

# The Creation of Neutron Stars and Black Stars and How the Chandrasekhar Limit Prevents the Creation of Intermediate Black Stars

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## Abstract

Using a program written in Excel, it was found that a supernova remnant, with a mass between 1.44 and 2 solar masses, contracts down to a neutron star. During the collapse, the decreasing gravitational potential slows time. Here, the pressure becomes high enough to stop the contraction. At greater than 2.2 solar masses, while the remnant is still contracting, the gravitational potential causes time to relatively freeze at the center, and stop the contraction before the pressure gets high enough to stop it, as it did in a neutron star. This also freezes the flow of information concerning the decrease in gravitational potential, thus, the frozen portions remain frozen and do not contract down any further and become imaginary. On top of this frozen center, additional matter physically and relatively contracts and the radius of the freeze point moves out. If the freeze made its way to the surface, it would meet the condition of a black hole, having a Schwarzschild radius; but it does not quite get there. The surface is not quite frozen. Even though these “almost black holes” do not have an event horizon, they are almost as small as that described by the Schwarzschild radius and due to the gravitational red shift, are very hard to see. A black star has been created. A contracting white dwarf at the Chandrasekhar limit (1.44 solar masses) has a density of about  $1 \times 10^9 \text{ kg/m}^3$ . After it cools and then collapses into a neutron star, it will have a minimum density of  $3.5 \times 10^{15} \text{ kg/m}^3$  near the surface. This article explains how these two densities relate to why there are no supernova created stellar black stars above 15 solar masses and why super massive black stars start at 50,000 solar masses? Extracting limits like these cannot be accomplished using the standard black hole model, but this black star model has revealed these size limits and a lot more.

## Keywords

Black Holes, White Dwarf, Chandrasekhar, Chandra, Excel

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## 1. Introduction

As one observes an object that is approaching the “event horizon” of a traditional black hole, it appears to flatten out on the event horizon and all motion comes to a dead stand still. Many believe that on the other side of the horizon, conditions become imaginary (due to the square root of a negative number). In a finite amount of time, the object crosses this event horizon and continues on its way to an infinitesimally small point called the singularity where it is crushed. “While a great deal is known about the properties of stationary black holes, we know very little about the process of black hole formation” [1]. Considering what happens during the growth of a black hole sheds some light on its makeup and what really happens. As a result, a black star is created which is made of matter throughout its volume, with no singularity or event horizon, and having a radius just slightly greater than the Schwarzschild radius.

The remnant core of a supernova explosion contracts in, at first, due to the pressure of the explosion, and then due to the increased gravity caused by the much higher density and smaller radius. An Excel program was developed to find out what happens during this collapse and growth of neutron stars and black holes. During this collapse, the physical contractions cause the gravitational potential to decrease causing time to relatively slow and come to a stop. Here, relativistic corrections derived from general relativity are factored into the physical contractions of the Newtonian model. These codes are viewed from a time coordinate position which follows the evolution from a position where the gravitational field is weak. Using these codes, the conditions for a black star were found rather than a black hole.

## 2. Supernova

When some large stars use up their nuclear energy released at a given temperature and pressure, they will cool and collapse. During this collapse, the release of gravitational energy reheats this matter to an even higher temperature and pressure. Lighter elements, just outside the energy spent core, react at this higher temperature and pressure and rapidly fuse, releasing energy in an explosion causing a supernova. During this explosion, if the remnant has a mass greater than the Chandrasekhar limit of 1.44 solar masses, the high pressure and gravity compresses the inner core to a density exceeding  $1 \times 10^9 \text{ kg/m}^3$ , which is the highest density supportable by electron degeneracy pressure. Here the electrons can no longer support the normal atomic structure, resulting in the pressure compressing the remaining core to a form of neutron matter, at densities starting at  $3.5 \times 10^{15} \text{ kg/m}^3$ . Below this density, the corresponding pressure would not be high enough to sustain neutron matter. The gap between these densities will be referred to as the Chandra gap which is the transition zone for matter as it is being compressed from degenerate white dwarf matter to neutron matter. See Section 9 for more details. Subrahmanyan Chandrasekhar liked to be called Chandra which is why I refer to this as the “Chandra gap”.

As the explosion progresses, the outer portion of the star acquires an outward

velocity greater than that required to escape its' bindings to the star and this outward momentum relieves pressure on the core. Some of the outer portions of the core, containing many of the newly formed heavy elements, like copper and gold, rebound and are blown away along with the outer portion of the star. This debris can be used in the formation of planetary star systems like ours. The remaining core matter continues to contract in, due to the increased gravity caused by the much higher density and smaller radius. Typically, a remnant between 1.4 and 2 solar masses forms a neutron star. We get a black star somewhere above that.

### 3. Freezing of Time

Relative to a remote point, time slows as one approaches or goes into a massive body. For this reason, we start the calculations on layers near the center and work our way to the surface. Here the relative slowing of time, the comparison of the rate of clock ticks, is used rather than dilation, the time between ticks. The expression

$$t/T = (1 + 2\Phi/C^2)^{1/2} \quad (1)$$

is used for the rate of time passage. This same factor is also used for relative length contractions or Lorentz type contractions. This equation comes from the metric tensor of space time in general relativity. The rate of time  $t$  passing, relative to a remote point in space  $T$ , slows as the gravitational potential  $\Phi$  (Greek phi) decreases. The gravitational potential is the gravitational potential energy per unit mass. Time freezes when  $t/T$  becomes 0 (zero). Substituting this 0 into  $t/T$  and solving for  $\Phi$  we find that time freezes at any radius  $r$  when the contraction causes the gravitational potential to drop down to

$$\Phi = -C^2/2. \quad (2)$$

The change in the gravitational potential across any layer  $\Delta r$  ( $\Delta$  Greek delta) is

$$\Delta\Phi = F\Delta r = -(m_{bm}G/r^2)\Delta r \quad (3)$$

at radius  $r$ , where  $F$  is the force per unit mass,  $\Delta r$  is the distance over which the force is applied, and  $m_{bm}$  is the total mass below the radius  $r$  at layer  $n$ . The gravitational potential, at any radius  $r$ , is expressed by

$$\Phi = -GM/R - \sum_R^r m_{bm}G\Delta r/r^2 \quad (4)$$

where  $M$  is the total remnant mass, and  $R$  is the outer radius.  $GM/R$  is the gravitational potential at the surface and  $\sum_R^r m_{bm}G\Delta r/r^2$  is the sum of the gravitational potentials from the surface  $R$  down to the radius  $r$ . Simply put, this gravitational potential at any radius  $r$  within the star remnant is the gravitational potential at the surface plus all the gravitational potential contributions down to that radius. As we go deeper into the remnant, the gravitational potential  $\Phi$  gets more negative. These relationships are explained in more detail in [Appendix B](#).

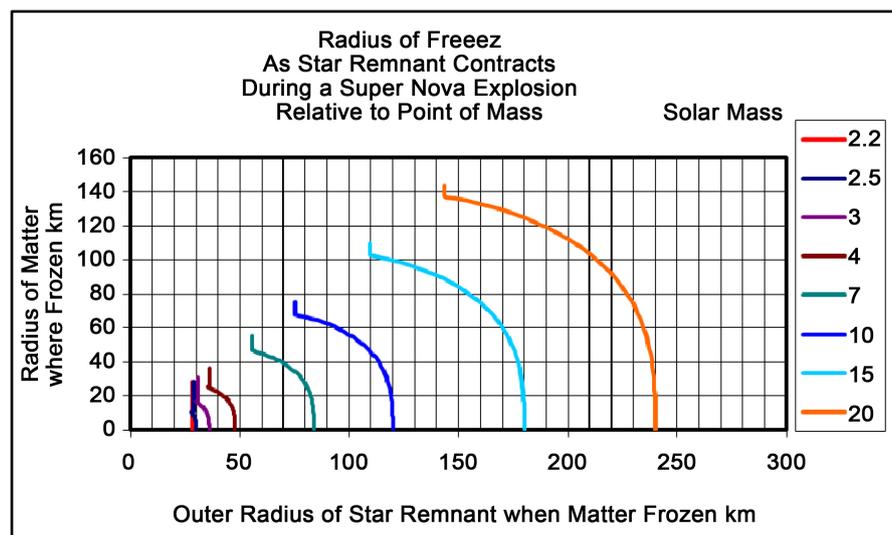
The gravitational potential gets more negative for each piece of mass as the remnant contracts. During this contraction, the thickness of all the  $\Delta r$  shells of matter that make up the star, contract together until time freezes at the center.

This is where you might expect a black hole to normally be formed; but it doesn't happen. The freezing of time relatively stops the contraction of neutron matter before the pressure/density gets high enough to cause a collapse and form a singularity. The results instead progress to the creation of a black star. The remaining shells which have not been frozen continue to contract until the next layer is frozen. All the remaining unfrozen shells will continue to contract and freeze each successive layer almost all the way to the surface. In **Figure 1**, the sum of all the shell thicknesses is the radius  $R$  at the surface. At a point near the surface, further contractions of the unfrozen matter cannot cause the gravitational potential to drop low enough to cause it to freeze. The freeze starts at the center of the remnant and as the remnant physically and relatively contracts in, the freeze moves out, almost all way to the surface, where a black star is left without an event horizon or singularity [2].

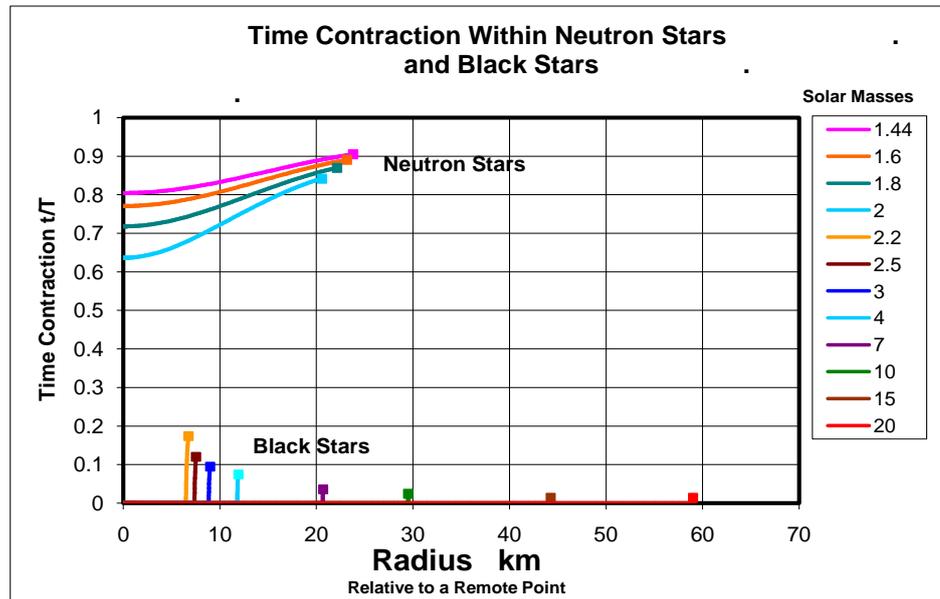
As matter physically contracts in and the density increases, the gravitational potential becomes more negative and time slows. Keep in mind that simultaneously, time and the dimensions of space and matter relatively contract together. You can see the results of both physical and relative contractions on the rate of time flow in **Figure 2**.

Even though these "almost black holes" do not have an event horizon, as black holes are conventionally expected too, they are almost as small and due to the gravitational red shift, are very hard to see. This red shift factor can be seen in **Figure 3**.

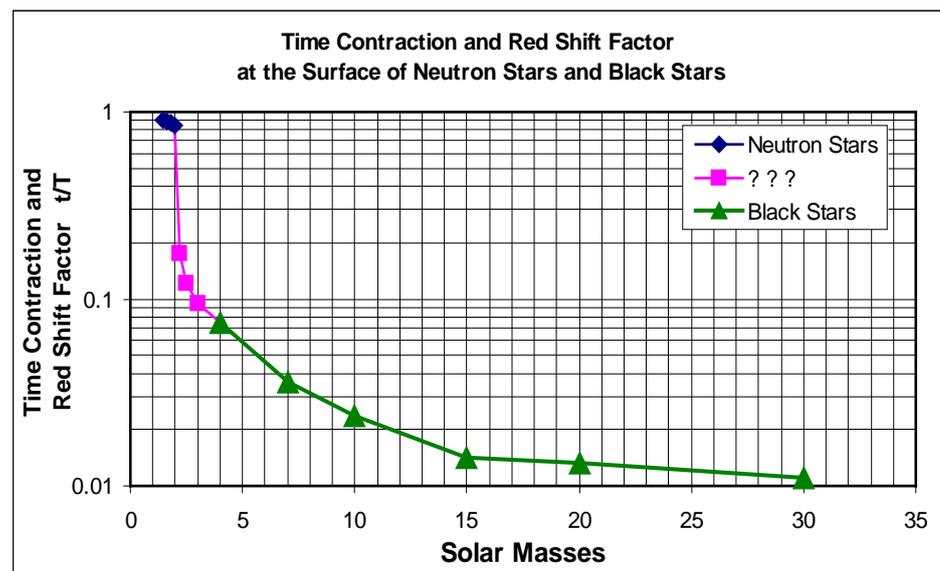
From our vantage point, if we could view any frozen point within a black star (maybe using something like an entangled transmitter receiver for instantaneous communication), time and movement would be at a dead standstill. On the other hand, relative to these frozen points within the black star, the rate of time flow



**Figure 1.** This graph shows that the freeze point starts at the center (on the y axis) and as the outer radius physically contracts in, the radius of the freeze point moves out. Note the small bends at the end of each line where the freeze ends near the surface, which prevents the creation of an event horizon. Here radial dimensions are relative to the point of mass and the freeze is relative to a remote location far out in space.



**Figure 2.** In a neutron star, as shown above by the colored lines that intercept the y axis, time slows more toward the center than it does near the surface (solid squares). Note that as the mass of a neutron star increases, the outer radius gets smaller. Time is not frozen anywhere within neutron stars; however, in black stars, it is frozen, throughout most of the volume. The freeze happens first at the center (0, 0) and then it progresses along the frozen x axis until it reaches a point close to the surface where the lack of contraction has not caused time to freeze. The surface of a black star is represented by a solid square with an almost vertical line of positive slope dropping down to the x axis. This almost frozen surface prevents the creation of an event horizon. These black stars start with a solar mass of 2.2 and are made up of neutron matter. Most of this matter is frozen in time, except near the surface where the physical pressure is not quite enough for the gravitational potential to cause a freeze. The radius, for black stars, ends up being just slightly greater than the Schwarzschild radius.



**Figure 3.** This graph shows what the value of time contraction would acquire at the surface of neutron stars and black stars. The frequency of the light emitted is gravitationally red shifted by this same factor. The rate of photon emission is also reduced by this factor. Because of these reasons and the small size, a black star is almost black and extremely hard to see.

for the rest of the universe would show events happening at a rate of perhaps billions of years per second [3].

#### 4. Freezing the Flow of Information

Relative to our remote point of view, as matter contracts in toward the center, it will freeze when the gravitational potential  $\Phi$  gets to  $-C^2/2$ . Since the gravitational potential at any point is the sum of all the gravitational potentials above, all points below the uppermost frozen point would meet the condition of  $\Phi$  being less than  $-C^2/2$ . This would make  $t/T$  imaginary if it wasn't for the freeze of information flow. As the freeze point progresses outward from the center, the freezing of the speed of light and the flow of information on gravitational potential prevent  $t/T$  from becoming imaginary. From our remote point of view, the inner frozen points do not receive the information about the outer matter contracting in and changing the gravitational potential. Relative to any frozen point  $r$ , the  $r$  and  $\Delta r$  dimensions above are greatly increased. This would relatively increase the gravitational potential of Equation (4) and prevent a local time freeze. Points within the frozen region are in a different position in space-time than the points above and are protected from becoming imaginary.

#### 5. Neutron Stars

The physical compressive forces during a supernova cause a remnant to physically and relatively contract down to neutron matter. Neutron star remnants, having a mass between 1.4 and 2 solar masses, physically contract down to where the pressure, beginning at the center, supports the incoming mass where the contraction stops. For these neutron stars, as shown in **Figure 2**, both the physical and relative contractions together cause time to slow, but will not cause it to freeze.

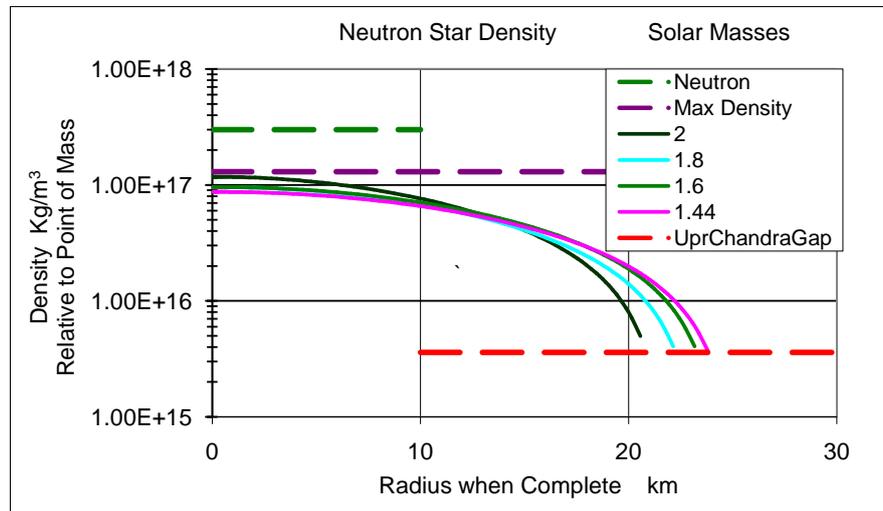
Derived from a graph found at the website by [4] (Hebeler, K. *et al.* 2013), and explained in more detail in **Appendix A**, the equation

$$\rho = 10^{(0.4838 \log P + 1.2372)} \quad (5)$$

expresses the relationship between the density  $\rho$  of neutron matter and the pressure  $P$ . This expression is used to determine the density as a function of radius as shown in **Appendix A**. In **Figure 4**, we see the equilibrium densities of neutron stars between 1.44 and 2 solar masses.

#### 6. Black Stars Greater Than 2.2 Solar Masses

For remnants, larger than 2.2 solar masses, the gravitational potential at the center approaches the value  $-C^2/2$  and the rate of time flow approaches 0 before the remnant has completed the physical contraction. As the center freezes, more matter from above physically and relatively contracts closer in. Relative to our remote point of view, the progression of this matter toward the center is stopped by this freezing of time before the increasing pressure can stop it. The gravitational potential of incoming matter from above also reduces to  $-C^2/2$  and freezes. More and more space and matter contract into a volume closer in and in a short



**Figure 4.** The density at the center of the largest neutron star ( $1.3 \times 10^{17} \text{ kg}\cdot\text{m}^{-3}$ ) is just short of that of a neutron ( $3 \times 10^{17} \text{ kg}\cdot\text{m}^{-3}$ ). As will be shown later, this density, being the highest found in matter, is also obtained at the center of the smallest black star. The smallest neutron star has just enough pressure near the surface to support the lowest density supportable by neutron matter ( $3.5 \times 10^{15} \text{ kg}/\text{m}^3$ ). This density is found at the upper edge of the Chandra gap discussed in Section 9. This lowest density is found in both the smallest neutron star and the largest stellar black star. Below this density, the corresponding pressure is not high enough to sustain neutron matter.

time the freeze cascades all the way to a point close to the surface to where pressure becomes the mechanism that supports the incoming matter. From this point on, only the relative contractions carry the freeze to a point a little closer to the surface, forming a black star without an event horizon. The freezing of time, at any point, limits the density from any further increases. We see in **Figure 5** that for increasing star remnant sizes, the relative freeze starts earlier in the contraction process, causing the density of the larger black stars to be less dense.

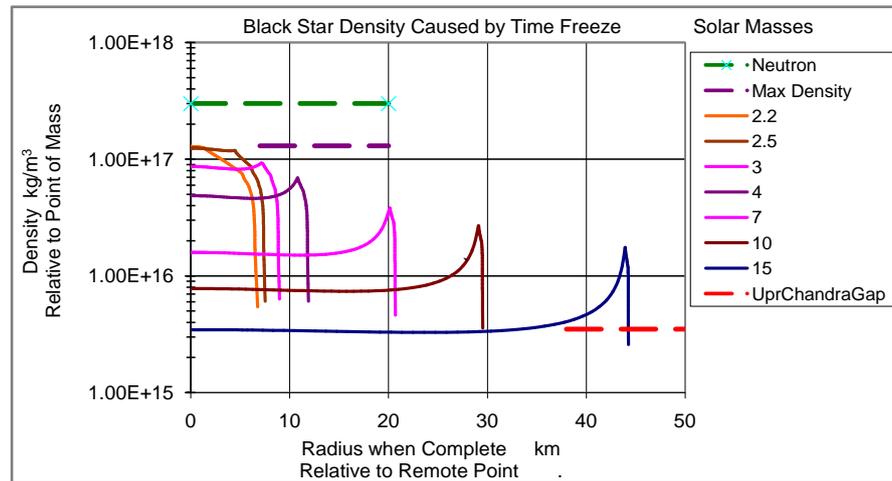
The upper edge of the Chandra gap, as described in Section 9, is the lowest density supportable by neutron matter. Below this density, the pressure is not high enough to sustain neutron matter, which limits the largest stellar black star to 15 solar masses.

## 7. Preventing Singularities

The creation of a singularity in a conventional black hole requires the rapid collapse of neutron matter toward the center. However, this fast collapsing process is interrupted by the slowing of time. As shown in Section 6, the decreasing gravitational potential causes time to relatively slow and then stop the progression of this matter toward the center. **Figure 5** shows where the matter is relatively stopped. This document views the creation of black stars from a remote location in space similar to that of earth.

## 8. Supermassive Black Stars (SMBS)s

The smallest SMBSs are made from the gas and dust compressed down to matter

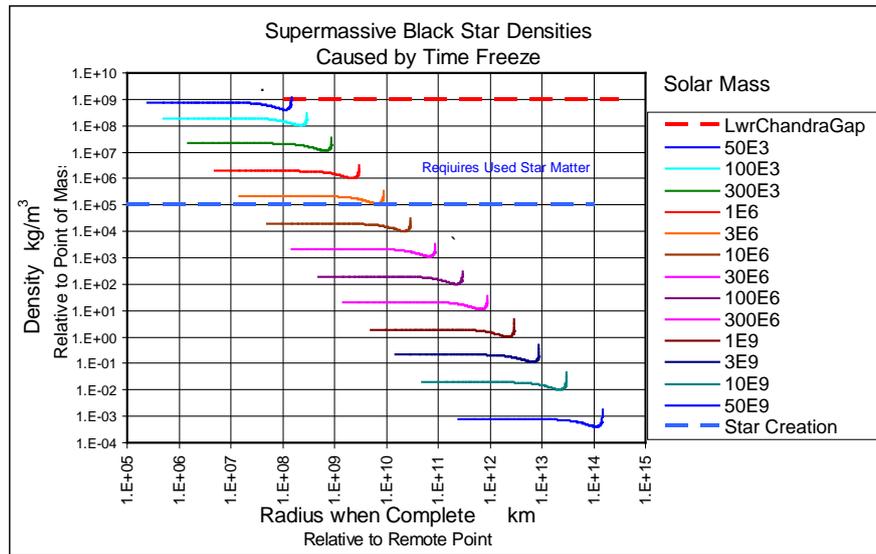


**Figure 5.** As a remnant contracts, the density is relatively frozen at a fairly uniform value where the freeze stops it before the pressure gets high enough to do so. Toward the surface, the contraction continues to a higher density peak where the freeze continues to stop it. At this peak, the pressure takes over as the mechanism that stops any further physical collapse. Here the density rapidly drops off while relative contractions cause the freeze to move almost to the surface. At a density of  $3.5 \times 10^{15} \text{ kg/m}^3$ , which is the upper edge of the Chandra gap, the maximum size of a black star is limited to around 15 solar masses (See Section 9). Below this density, as would be seen if black stars were larger, the pressure would not be high enough to sustain neutron matter.

similar to that of a white dwarf. This ball of matter needs to be big enough so that when it contracts and the gravitational potential at the center causes time to freeze, the pressure has NOT obtained a value high enough to overcome the electron degeneracy pressure and cause it to collapse down to neutron matter. This happens when the density gets near  $1 \times 10^9 \text{ kg/m}^3$  and the size is 50,000 solar masses, creating the smallest SMBS. At a radius, just outside the initially frozen center, the matter continues to physically and relatively contract in and due to the decreasing gravitational potential, it also freezes. Relative to our point of view, the progression of this matter toward the center is stopped by the freezing of time. More and more space and matter relatively contract into a volume closer in and the freeze cascades almost all the way to the surface, forming a SMBS. With larger balls of dust and gas, the more negative gravitational potential causes this freeze to happen even earlier in the contraction, causing it to have an even lower density, as shown in **Figure 6**.

Stars made of hydrogen and helium, contract and typically ignite at a temperature of  $1.6 \times 10^7 \text{ K}$  and at density of  $1.5 \times 10^5 \text{ kg/m}^3$ . SMBSs above the “Star Creation” line in **Figure 6**, like the 3 million solar mass one which has a density of  $2 \times 10^5 \text{ kg/m}^3$ , cannot be made before the creation of the first-generation stars in a galaxy. Doing so, with a density above  $1.5 \times 10^5 \text{ kg/m}^3$ , would start the fusion of hydrogen and release excess heat which would reverse the contraction and growth of the SMBS. Instead, these SMBSs are made from dust and gas left over from dying star explosions. This used star matter is ideal because it will not form active stars that would hinder the formation of SMBSs.

When SMBSs larger than 4 million solar masses start their growth below the



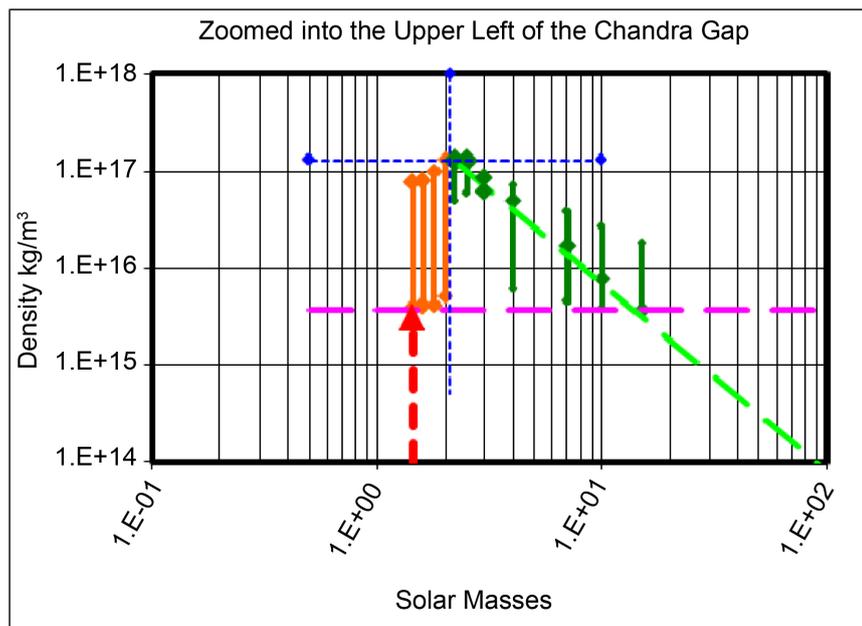
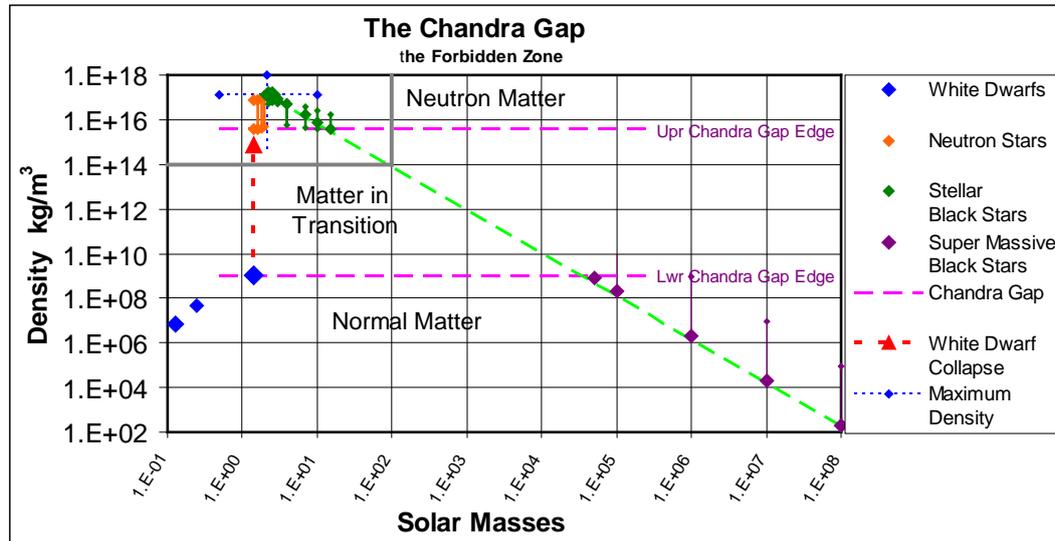
**Figure 6.** Along the horizontal line representing each black star, the density is relatively frozen at a fairly uniform value. Near the surface, the density climbs due to a decrease in the gravitational potential not being large enough to maintain the uniform density. The smallest SMBS has a size of 50,000 solar masses and a main core density of slightly less than  $1 \times 10^9 \text{ kg/m}^3$ . This density is on the “Lower Chandra Gap” edge as seen in Figure 7. The smallest one found to date is in a dwarf galaxy RGG 118 [5], and has a mass of 50,000 solar masses. SMBs created below the “Lower Chandra Gap” and above the “Star Creation” line, can only be made from “Used Star Matter” or dust and gas from exploded stars. They cannot be created by active star forming matter like hydrogen because the heat created from fusion would prevent the contraction and growth of the SMBS. Larger SMBs created below the “Star Creation” line do not have this problem and could be made either before or after the existence of any first-generation stars. Note: Some of the data near the surface of the SMBSs could not be obtained due to the 15-digit accuracy limit of Excel.

“Star Creation” line, the density is less than  $1.5 \times 10^5 \text{ kg/m}^3$ . Even in the presence of pure hydrogen, this density is low enough to prevent the formation of active stars; thus, allowing the formation of black stars. Here time freezes before the temperature and density gets high enough to fuse and form stars. This could even allow these larger SMBSs to be created before the birth of any first-generation stars in the galaxy.

### 9. The Chandra Gap

When either the mass of a supernova remnant or a white dwarf just gets up to the Chandrasekhar limit (1.44 solar masses), the electron degeneracy pressure will be overcome and it will collapse into the smallest neutron star of the same mass [6]. The high pressure and gravity compresses the core to a density exceeding  $1 \times 10^9 \text{ kg/m}^3$ , which is the highest density supportable by electron degeneracy pressure. Here the electrons can no longer support the normal atomic structure, resulting in the pressure compressing the remaining core to a form of neutron matter, at densities starting at  $3.5 \times 10^{15} \text{ kg/m}^3$ . Below this density, the corresponding pressure would not be high enough to sustain neutron matter.

The Chandra gap is the transition zone for matter as it is being compressed from degenerate white dwarf matter to neutron matter. Additional limits can be extrapolated from the collapse of this white dwarf as seen in **Figure 7** and **Table 1**.



**Figure 7.** Above the Chandrasekhar limit (1.44 solar masses), a white dwarf is compressed to a neutron star, and as a result (red arrow), it crosses the Chandra Gap. The Chandra gap is the transition zone for matter as it is being compressed from degenerate white dwarf matter to neutron matter. All neutron matter is found above the Chandra Gap and the normal type matter is found below. Starting at 2.2 solar masses, stellar black stars populate the graph up to 15 solar masses [7]. Some of the larger observations could have been due to binary paired black star mergers [8]. The density of a 15-solar mass black star gets down to  $3.5 \times 10^{15}$  kg/m<sup>3</sup>, which is on the upper edge of the Chandra Gap. Below this density, the pressure is not high enough to sustain neutron matter; therefore, 15 is the upper limit of a stellar black star created by a super nova. In Section 8 it was found that the smallest SMBS that can be created is 50,000 solar masses, which has a density just below  $1 \times 10^9$  kg/m<sup>3</sup>. This is the highest density supportable by electron degeneracy pressure, which causes the smallest SMBS found at the lower edge of the Chandra gap.

**Table 1.** List of extremes.

Star Extremes	Solar Masses	Density kg/m <sup>3</sup>	Gap Edge	Figure
Largest White Dwarf	1.44	$1 \times 10^9$	Lower	7
Smallest Neutron Star	1.44	$3.6 \times 10^{15}$	Min Upper	4
Largest Neutron Star	2.0	$1.3 \times 10^{17}$	Max at Center	4
Smallest Stellar Black Star	2.2	$1.3 \times 10^{17}$	Max at Center	5
Largest Stellar Black Star	15	$3.5 \times 10^{15}$	Upper	5
Smallest Supermassive Black Star	50,000	$1 \times 10^9$	Lower	6

## 10. Conclusions

Using the model created in Excel, it was found that during a supernova explosion, the remnant core, with a mass between 1.4 and 2 solar masses, would contract down to a neutron star. Stellar black stars are found between 2.2 and 15 solar masses. SMBs are found between 50 thousand and 50 billion solar masses. Here time begins freezing at the center before the remnant physically contracts down to the point where the pressure can stop the contraction. When time contracts all the way to 0 and freezes, the flow of information also freezes and prevents the remnant from contracting any further and becoming imaginary. The relative and physical contractions simultaneously cause the time freeze to work its way to a point just short of the surface creating a black star.

By extracting information from the Chandrasekhar limit, it was found that there is a density gap of unstable matter now referred to as the Chandra gap. This is the transition zone for matter as it is being compressed from degenerate white dwarf matter to neutron matter. This Chandra gap explains the gap of missing black star sizes between the largest stellar black star and the smallest SMBS.

In a quick summary, relative to a remote point in space:

- 1) Natural black holes might not exist.
- 2) Stellar black stars are made of solid neutron type matter throughout and thus, do not have a singularity.
- 3) The highest density ( $1.3 \times 10^{17} \text{ kg}\cdot\text{m}^{-3}$ ), is found in the largest neutron stars and the smallest black stars.
- 4) "Time freeze" limits the maximum density of matter to being less than that of a neutron.
- 5) "Time freeze" protects neutron matter from being crushed by gravity.
- 6) Black stars freeze in time from the inside out to a point near the surface.
- 7) Black stars are not frozen at the surface, and thus do not have an event horizon.
- 8) A time freeze contraction does not relatively drop below 0 and become imaginary.
- 9) The speed of light and information in frozen matter is 0.
- 10) A black star cannot be naturally created smaller than 2.2 solar masses.
- 11) Supernova created black stars are limited to a maximum of about 15 solar

masses.

- 12) SMBs are not made of neutron matter.
- 13) SMBs are made of normal type matter that is frozen in time.
- 14) The smallest SMBS is 50,000 solar masses and has a density less than  $1 \times 10^9 \text{ kg/m}^3$ .
- 15) Larger black stars are less dense than smaller ones.
- 16) All black stars, less than 4 million solar masses, are made from used star matter.
- 17) Black stars greater than 4 million solar masses are made either before or after first-generation galaxy stars.

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## Appendix/Mathematical Derivations

### Appendix A. Density Setup

In order to predict what would happen to the remnant core of a supernova, an Excel program was written to create models of supernova remnants ranging in size from 1 to 30 solar masses. This program created all the graphs in this document. It is made up of rows of fixed mass and columns of calculation steps. Dividing the star remnant into 500 equal thickness layers of uniform density and then adjusting these thicknesses, allows the density, gravitational potential, or radius to be adjusted. Derived from a graph found at the website by [4] (Hebeler, K *et al.* 2013), the equation

$$\rho = 10^{(m \log P + b)} \quad (6)$$

expressing the relationship between the density  $\rho$  of neutron matter and the pressure P, determines the density of each layer, where  $m = 0.4838$  and  $b = 1.7258$ . This equation helped set up the foundation of the pressure and density. In order to be representative of the neutron stars and black holes that have been observed, the intercept constant b was changed to 1.2372. This may be a more realistic value since it prevents black stars below 2.2 solar masses from being created. Equalization of the pressure and density is obtained by several iterations of the following equations:

From the mass of each shell, calculate the contribution of pressure of that shell.

$$\Delta P = (\Delta m_n) \times (m_{br}) G / ((r^2)(4\pi r^2)) \quad (7)$$

where  $m_{br}$  is the mass below the radius  $r$ .

Sum the contributions in pressure of each shell.

$$P_r = \sum_n^N \Delta P \quad (8)$$

Calculate the density for each shell.

$$\rho = 10^{(0.4838 \log P + 1.2372)} \quad (9)$$

Calculate the new radius for each shell.

$$r_n = \left( (m_{brn} - m_{br(n-1)}) / (\rho (4/3) \pi) + r_{(n-1)}^3 \right)^{1/3} \quad (10)$$

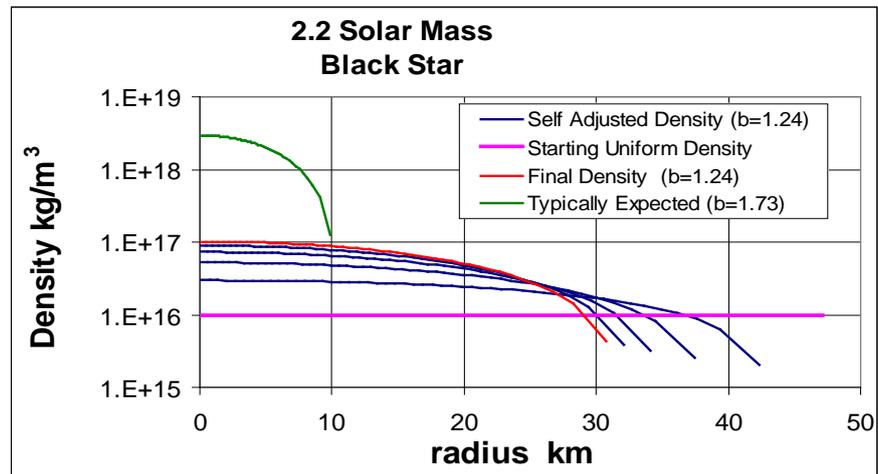
which is derived from

$$\rho = (m_{brn} - m_{br(n-1)}) / (4/3 \pi (r_n^3 - r_{(n-1)}^3)) \quad (11)$$

Use the new value of  $r_n$  to iterate (recalculate) the above steps 4 more times.

Starting at a uniform density, it can be seen in **Figure 8**, how each iteration settles in closer to the “Final Density” at equilibrium, which is done before any relative contractions are applied. To improve the accuracy, either more iterations can be used or the “Starting Uniform Density” can be put closer to the “Final Density” than what is shown.

Pressure and Density Calculations in Excel



**Figure 8.** Notice the equilibrium density decreases when  $b$  reduces from 1.7258 (typically expected by others as in the above web site), to 1.2372. To date, the largest neutron star observed is 2 solar masses and the smallest black hole is 4 solar masses. If it is found that the extreme sizes for black stars is not 2.2 and 15 solar masses, the constant  $b$  can be adjusted [7]. For example, by making  $b$  equal to 0.987765, the smallest allowable black star would be 3.9 solar masses, which is just short of the smallest one observed. The largest stellar black star prior to any merger would then become 32 solar masses which could relate to the LIGO incident [8]. The upper Chandra gap edge would then drop down to  $7.5 \times 10^{14} \text{ kg/m}^3$ .

In Excel, out of the 500 rows (31-530)-row35 equations are expressed here. Each equation is relatively copied to all cells in the column.

Equalization of the pressure and density is obtained by several iterations of columns F thru I. Multiple iterations are obtained by making concurrent copies of the column sets.

Previous  $r$  with uniform density. (XL cell E35)

Equations (7)-(11) above are used in Excel Columns (F-I) below.

From the mass of each shell, calculate the contribution of pressure of that shell.

$$\Delta P = (\Delta m_n) \times (m_{br}) G / ((r^2)(4\pi r^2))$$

$$= (\$D35 - \$D34) * \$D34 * \$G\$22 / (4 * PI() * E35^4) \quad \text{(XL cell F35)}$$

where  $m_{br}$  is the mass below that radius found in column D.

Sum the contributions in pressure of each shell.

$$P_r = \sum_n^N \Delta P$$

$$= \text{SUM}(F35:F530) \quad \text{(XL cell G35)}$$

Calculate the density for each shell.

$$\rho = 10^{(0.4838 \log P + 1.2372)}$$

$$= 10^{(\$I\$22 * LOG(G35) + \$J\$22)} \quad \text{(XL cell H35)}$$

Adjust the shell thickness/radius to obtain the new density.

$$r_n = \left( (m_{bm} - m_{br(n-1)}) / (\rho (4/3) \pi) + r_{(n-1)}^3 \right)^{1/3}$$

$$= ((\$D35 - \$D34) / (4/3 * PI() * H35) + I34^3)^{(1/3)} \quad \text{(XL cell I35)}$$

Use this new value of  $r_n$  to recalculate the above steps 4 more times.

(Row 22 contains constants.  $\$G\$22$  is  $G$ ,  $\$I\$22$  is constant  $m$ ,  $\$J\$22$  is con-

stant  $b$  (1.2372).)

### Appendix B. Freezing of Time (General Relativistic Effects)

The rate of time  $t$  passing, relative to a remote point in space  $T$ , slows as the gravitational potential (gravitational potential energy per unit mass)  $\Phi$  (Greek phi) decreases. Time slows as one approaches or goes into a massive body. For this reason, we start the calculations on layers near the center and work our way to the surface. Here the relative slowing of time, the comparison of the rate of clock ticks, is used rather than dilation, the time between ticks. Using Equation (1)

$$t/T = \left(1 + 2\Phi/C^2\right)^{1/2}$$

for the rate of time passage, leads to the prime characteristics of a black star. This equation comes from the metric tensor of space time in general relativity. Time freezes when  $t/T$  gets to 0 (zero). Substituting this 0 into  $t/T$  and solving for  $\Phi$  we find that time freezes at any radius  $r$  when the contraction causes the gravitational potential to get down to Equation (2)

$$\Phi = -C^2/2.$$

Substituting the gravitational potential at the surface of a spherical object

$$\Phi = -GM/R \quad (12)$$

into Equation (2) and solving for  $R$ , you get the Schwarzschild radius

$$R_s = 2GM/C^2. \quad (13)$$

which is the radius of an event horizon of a black hole. Within a spherical object, the equation for the force per unit mass is

$$F = -m_{brn}G/r^2 \quad (14)$$

at a radius  $r$ , where  $m_{brn}$  is the total mass below the radius  $r$ . The change in the gravitational potential across any  $\Delta r$  layer is then

$$\Delta\Phi = F\Delta r = -m_{brn}G\Delta r/r^2 \quad (15)$$

where  $\Delta r$  ( $\Delta$  Greek delta) is the distance over which the force is applied. The gravitational potential at any radius  $r$ , within the remnant is

$$\Phi = -GM/R - \sum_R^r \Delta\Phi \quad (16)$$

where  $M$  is the total remnant mass, and  $R$  is the outer radius.  $GM/R$ , as mentioned in Equation (12), is the gravitational potential at the surface. As we go deeper, the gravitational potential  $\Phi$  gets more negative with depth. The total gravitational potential down to any radius  $r$  within the star remnant is the gravitational potential at the surface plus all the gravitational potential contributions down to that radius. In the square of Equation (1),

$$(t/T)^2 = \left(1 + 2\Phi/C^2\right), \quad (17)$$

at no time does the value in a real supernova remnant drop below 0 and cause  $t/T$  to become imaginary. As the contraction of  $(t/T)$  takes place, it not only

contracts time, but it also contracts the relative dimensions of  $\Delta r$ . Since  $(t/T)^2 > 0$  then the new contracted delta r becomes

$$\Delta r = (t/T) \Delta r_{(\text{density settled } \Delta r)} \quad (18)$$

where  $\Delta r_{(\text{density settled } \Delta r)} = (r_n - r_{(n-1)})_{(\text{density settled } r)}$ . This is where relativistic corrections convert the Newtonian calculations to results compatible with general relativity. This is also shown in the Excel cell BL35 calculation below. Relative contractions, sometimes called length contractions or Lorentz type contractions do not cause local stresses on matter such as those caused by physical contractions, like increasing pressure and density. Due to the  $\Delta r$  contractions below the radius  $r_n$  the new value of  $r_n$  is the sum of the  $\Delta r$ s from the center

$$r_n = \sum_0^n \Delta r_n . \quad (19)$$

These changes, in turn, relatively change the gravitational potential, which leads to another contraction as described by Equations (15) thru (19). The values obtained in Equations (18) and (19) are applied to Equation (15) in order to form additional iterations of Equations (15) thru (19). These contractions asymptotically approach a lower value. See **Figure 2** for examples of these asymptotic values obtained in neutron stars. When the physical pressure at radius  $r$ , reaches the point where it can support the weight of the above neutron matter, and  $(t/T)^2$  settles in at a value greater than 0, as in neutron stars, then this pressure stops the contraction factor at the value of  $t/T$ . The density has come to an equilibrium with pressure and the contraction has stopped. (Note: If  $r_n$  at any point causes a time freeze,  $r_n$  is then also frozen before the application of another contraction.)

For remnants, greater than 2.2 solar masses, in order to maintain the relationship between pressure and density, the density is first allowed to come to equilibrium with pressure. This causes the center of the remnant to go below the freeze point and become imaginary, which never actual happens. We therefore need to go back in time to where the freeze first took place-while the remnant was larger and still contracting. The value of R and all its' delta r values are then increased by an amount that just allows the center to get to the freeze point. A multiplication factor  $K_1$  is used to increase all the delta r values ( $K_n \Delta r_n$ ). Since  $K_n = K_{n-1}$ , all the delta r values ( $\Delta r_n$ ) are increased, all the way to the surface, by the same factor  $K_1$ . These  $\Delta r$  values are changed at the point in the calculation which is just before the first contraction of time. An Excel visual basic program searches to find a value of  $K_1$  that will cause the  $(t/T)^2$  for the 1<sup>st</sup> layer (at the center) to become 0 and freeze.  $K_1$  is also frozen by replacing the cell equation ( $=K_{n-1}$ ) by the value that causes the freeze. The radius  $r_n$  is also relatively frozen at this point. This is like going back time to where the remnant first becomes frozen at the center.  $K_2$  is now slightly decremented until  $(t/T)^2$  for layer 2 becomes 0. This decrease continues with each layer by decreasing  $K_n$  until each  $(t/T)_n^2$  is 0. When the layer being worked on can support the matter above, the decreasing of  $K_n$  is stopped and the density is allowed to come to equilibrium

with pressure. The rest or the remnant is allowed to relatively contract until the freeze almost makes its way to the surface.

Relative Time and Length Contraction Calculations in Excel

In Excel, out of the 500 rows (31-530)-row35 equations are expressed here. Each equation is relatively copied to all cells in the column.

Relative time and length contractions are obtained by several iterations of columns BG thru BM. Multiple iterations are obtained by making copies of the column sets.

Previous  $\Delta r$  after adjustments for density settlement (XL cell BE35)

Previous  $r$  after adjustments for density settlement (XL cell BF35)

Excel Equations (15)-(19) used in Excel Columns BG-BM below

$\Delta PE$ /unit mass across  $\Delta r @ r$

$$\Delta\Phi = mG\Delta r / r^2$$

$$= \$D35 * \$G\$22 * BE35 / BF35^2 \quad \text{(XL cell BG35)}$$

where  $m_{br}$  is the mass below that radius found in column D.

$PE$ /unit mass down to  $r \Phi$

$$\Phi = -MG/R - \sum_R^r \Delta PE m$$

$$= -\$E\$22 * \$G\$22 / BF\$530 - SUM(BG35:BG\$530) \quad \text{(XL cell BI35)}$$

Contraction squared  $(t/T)^2$

$$(t/T)^2 = 1 + 2\Phi / C^2$$

$$= 1 + 2 * BI35 / \$BM\$15^2 \quad \text{(XL cell BJ35)}$$

$t/T$  Time Contraction

$$(t/T) = \text{IF}((BJ35) < 0, 0, \text{SQRT}(BJ35)) \quad \text{(XL cell BK35)}$$

Newtonian theory to general relativity by relativistic correction

1<sup>st</sup> Iteration relatively contracted

$$\Delta r = \Delta R * (t/T)$$

$$= \text{IF}(BJ35 < 0.000001, BE35, \$BE35 * BK35) \quad \text{(XL cell BL35)}$$

Each contraction iteration references the initial column BE.

$r$  summed from center

$$r_n = \sum_0^n \Delta r_n$$

$$= \text{IF}(BJ35 < 0.000001, BF35, \text{SUM}(BL\$31:BL35)) \quad \text{(XL cell BM35)}$$

(Each Cell in D column \$D  $n$  is the total mass below row  $n$ .)

(Row 22 contains constants. \$G\$22 is G, \$E\$22 is the Total Mass.)