18\times10^{-5}\,\text{g}$ is the Planck mass and $\ell = 1.616\times10^{-33}\,\text{cm}$ is the Planck length. This value is well supported by both theory and experimental results [17]-[23].

The cosmological vacuum energy density determined from observations, $\rho_{vac} = 5.83\times10^{-30}\ \text{g/cm}^3$, is therefore in disagreement with the vacuum energy density at the Planck cutoff, predicted by quantum field theory, $\rho_{vac} = 5.16\times10^{93}\ \text{g/cm}^3$. This discrepancy is a significant 122 orders of magnitude and is thus known as the "vacuum catastrophe".

Possible attempts to solve this discrepancy, as reviewed by Weinberg [24], include introducing a scalar field coupled to gravity in such a way that ρ_{vac} is automatically cancelled when the scalar field reaches equilibrium [25]. A second approach imagines a deep symmetry that isn't apparent in the effective field theory but nevertheless constrains parameters of this effective theory so that ρ_{vac} is zero or small [26]. Then there is the idea of quintessence which states that the cosmological constant is small because the universe is old and thus imagines a scalar field that rolls down a potential governed by a field equation [27] [28] [29]. When such a slowly varying scalar field is minimally coupled to gravity it can lead to the observed acceleration of the Universe [30]. This idea of quintessence has further been supported by the recent conjecture offered by Obied [31] to explain why string theory has not been able to construct a meta-stable de Sitter vacuum. They found that the resulting "allowed" universe points to an expanding universe in which the vacuum energy decreases at a rate above a specific lower limit *i.e.* a quintessent universe [31] [32] [33].

Finally, anthropic considerations apply an anthropic bound on +ve ρ_{vac} by setting the requirement that it should not be so large as to prevent the formation of galaxies [34]. Using a simple spherical in-fall model of Peebles [35] the upper bound gives ρ_v as being no larger than the cosmic mass density at the time of earliest galaxy formation ($z = 5$), which is approximately 200 times the present mass density and thus a big improvement from the 122 orders of magnitude. Therefore, as yet, the "vacuum catastrophe" is unresolved.

2. The Generalized Holographic Model

In previous work [36] [37], a quantized solution to gravity is given in terms of Planck Spherical Units (PSU) in a generalized holographic approach. A brief description of this solution is given below.

Following the holographic principle of 't Hooft [38], based on the Bekenstein-Hawking formulae for the entropy of a black hole [39] [40], the surface and volume entropy of a spherical system is explored. The holographic bit of information is defined as an oscillating Planck spherical unit (PSU), given as,

$$PSU = \frac{4}{3}\pi r_\ell^3 \tag{9}$$

where $r_\ell = \frac{\ell}{2}$.

These PSUs, or Planck "voxels", tile along the area of a spherical surface horizon, producing a holographic relationship with the interior information mass-energy density (see **Figure 1**).

In this generalized holographic approach, it is therefore suggested that the information/entropy of a spherical surface horizon should be calculated in spherical bits and thus defines the surface information/entropy in terms of PSUs, such that,

$$\eta = \frac{A}{\pi r_\ell^2} \tag{10}$$

where the Planck area, taken as one unit of information/entropy, is the equatorial disk of a Planck spherical unit, πr_ℓ^2 and A is the surface area of a spherical system. We note that in this definition, the entropy is slightly greater (~ 5 times) than that set by the Bekenstein bound, and the proportionality constant is taken to be unity (instead of 1/4 as in the Bekenstein-hawking entropy). It has been previously suggested that the quantum entropy of a black hole may not exactly equal A/4 [41]. To differentiate between models, the information/entropy S, encoded on the surface boundary in the generalized holographic model is termed, $\eta \equiv S$.

As first proposed by 't Hooft the holographic principle states that the description of a Volume of space can be encoded on its surface boundary, with one discrete degree of freedom per Planck area, which can be described as Boolean variables evolving with time [42].

Following the definition for surface information η, the information/entropy within a volume of space is similarly defined in terms of PSU as,

$$R = \frac{V}{\frac{4}{3}\pi r_\ell^3} = \frac{r^3}{r_\ell^3} \tag{11}$$

where V is the volume of the spherical entity and r is its radius.

In previous work [36] [37], it was demonstrated that the holographic relationship between the transfer energy potential of the surface information and the volume information, equates to the gravitational mass of the system. It was thus found that for any black hole of Schwarzschild radius r_S the mass m_S can be given as,

$$m_S = \frac{R}{\eta} m_\ell \tag{12}$$

where η is the number of PSU on the spherical surface horizon and R is the number of PSU within the spherical volume. Hence, a holographic gravitational mass equivalence to the Schwarzschild solution is obtained in terms of a discrete granular structure of spacetime at the Planck scale, giving a quantized solution to gravity in terms of Planck spherical units (PSUs). It should be noted that this view of the interior structure of the black hole in terms of PSUs, is supported by the concept of black hole molecules and their relevant number densities as proposed by Miao and Xu [43] and Wei and Lui [44]. As well, the relationship between the interior structure in terms of "voxels" and the connecting horizon pixels is discussed in the work of Nicolini [45].

Of course, these considerations lead to the exploration of the clustering of the structure of spacetime at the nucleonic scale, where it was found that a precise value for the mass m_p and charge radius r_p of a proton can be given as,

$$m_p = 2\frac{\eta}{R} m_\ell = 2\phi m_\ell \tag{13}$$

$$r_p = 4\ell \frac{m_\ell}{m_p} = 0.841236(28) \times 10^{-13}\,\text{cm} \tag{14}$$

where $\phi = \frac{\eta}{R}$ is defined as a fundamental holographic ratio. Significantly, this value is within an 1σ agreement with the latest muonic measurements of the charge radius of the proton [36] [37], relative to a 7σ variance in the standard approach [46].

3. Resolving the Vacuum Catastrophe

To resolve the vacuum catastrophe, we must first understand where the value for the vacuum energy density at the Planck scale is coming from. As was previously defined [36] [37], and summarized above, the physical structure and thus energy density at this scale is more appropriately represented in terms of PSUs, such that the vacuum energy density at the Planck scale ρ_ℓ, can be given as,

$$\rho_\ell = \frac{m_\ell}{PSU} = 9.86 \times 10^{93}\ \text{g/cm}^3 .$$

The vacuum energy density at the quantum scale is thus $\rho_\ell = 9.86 \times 10^{93}\ \text{g/cm}^3$ instead of the value $\rho_{vac} = 5.16 \times 10^{93}\ \text{g/cm}^3$ given in Equation (8).

The generalized holographic model describes how any spherical body can be considered in terms of its PSU packing, or volume entropy, *R*. The mass-energy M_R, in terms of PSU, can therefore be given as $M_R = Rm_\ell$ and the mass-energy density is given as, $\rho_R = \frac{M_R}{V}$.

In the case of the proton, the mass-energy in terms of Planck mass was calculated as $M_R = Rm_\ell = 2.45 \times 10^{55}\ \text{g}$, which is equivalent to the mass of the observable universe (*i.e.* $M_u = 136 \times 2^{256} \times m_p = N_{Edd} m_p = 2.63 \times 10^{55}\ \text{g}$ in terms of the Eddington number; and $M_u \approx 3.63 \times 10^{55}\ \text{g}$ from density measurements). Since these values for the mass of the observable universe are just approximations, we will take the mass of the observable universe to be the mass-energy of the proton, as calculated above. The mass-energy density of the universe can thus be defined in terms of the mass-energy density of the proton. Thus, at the cosmological scale the mass-energy density, or vacuum energy density, is calculated to be,

$$\rho_u = \rho_R = \frac{M_R}{V_U} = \frac{Rm_\ell}{V_U} = 2.26 \times 10^{-30}\ \text{g/cm}^3 = 0.265 \rho_{crit} \quad (15)$$

where $V_U = 1.08 \times 10^{85}\ \text{cm}^3$ and was found by taking r_U as the Hubble radius $r_H = c/H_o = 1.37 \times 10^{28}\ \text{cm}$. Thus, when the vacuum energy density of the Universe is considered in terms of the proton density and the protons PSU packing (*i.e.* its volume entropy, *R*) we find the density scales by a factor of 10^{122}. As well, it should be noted that this value for the mass-energy density is found to be equivalent to the dark matter density, $\rho_d = 0.268 \rho_{crit}$.

Similarly, the vacuum energy density can be considered in terms of the PSU surface tiling (*i.e.* its surface entropy, η), as the radius expands from the Planck scale ρ_ℓ to the cosmological scale. The vacuum density at the cosmological scale is thus given as,

$$\rho_u = \frac{\rho_\ell}{\eta} = 8.53 \times 10^{-30}\ \text{g/cm}^3 \left(= \rho_{crit}\right) \quad (16)$$

where η is found by assuming a spherical shell Universe of radius $r_U = r_H$. The resulting change in density, from the vacuum density at the Planck scale to that at the cosmological scale yields an exact equivalent to the currently observed critical density of the universe, ρ_{crit}. Thus, when we consider the generalized holographic approach, which describes how any spherical body can be considered in terms of its PSU packing, we show the scale relationship between the PSUs and a spherical shell universe and resolve the 122 orders of magnitude discrepancy between the vacuum energy density at the Planck scale and the vacuum energy density at the cosmological scale.

The solution presented here is in line with the ideas of quintessence in which the mass-energy density is governed by the scale factor η_{φ}^{-1}, such that $\rho_{\varphi} = \frac{\rho_{\ell}}{\eta_{\varphi}}$ for $\eta_{\varphi} > \eta_{\ell}$. Following this approach, the Friedman equation can then be written in the form:

$$H_{\varphi}^{2} = \frac{8\pi G}{3}\rho_{\varphi} = \frac{8\pi G}{3}\frac{\rho_{\ell}}{\eta_{\varphi}} \tag{17}$$

which can also be given in terms of the varying radius, such that $\rho_{\varphi} = \frac{\rho_{\ell}}{4}\left(\frac{r_{\ell}}{r_{\varphi}}\right)^{2}$ for $r_{\varphi} > r_{\ell}$ and the Friedman equation becomes:

$$H_{\varphi}^{2} = \frac{8\pi G}{3}\rho_{\varphi} = \frac{8\pi G}{3}\frac{\rho_{\ell}}{4}\left(\frac{r_{\ell}}{r_{\varphi}}\right)^{2} = \frac{2\pi G}{3}\rho_{\ell}\left(\frac{r_{\ell}}{r_{\varphi}}\right)^{2} \tag{18}$$

These findings are in agreement with those of Ali and Das [47] who, in an attempt to resolve the current problems of cosmology, interpret one of the quantum correction terms in the second order Friedman equation as dark energy. From the quantum corrected Raychaudhuri equations they find the first correction term $\Lambda_{Q} = 1/L_{0}^{2}$ where L_{0} is identified as the current linear dimension of our observable universe, such that $\lambda_{Q} = 10^{-123}$ in planck units.

Essentially, they are adding the correction term $\Lambda_{Q} = r_{\ell}^{2}/L_{0}^{2}$ whereas we include the scale factor $r_{\ell}^{2}/r_{\varphi}^{2}$. However, their solution describes a purely quantum mechanical description of the universe assuming quantum gravity affects are practically absent, whereas the results described here show how, as the density changes with radius we have a scaler field that is coupled to gravity and thus rolls down a potential governed by a generalized holographic quantized solution to gravity [36].

Similar scale-invariant models have also been proposed by Maeder [48] [49] [50] who much like Milgrom's modified Newtonian dynamics (MOND) [51] [52] [53] defines a limit where scale invariance is applicable at large scales (*i.e.* low accelerations in MOND). In his model Maeder utilizes a new co-ordinate system, derived from scale invariant tensor analysis, and much like Milgrom and Verlinde [54] he finds an additional factor κ_{v} that opposes gravity. Interestingly, and in line with our findings, Maeder notes that with this new co-ordinate system, both the pressure and density are not scale invariant.

It should as well be noted that the equivalence found between the critical density and that found from the surface entropy (Equation (16)) yields a critical mass that obeys the Schwarzschild solution for a universe with a radius of the Hubble radius,

$$M_{crit} = \frac{\rho_{\ell}}{\eta}V_{u} = \frac{m_{\ell}}{\phi} = 9.24\times10^{55}\,\text{g}\left(\equiv\frac{r_{s}c^{2}}{2G}\right) \tag{19}$$

The idea that the observable universe is the interior of a black hole was originally put forward by Pathria [55] and Good [56] and more recently by Poplowski [57]. If such a solution holds true, then this would give us the prefect opportunity to study the interior of a black hole.

4. Discussion

Previous attempts to resolve the vacuum catastrophe include large quantum corrections (e.g. [47] [58]). However such theories offer no physical explanation and although solutions such as Zlatev [59] [60] do not depend on any fine tuning of the initial conditions, fine-tuning is still required to set the energy density of the scalar field to equal the energy density of matter and radiation at the present time *i.e.* at the cross-over from matter dominated to scalar field (or vacuum) dominated. This was the weak point of Hoyle's [12] steady state universe, as although he was able to show expansion properties with the introduction of the space-time vector C, no physical explanation was proposed.

The solution described in this paper utilizes the generalized holographic approach [36], offering a physical explanation which is thus inherent within the equations of general relativity such that no correction terms are necessary. Renormalization still occurs, where the cutoff for renormalization is the Planck unit (PSU) which is based on the fundamental constants of nature (within our universe at least).

Similarly, Huang [61] who presents a super-fluid model of the universe attempts to solve the fine-tuning problem by assuming a self-interacting complex scalar field that emerges with the big bang. The potential (defined as the Halpern-Huang potential) then grows from zero as the length scale expands (*i.e.* it should be asymptotically free) and the cosmological constant, in terms of a high-energy cut-off decreases with the expanding universe.

The nature of the fundamental constants and the large dimensionless numbers resulting from their relationships has been a long-standing puzzle (e.g. [62]-[69]), and concepts such as a variable G [66] [67] [68] [70] and continuous matter creation have been introduced [66]. The relationship between the number of particles in the universe and Weyls ratio [62] [71] showed that the number of particles in the universe should be increasing proportionally to the square of the age of the universe and therefore matter must be continually created. Steady state cosmology, previously suggested by Hoyle [12] and Einstein [72], offered such a concept, but with a constant G, as oppose to Dirac and his variable G. In previous work [73] this was resolved by suggesting that it is the mass-energy density that is changing and not G. In this paper we show that the mass-energy density decreases with the increasing size of the universe, so although the number of particles in the universe is increasing, with continuous matter creation the energy/information is conserved *i.e.* particles passing out of the observable universe are compensated by the creation of new particles where it is only through the creation of matter that an expanding universe can be consistent with the conservation of mass within the observable universe.

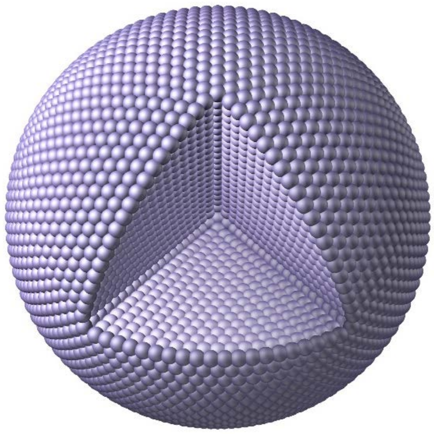

Figure 1. Schematic to illustrate the Planck Spherical Units (PSU) packed within a spherical volume.

The standard model of the universe (*i.e.* concordance ΛCDM) explains the accelerated expansion of the universe in terms of a negative pressure generated by the so-called dark energy. However, although in good agreement with CMB, large scale structure and SNeIa data, it is not yet able to explain the coincidence (fine-tuning) or the cosmological problem. As noted by Corda (2009) [74] extended theories of gravity (e.g. theories of gravity where the Lagrangian is modified by adding high-order terms in the curvature invariants or terms with scalar fields non-minimally coupled to geometry) generate inflationary frameworks which solve many of the problems, including the accelerated expansion. This is in agreement with the theory presented here where the acceleration of the universe can be explained in terms of a pressure gradient due to the information transfer potential at the horizon. The details of this are beyond the scope of this paper and will be addressed in a follow-up paper.

In summary we have shown how the generalized holographic model resolves the 122 orders of magnitude discrepancy between the vacuum energy density at the Planck scale and the vacuum energy density at the cosmological scale. Thus, not only resolving this long-standing problem in physics but also validating this geometrical approach. The details in terms of matter creation and the expansion rate are beyond the scope of this paper and will be addressed in a forth coming paper. The results presented here have profound implications for astrophysics, cosmogenesis, universal evolution and quantum cosmology giving incentive to further exploration and developments.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Adler, R. J., Casey, B. and Jacob, O.C. (1995) Vacuum Catastrophe: An Elementary Exposition of the Cosmological Constant problem. *American Journal of Physics*, **63**, 620-626. https://doi.org/10.1119/1.17850

[2] Hubble, E.P. (1929) A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae. USA, (1929). *Proceedings of the National Academy of Sciences of the United States of America*, **15**, 168-173 https://doi.org/10.1073/pnas.15.3.168

[3] Reiss, A.G., Filippenko, A.V., Challis, P., Clocchiattia, A. and Diercks, A. (1998) Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal*, **116**, 1009-1038. https://doi.org/10.1086/300499

[4] Schmidt, B.P., Suntzeff, N.B., Phillips, M.M., Schommer, R.A. and Clocchiatti, A. (1998) The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Curvature of the Universe Using Type Ia Supernovae. *APJ*, **507**, 46-63. https://doi.org/10.1086/306308

[5] Perlmutter, G., Aldering, G., Goldhaber, R.A., Knop, P. and Nugent, P.G. (1999) Measurements of Ω and Λ from 42 High-Redshift Supernovae. *APJ*, **517**, 565-586. https://doi.org/10.1086/307221

[6] Spergel, D.N., Verde, L., Peiris, H.V., Komatsu, E., Nolta, M.R., Bennett, C.L., Halpern, M., Hinshaw, G., Jarosik, N., Kogut, A., Limon, M., Meyer, S.S., Page, L., Tucker, G.S., Weiland, J.L., Wollack, E. and Wright, E.L. (2003) First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters. *APJ*, **148**, 175-194.

[7] Eisenstein, D.J., Zehavi, I., Hogg, D.W., Scoccimarrow, R. and Blanton, M.R. (2005) Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies. *APJ*, **633**, 560-574. https://doi.org/10.1086/466512

[8] Hinshaw, G., Larson, D., Komatsu, E., Spergel, D.N. and Bennett, C.L. (2013) Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results. *APJ*, **208**, 19H. https://doi.org/10.1088/0067-0049/208/2/19

[9] Zel'dovich, Y.B. (1968) The Cosmological Constant and the Theory of Elementary Particles. *Soviet Physics Uspekhi*, **11**, 381-393. https://doi.org/10.1070/PU1968v011n03ABEH003927

[10] Bludman, S.A. and Ruderman, M.A. (1977) Induced Cosmological Constant Expected above the Phase Transition Restoring the Broken Symmetry. *Physical Review Letters*, **38**, 255-257. https://doi.org/10.1103/PhysRevLett.38.255

[11] Carroll, S.M. (2001) The Cosmological Constant. *Living Reviews in Relativity*, **4**, 1. https://doi.org/10.12942/lrr-2001-1

[12] Hoyle, F. (1948) A New Model for the Expanding Universe. *MNRAS*, **108**, 372-382. https://doi.org/10.1093/mnras/108.5.372

[13] Guth, A.H. (1981) Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems. *Physical Review*, **23**, 347-356.

[14] Aghanim, N., Akrami, Y., Ashdown, M., Aumont, J., *et al.* Planck 2018 Results. VI.

Cosmological Parameters. Planck Collaboration, arXiv:1807.06209v1.

[15] Sparnaay, M.J. (1958) Measurements of Attractive Forces between Flat Plates. *Physica*, **24**, 751-764. https://doi.org/10.1016/S0031-8914(58)80090-7

[16] Wheeler, J.A. (1962) Geometrodynamics. Academic Press, New York and London.

[17] Sabisky, E.S. and Anderson, C.H. (1973) Verification of the Lifshitz Theory of the van der Waals Potential Using Liquid-Helium Films. *Physical Review A*, **7**, 790-806. https://doi.org/10.1103/PhysRevA.7.790

[18] Eberlein, C. (1996) Sonoluminescence as Quantum Vacuum Radiation. *Physical Review Letters*, **76**, 3842-3845. https://doi.org/10.1103/PhysRevLett.76.3842

[19] Lamoreaux, S.K. (1997) Demonstration of the Casimir Force in the 0.6 to 6 μm Range. *Physical Review Letters*, **78**, 5-8. https://doi.org/10.1103/PhysRevLett.78.5

[20] Bordag, M., Mohideen, U. and Mostepanenko, V.M. (2001) New Developments in the Casimir Effect. *Physics Reports*, **353**, 1-205. https://doi.org/10.1016/S0370-1573(01)00015-1

[21] Becka, C. and Mackey, M.C. (2007) Measurability of Vacuum Fluctuations and Dark Energy. *Physica A*: *Statistical Mechanics and Its Applications*, **379**, 101-110. https://doi.org/10.1016/j.physa.2006.12.019

[22] Capasso, F., Munday, J.N. and Parsegian, V.A. (2009) Measured Long-Range Repulsive Casimir-Lifshitz Forces. *Nature*, **457**, 170-173. https://doi.org/10.1038/nature07610

[23] Wilson, C.M., Johansson, G., Pourkabirian, A., Simoen, M., Johansson, J.R., Duty, T., Nori, F. and Delsing, P. (2011) Observation of the Dynamical Casimir Effect in a Superconducting Circuit. *Nature*, **479**, 376-379. https://doi.org/10.1038/nature10561

[24] Weinberg, S. (2000) Approaches to the Cosmological Constant Problem. 4*th International Symposium on Sources and Detection of Dark Matter*/*Energy in the Universe*, Los Angeles.

[25] Weinberg, S. (1989) The Cosmological Constant Problem. *Reviews of Modern Physics*, **61**, 1. https://doi.org/10.1103/RevModPhys.61.1

[26] Witten, E. (1995) Is Supersymmetry Really Broken? *International Journal of Modern Physics*, **10**, 1247-1248. https://doi.org/10.1142/S0217751X95000590

[27] Peebles, P.J.E. and Ratra, B. (1988) Cosmology with a Time-Variable Cosmological Constant. *The Astrophysical Journal*, **325**, L17-L20. https://doi.org/10.1086/185100

[28] Ratra, B. and Peebles, P.J.E. (1988) Cosmological Consequences of a Rolling Homogeneous Scalar Field. *Physical Review D*, **37**, 3406. https://doi.org/10.1103/PhysRevD.37.3406

[29] Wetterich, C. (1988) Cosmology and the Fate of Dilatation Symmetry. *Nuclear Physics B*, **302**, 668-696. https://doi.org/10.1016/0550-3213(88)90193-9

[30] Tsujikawa, S. (2013) Quintessence: A Review. *Classical and Quantum Gravity*, **30**, 214003. https://doi.org/10.1088/0264-9381/30/21/214003

[31] Obied, G., Ooguri, H., Spodneiko, L. and Vafa, C. De Sitter Space and the Swampland. arXiv:1806.08362.

[32] Denef, F., Hebecker, A. and Wrase, T. (2018) de Sitter Swampland Conjecture and the Higgs Potential. *Physical Review D*, **98**, Article ID: 086004. https://doi.org/10.1103/PhysRevD.98.086004

[33] Agrawal, P., Obied, G., Steinhardt, P. and Vafa, C. (2018) On the Cosmological Implications of the String Swampland. *Physics Letters B*, **784**, 271-276.

https://doi.org/10.1016/j.physletb.2018.07.040

[34] Weinberg, S. (1987) Anthropic Bound on the Cosmological Constant. *Physical Review Letters*, **59**, 2607. https://doi.org/10.1103/PhysRevLett.59.2607

[35] Peebles, P.J.E. (1967) The Gravitational Instability of the Universe. *The Astrophysical Journal*, **147**, 859. https://doi.org/10.1086/149077

[36] Haramein, N. (2013) Quantum Gravity and the Holographic Mass. *Physical Review & Research International*, **3**, 270-292.

[37] Haramein, N. (2013) Addendum to "Quantum Gravity and the Holographic Mass in View of the 2013 Muonic Proton Charge Radius Measurement. https://osf.io/4uhwp

[38] 't Hooft, G. (2000) The Holographic Principle. in Basics and Highlights in Fundamental Physics. *Proceedings of the International School of Subnuclear Physics*, Erice, 1-15.

[39] Bekenstein, J. (1973) Black Holes and Entropy. *Physical Review D*, **7**, 2333-2346. https://doi.org/10.1103/PhysRevD.7.2333

[40] Hawking, S. (1975) Particle Creation by Black Holes. *Communications in Mathematical Physics*, **43**, 199-220. https://doi.org/10.1007/BF02345020

[41] Dabholkar, A. (2005) Exact Counting of Black Hole Microstates. https://arxiv.org/pdf/hep-th/0409148.pdf

[42] 't Hooft, G. (1993) Dimensional Reduction in Quantum Gravity. arXiv:gr-qc/9310026

[43] Miao, Y.-G. and Xu, Z.-M. (2019) Interaction Potential and Thermo-Correction to the Equation of State for Thermally Stable Schwarzschild Anti-de Sitter Black Holes. *Science China Physics*, *Mechanics and Astronomy*, **62**, 10412. https://doi.org/10.1007/s11433-018-9254-9

[44] Wei, S.-W. and Liu, Y.-X. (2015) Insight into the Microscopic Structure of an AdS Black Hole from Thermodynamical Phase Transition. *Physical Review Letters*, **115**, Article ID: 111302. https://doi.org/10.1103/PhysRevLett.115.111302

[45] Nicolini, P. and Singleton, D. (2014) Connecting Horizon Pixels and Interior Voxels of a Black Hole. *Physics Letters B*, **738**, 213-217. https://doi.org/10.1016/j.physletb.2014.09.038

[46] Antognini, A., Nez, F., Schuhmann, K., Amaro, F.D., *et al.* (2013) Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen. *Science*, **339**, 417-420. https://doi.org/10.1126/science.1230016

[47] Ali, F.A. and Das, S. (2015) Cosmology from Quantum Potential. *Physics Letters B*, **741**, 276-279. https://doi.org/10.1016/j.physletb.2014.12.057

[48] Maeder, A. (2017) An Alternative to the Lambda CDM Model: The Case of Scale Invariance. *APJ*, **834**.

[49] Maeder, A. (2017) Scale Invariant Cosmology and CMB Temperatures as a Function of Redshifts. *APJ*, **847**,

[50] Maeder, A. (2017) Dynamical Effects of the Scale Invariance of the Empty Space: The Fall of Dark Matter? *APJ*, **849**.

[51] Milgrom, M. (1983) A modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis. *APJ*, **270**.

[52] Milgrom, M. (1983) A Modification of the Newtonian Dynamics—Implications for Galaxies. *APJ*, **270**.

[53] Milgrom, M. (1983) Emergent Gravity and the Dark Universe. *APJ*, 270, 384, 270.

[54] Verlinde, E.P. (2017) Emergent Gravity and the Dark Universe. *SciPost: SciPost Physics*, **2**.

[55] Pathria, R.K. (1972) The Universe as a Black Hole. *Nature*, **240**, 298-299. https://doi.org/10.1038/240298a0

[56] Good, I.J. (1972) Chinese Universes. *Physics Today*, **25**, 15. https://doi.org/10.1063/1.3070923

[57] Poplawski, N.J. (2010) Radial Motion into an Einstein-Rosen Bridge. *Physics Letters B*, **687**, 110-113.

[58] Ashtekar, A., Pawlowski, T. and Singh, P. (2006) Quantum Nature of the Big Bang. *Physical Review Letters*, **96**, Article ID: 141301. https://doi.org/10.1103/PhysRevLett.96.141301

[59] Zlatev, I., Wang, L. and Steinhardt, P.J. (1999) Quintessence, Cosmic Coincidence, and the Cosmological Constant. *Physical Review Letters*, **82**, 896. https://doi.org/10.1103/PhysRevLett.82.896

[60] Zlatev, I., Wang, L. and Steinhardt, P.J. (1999) Cosmological Tracking Solution. *Physical Review D*, **59**.

[61] Huang, K. (2013) Dark Energy and Dark Matter in a Superfluid Universe. *International Journal of Modern Physics A*, **28**, Article ID: 1330049. https://doi.org/10.1142/S0217751X13300494

[62] Weyl, H. (1917) On the Theory of Gravitation. *Annals of Physics*, **54**, 117.

[63] Eddington, A. (1931) Preliminary Note on the Masses of the Electron, the Proton, and the Universe. *Mathematical Proceedings of the Cambridge Philosophical Society*, **27**, 15-19. https://doi.org/10.1017/S0305004100009269

[64] Eddington, A. (1936) Relativity Theory of Protons and Electrons. Cambridge University Press, Cambridge.

[65] Eddington, A. (1946) Fundamental Theory. Cambridge University Press, Cambridge.

[66] Dirac, P.A.M. (1937) The Cosmological Constants. *Nature*, **139**, 323.

[67] Dirac, P.A.M. (1938) A New Basis for Cosmology. *Proceedings of the Royal Society A*, **165**, 199-208.

[68] Dirac, P.A.M. (1974) Cosmological Models and the Large Numbers Hypothesis. *Proceedings of the Royal Society A*, **338**, 439-446.

[69] Funkhouser, S. (2006) The Large Number Coincidence, the Cosmic Coincidence and the Critical Acceleration. *Proceedings of the Royal Society A*, **462**, 3657-3661.

[70] Milne, E.A. (1935) Relativity, Gravity and World Structure. Oxford University Press, Oxford.

[71] Weyl, H. (1919) New Extension of Relativity Theory. Annalen der Physik, **59**.

[72] O'Raifeartaigh, C., McCann, B., Nahm, W. and Mitton, S. (2014) Einstein's Steady-State Theory: An Abandoned Model of the Cosmos. *The European Physical Journal H*, **39**, 353-367. https://doi.org/10.1140/epjh/e2014-50011-x

[73] Haramein, N., Rauscher, E.A. and Hyson, M. (2008) Scale Unification: A Universal Scaling Law. *Proceedings of the Unified Theories Conference*, Budapest, 2008.

[74] Corda, C. (2009) Interferometric Detection of Gravitational Waves: The Definitive Test for General Relativity. *International Journal of Modern Physics D*, **18**, 2275-2282.