

A Hydrodynamic Model Including Phase Transition and the Thermal Motion Induced Transverse Momentum Distributions in p - p Collisions at LHC Energies

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Abstract

It is widely believed that the matter created in p - p collisions exhibits a similar collective behavior as that formed in heavy ion collisions. In this paper, by taking into account the effects of thermal motion, the transverse momentum distributions of identified charged particles are discussed in the scope of the hydrodynamic model including phase transition. The theoretical model gives a good description to the data collected in p - p collisions at LHC energies for the transverse momentum up to about $p_T = 1.0 \text{ GeV}/c$.

Keywords

Thermal Motion, Relativistic Hydrodynamics, Phase Transition, Transverse Momentum Distribution

1. Introduction

In the past decades, a number of bulk observables about charged particles, such as the Fourier coefficients v_n of azimuth-angle distributions [1] [2], transverse momentum spectra [3]-[8] and pseudorapidity distributions [9] [10], have experienced a series of extensive investigations in both nucleus and hadron collisions. These investigations have confirmed two facts. One is that the quark-gluon plasma (QGP) might have come into being in nucleus collisions at current RHIC or LHC energies [11] [12] [13] [14] [15]. This also might be true even in hadron collisions at the early lower energies of Intersecting Storage Rings (ISR) and Super Proton Synchrotron (SPS) at CERN [16] [17] [18] [19] [20]. The other is that the matter created in nucleus or hadron collisions shows a clear feature of collective flow, expanding

nearly like a perfect fluid with very low viscosity [21]-[47].

Apart from collective movement, the quanta of produced matter also have the components of thermal motion. The evolution of the produced matter is the convolution of the collective movement and thermal motion. To clarify the role of thermal motion in the expansions of the produced matter in p - p collisions at LHC energies is the major subject of this paper. To this end, we may as usual ignore the minor collective flow in the transverse directions. The transverse movement of the produced matter is therefore only induced by the thermal motion.

The collective movement of produced matter in the longitudinal direction can be solved analytically. There are a number of schemes in dealing with such precise calculations [21]-[35]. In this paper, the hydrodynamic model proposed by N. Suzuki is employed [27]. Besides the analytical characteristics, the other typical feature of this theoretical model is that it incorporates the effects of phase transition into solutions. This coincides with the current experimental observations as mentioned above. Hence, the employed model is more in line with the realistic situations. In addition, the model is related to the initial temperature of QGP, the sound speed in both partonic and hadronic media, the baryochemical potential, as well as the critical temperature of phase transition. This work may therefore help us understand various transport coefficients of expanding system.

In Section 2, a brief introduction is given to the theoretical model [27], presenting its analytical solutions. The solutions are then used in Section 3 to formulate the invariant multiplicity distributions of charged particles produced in collisions which are in turn compared with the measurements performed by LHC-CMS Collaboration in p - p collisions at LHC energies of $\sqrt{s} = 0.9, 2.76$ and 7 TeV [3], respectively. The last Section 4 is about conclusions.

2. A Brief Introduction to the Model

Here, for the purpose of completion and application, we shall list the key ingredients of the adopted model [27].

- 1) The movement of fluid meets continuity equation

$$\frac{\partial T^{\mu\nu}}{\partial x^\nu} = 0, \quad \mu, \nu = 0, 1 \quad (1)$$

where $x^\nu = (x^0, x^1) = (t, z)$, t is the time and z is the longitudinal coordinate along beam direction. $T^{\mu\nu}$ is the energy-momentum tensor, which, for a perfect fluid, takes the form

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}, \quad (2)$$

where $g^{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1)$ is the metric tensor.

$$u^\mu = (u^0, u^1) = (\cosh y_F, \sinh y_F), \quad u^\mu u_\mu = 1, \quad (3)$$

is the 4-velocity of fluid, y_F is its rapidity. ε and p in Equation (2) are the energy density and pressure of fluid, which fit thermodynamic relations

$$\varepsilon + p = Ts, \quad d\varepsilon = Tds, \quad dp = sdT, \tag{4}$$

where T and s are the temperature and entropy density of fluid, respectively. To close Equation (1), another relation, namely the equation of state

$$\frac{dp}{d\varepsilon} = \frac{sdT}{Tds} = c_s^2 \tag{5}$$

is needed, where c_s is the sound speed of fluid, which takes different values in QGP and in hadronic phase.

2) Project Equation (1) to the direction of u_μ and the direction perpendicular to u_μ , respectively. This leads to equations

$$\begin{aligned} \frac{\partial(su^\nu)}{\partial x^\nu} &= 0, \\ \frac{\partial(T \sinh y_F)}{\partial t} + \frac{\partial(T \cosh y_F)}{\partial z} &= 0. \end{aligned} \tag{6}$$

The first formula is the continuity equation for entropy conservation. The second one means the existence of a scalar function ϕ satisfying relations

$$\frac{\partial\phi}{\partial t} = T \cosh y_F, \quad \frac{\partial\phi}{\partial z} = -T \sinh y_F. \tag{7}$$

From ϕ and Legendre transformation, Khalatnikov potential χ is introduced via

$$\chi = \phi - tT \cosh y_F + zT \sinh y_F. \tag{8}$$

In terms of χ , the variables t and z can be expressed as

$$\begin{aligned} t &= \frac{e^\theta}{T_0} \left(\frac{\partial\chi}{\partial\theta} \cosh y_F + \frac{\partial\chi}{\partial y_F} \sinh y_F \right), \\ z &= \frac{e^\theta}{T_0} \left(\frac{\partial\chi}{\partial\theta} \sinh y_F + \frac{\partial\chi}{\partial y_F} \cosh y_F \right), \end{aligned} \tag{9}$$

where T_0 is the initial temperature of fluid and $\theta = \ln(T_0/T)$. Through above equations, the coordinate base of (t, z) is transformed to that of (θ, y_F) , and the first formula of Equation (6) is translated into the so called telegraphy equation

$$\frac{\partial^2\chi}{\partial\theta^2} - 2\beta \frac{\partial\chi}{\partial\theta} - \frac{1}{c_s^2} \frac{\partial^2\chi}{\partial y_F^2} = 0, \quad \beta = \frac{1-c_s^2}{2c_s^2}. \tag{10}$$

3) Along with the expansions of matter created in collisions, it becomes cooler and cooler. As its temperature drops from the initial T_0 to the critical T_c , phase transition occurs. The matter transforms from QGP state to hadronic state. The produced hadrons are initially in the violent and frequent collisions. The major part of these collisions is inelastic. Hence, the abundances of identified hadrons are in changing. Furthermore, the mean free paths of these primary hadrons are very short. The movement of them is still like that of a fluid meeting Equation (10) with only difference being the value of c_s . In QGP, $c_s = c_0 = 1/\sqrt{3}$, which is the sound speed of a massless perfect fluid, being the maximum of c_s . In the

hadronic state, $c_s = c_h < c_0$. At the point of phase transition, that is as $T = T_c$, c_s is discontinuous.

4) The solution of Equation (10) for the sector of QGP is [27]

$$\begin{aligned} \chi_0(\theta, y_F) &= \frac{q_0 c_0}{2} e^{\beta_0 \theta} I_0 \left(\beta_0 c_0 \sqrt{y_0^2(\theta) - y_F^2} \right), \\ \beta_0 &= \frac{1 - c_0^2}{2c_0^2} = 1, \quad y_0(\theta) = \frac{\theta}{c_0}, \end{aligned} \tag{11}$$

where q_0 is a constant determined by tuning the theoretical results to experimental data. I_0 is the 0th order modified Bessel function of the first kind.

In the sector of hadrons, the solution of Equation (10) is [27]

$$\chi_h(\theta, y_F) = \frac{q_0 c_0}{2} B(\theta) I_0 \left[\lambda(\theta, y_F) \right], \tag{12}$$

where

$$\begin{aligned} B(\theta) &= e^{\beta_h(\theta - \theta_c) + \beta_0 \theta_c}, \quad \lambda(\theta, y_F) = \beta_h c_h \sqrt{y_h^2(\theta) - y_F^2}, \\ \beta_h &= \frac{1 - c_h^2}{2c_h^2}, \quad y_h(\theta) = \frac{\theta - \theta_c}{c_h} + \frac{\theta_c}{c_0}, \quad \theta_c = \ln \left(\frac{T_0}{T_c} \right). \end{aligned} \tag{13}$$

It is evident that, if $c_h = c_0$, then $\beta_h = \beta_0$, $B(\theta) = e^{\beta_0 \theta}$, $y_h = y_0$, and thus $\chi_h = \chi_0$. At the point of phase transition, $T = T_c$, $\theta = \theta_c$, $B(\theta) = e^{\beta_0 \theta_c}$, $y_h = y_0 = \theta_c / c_0$. Then

$$\begin{aligned} \chi_0(\theta, y_F) \Big|_{T=T_c} &= \frac{q_0 c_0}{2} e^{\beta_0 \theta_c} I_0 \left(\beta_0 \sqrt{\theta_c^2 - c_0^2 y_F^2} \right), \\ \chi_h(\theta, y_F) \Big|_{T=T_c} &= \frac{q_0 c_0}{2} e^{\beta_0 \theta_c} I_0 \left(\frac{\beta_h c_h}{c_0} \sqrt{\theta_c^2 - c_0^2 y_F^2} \right), \end{aligned} \tag{14}$$

$\chi_h(\theta, y_F) \Big|_{T=T_c} \neq \chi_0(\theta, y_F) \Big|_{T=T_c}$. That is, the potential χ is discontinuous at point of $T = T_c$.

3. The Thermal Motion Induced Transverse Momentum Distributions of Identified Charged Particles

With Khalatnikov potential χ , the rapidity distributions of fluid read as [26]

$$\frac{dN}{dy_F} = \frac{q_0 c_0}{2} A \left(\cosh y \frac{dz}{dy_F} - \sinh y \frac{dt}{dy_F} \right), \tag{15}$$

where A is the cross area of overlap region of collisions. Inserting Equation (9) into above equation, the part in the round brackets becomes

$$\begin{aligned} &\cosh y \frac{dz}{dy_F} - \sinh y \frac{dt}{dy_F} \\ &= \frac{1}{T} c^2 \frac{\partial}{\partial \theta} \left(\chi + \frac{\partial \chi}{\partial \theta} \right) \cosh(y - y_F) - \frac{1}{T} \frac{\partial}{\partial y_F} \left(\chi + \frac{\partial \chi}{\partial \theta} \right) \sinh(y - y_F). \end{aligned} \tag{16}$$

Along with the expansions of hadronic matter, its temperature continues becoming lower. According to the prescription of Cooper-Frye [26], as the temperature drops to the so called chemical freeze-out temperature T_{FO} , the inelastic collisions

among hadrons cease. The yields of identified hadrons maintain unchanged becoming the measured results in experiments. Convoluting Equation (15) with the contributions from thermal motion, the invariant multiplicity distributions of charged particles equal [26] [27] [28]

$$\frac{d^2N}{2\pi p_T dy dp_T} = \frac{1}{(2\pi)^3} \int \frac{dN}{dy_F} \frac{m_T \cosh(y - y_F)}{\exp\{[m_T \cosh(y - y_F) - \mu_B]/T\} + \delta} \Big|_{T=T_{FO}} dy_F, \quad (17)$$

where $m_T = \sqrt{m^2 + p_T^2}$ is the transverse mass of produced charged particle with rest mass m . μ_B in Equation (17) is the baryochemical potential. For Fermi charged particles, $\delta = 1$ in the denominator of Equation (17), and for Bosons, $\delta = -1$. The meaning of Equation (17) is evident. It is the convolution of dN/dy_F with the energy of the charged particles in the state with temperature T . The former comes from the longitudinal collective flow. The latter stems from the thermal motion.

The right hand side of Equation (17) is evaluated with $T = T_{FO}$. At this moment, the fluid freezes out into the charged particles. Replacing χ in Equation (16) by χ_h of Equation (12), it turns into

$$\left(\cosh y \frac{dz}{dy_F} - \sinh y \frac{dt}{dy_F} \right) \Big|_{T=T_{FO}} \quad (18)$$

$$= \frac{1}{T_{FO}} (\beta_h c_h)^2 B(\theta_{FO}) [S(\theta_{FO}, y_F) \sinh(y - y_F) + C(\theta_{FO}, y_F) \cosh(y - y_F)],$$

where

$$S(\theta_{FO}, y_F) = \frac{\beta_h y_F}{\lambda(\theta_{FO}, y_F)} \left\{ \frac{\beta_h c_h y_h(\theta_{FO})}{\lambda(\theta_{FO}, y_F)} I_0[\lambda(\theta_{FO}, y_F)] \right. \quad (19)$$

$$\left. + \left[\frac{\beta_h + 1}{\beta_h} - \frac{2\beta_h c_h y_h(\theta_{FO})}{\lambda^2(\theta_{FO}, y_F)} \right] I_1[\lambda(\theta_{FO}, y_F)] \right\},$$

$$C(\theta_{FO}, y_F) = \left\{ \frac{\beta_h + 1}{\beta_h} + \frac{[\beta_h c_h y_h(\theta_{FO})]^2}{\lambda^2(\theta_{FO}, y_F)} \right\} I_0[\lambda(\theta_{FO}, y_F)] \quad (20)$$

$$+ \frac{1}{\lambda(\theta_{FO}, y_F)} \left\{ \frac{y_h(\theta_{FO})}{c_h} + 1 - \frac{2[\beta_h c_h y_h(\theta_{FO})]^2}{\lambda^2(\theta_{FO}, y_F)} \right\} I_1[\lambda(\theta_{FO}, y_F)],$$

where I_1 is the 1st order modified Bessel function of the first kind.

The integral interval of y_F in Equation (17) is $[-y_h, y_h]$. By using Equations (15), (17), and (18)-(20), together with the definitions in Equation (13), we can get the thermal motion induced transverse momentum distributions of identified charged particles as shown **Figure 1**. This figure shows the transverse momentum distributions of π^\pm , K^\pm and $p(\bar{p})$ at mid-rapidity of $|y| < 1$ in p - p collisions at $\sqrt{s} = 0.9, 2.76$ and 7 TeV, respectively. The solid dots are the experimental data [3]. The solid curves are the results of Equation (17). The left and right columns are for the positive and negative charged particles, respectively.

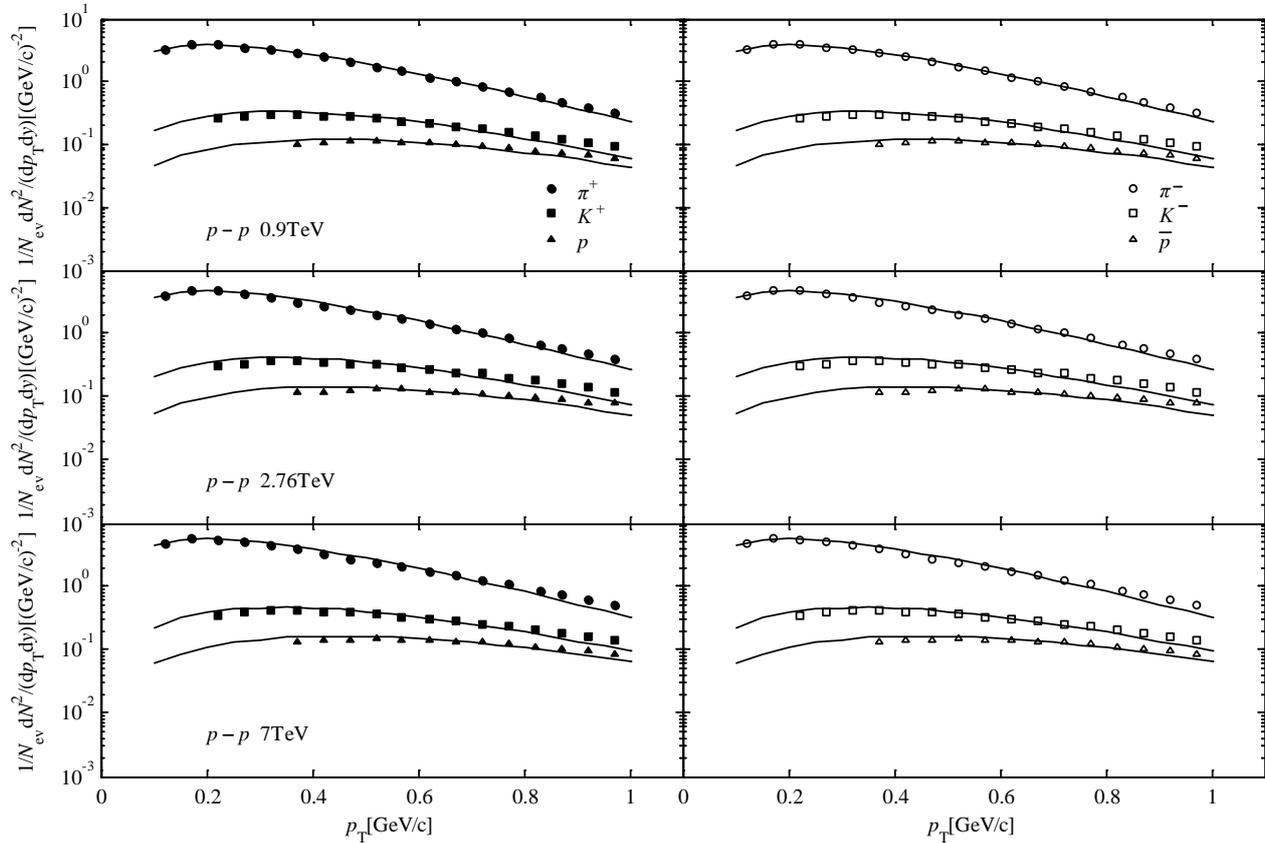


Figure 1. The transverse momentum spectra of identified charged particles produced at mid-rapidity of $|y| < 1$ in p - p collisions at $\sqrt{s} = 0.9, 2.76$ and 7 TeV (from top to bottom). The solid dots are the experimental data [3]. The solid curves are the results of Equation (17). The left and right columns are for the positive and negative charged particles, respectively.

It can be seen that the theoretical results are in good accordance with experimental data for transverse momentum up to about $p_T = 1.0$ GeV/ c .

In calculations, T_c in Equation (13) takes the well-recognized value of $T_c = 180$ MeV. c_h in Equation (13) takes the value of $c_h = 0.35$ from the investigations of Refs. [28] [48] [49] [50]. Referring to the measurements of Ref. [4], the chemical freeze-out temperature T_{FO} takes the value of 160 MeV for π in the three different incident energies of 0.9, 2.76 and 7 TeV. For K and p , T_{FO} takes 170 and 175 MeV, respectively, in the three cases of collisions. The baryochemical potential μ_B in Equation (17) takes the value of $\mu_B = 0$ since in the considered collisions, the abundances of particle and antiparticle are approximately equal to each other [3]. This can be clearly seen in **Figure 1**. As for the initial temperature T_0 in Equation (13), there is no widely accepted value so far. In this paper, consulting the results presented in Ref. [28], T_0 takes the values of 3.0, 6.5 and 7.8 GeV for incident energies equaling 0.9, 2.76 and 7 TeV, respectively.

4. Conclusions

In order to see the importance of thermal motion in the expansions of the matter

produced in p - p collisions, the minor transverse collective flow is as usual ignored. The movement of matter in the transverse directions is therefore only induced by the thermal motion. Convoluting it with longitudinal collective movement, we can get the invariant multiplicity distributions of charged particles.

The longitudinal collective flow follows the relativistic hydrodynamics. The most striking feature of the model used in this paper is that it takes the phase transition into consideration, which might have appeared in the current experiments in nuclear or even in hadronic collisions. Hence, the adopted model might reflect a more realistic process of spatiotemporal evolution of matter created in collisions.

The model contains a rich information about the transport coefficients of fluid, such as the sound speed in QGP c_0 , the sound speed in hadronic state c_h , the phase transition temperature T_c , the chemical freeze-out temperature T_{FO} , the baryochemical potential μ_B and the initial temperature T_0 . Exception for T_0 , the other five coefficients take the values either from the well-known theoretical results or from experimental measurements. As for T_0 , there is no much knowledge about it at present. In our calculations, T_0 is estimated referring to other researches.

The present investigations show that the p_T distributions of charged particles measured in p - p collisions at LHC energies of $\sqrt{s} = 0.9, 2.76$ and 7 TeV can be well described by the theoretical model for p_T up to about $p_T = 1.0$ GeV/ c . Certainly, the transverse collective flow objectively exists even though it is minor. It should affect the motion of the matter. The existing investigations have shown that if the transverse collective flow is taken into account, the describable region is extended to about $p_T < 2.0$ GeV/ c [5] [6] [7]. Furthermore, to describe the experimental data in the region of $p_T \geq 2.0$ GeV/ c , some other mechanisms, such as the method of blast-wave and perturbative QCD [8], should be introduced. Considering that the overwhelming majority of charged particles situates in the region of $p_T < 1.0$ GeV/ c , we may conclude that the thermal motion plays a non-ignorable role in the transverse expansions of the matter produced in p - p collisions at LHC energies.

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