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Quantum Neutron Unit Gravity

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Abstract

Quantum gravity and the transformation of a neutron star or the merger of two neutron stars into a black hole are important topics in cosmology. According to the Schwarzschild radius relationship, a black hole arises when two times of the gravitational binding energy of the gravitational system, *GBE*, equal the annihilation energy of its total mass. From a quantum perspective, the integer number of neutrons defines the *GBE* and mass in the merger of binary pure neutron stars transforming to a black hole. Therefore, one can scale all gravitational binding energy relationships by using neutron mass, energy, distance, time, or frequency equivalents. We define $GBE_{n^0\_\text{unit}}$ of the neutron as the binding energy, 1.4188×10^{-49} J, of a virtual system of two neutrons separated by the neutron Compton wavelength. The $GBE_{n^0\_\text{unit}}$ divided by a neutron's rest mass energy represents a fundamental, dimensionless proportionality constant, 9.4252×10^{-40}, $k_{n^0\_\text{unit}}$. The square root of $2k_{n^0\_\text{unit}}$, α_G, which we introduce here as a coupling constant, is identical in concept to the fine structure constant found in electromagnetic physics, but for gravity. Both α_G and $GBE_{n^0\_\text{unit}}$ inter-relate the neutron, proton, electron, Bohr radius, speed of light, Planck's constant, *GBE* of the electron in hydrogen, and Planck time. This paper demonstrates a direct conceptual and computational rationale of why the neutron and its negative beta decay quantum products accurately can represent a quantum gravitational natural unit system.

Keywords

Quantum Gravity, Neutron, Black Holes, Neutron Stars

1. Introduction

Developing a quantum model of gravity has been an important conceptual goal in physics [1]-[6]. Uniting cosmologic physics and quantum physics is a contin-

uing incomplete process. Many different models have been proposed, but none have seamlessly united quantum, classical and cosmologic phenomena [7]. Neutron, n^0, neutron stars, NS, and black holes, BH, are ideal conceptual, computational, and observational settings to interrogate gravitational properties at the extremes of short and long distances, and high gravitational binding energies, *GBE*s [8]-[14]. Many new observations are being acquired that lend themselves to interpretations of the basic properties and scaling of gravitational systems. Recent gravity wave measurements are dependent upon a robust gravity model [15] [16].

A NS is related to the transformation of atomic matter, in the simplest case hydrogen, *H*, to a degenerate n^0 state driven by gravity. NSs can be characterized by a few physical factors since they are composed predominately of neutrons, and these are referred to as equations of state [17] [18]. Some theoretical studies consider NSs as pure n^0 matter [19]. Some models describe the core as superfluid neutron-degenerate matter (mostly neutrons, with some protons and electrons). About 5% of all known NSs are in binary systems [20]. In some theories of binary evolution it is expected that NSs also exist in binary systems with a BH. It has been proposed that coalescence of binaries consisting of two NSs may be responsible for producing short gamma-ray bursts [21].

A BH is a mathematically defined region of spacetime exhibiting such a powerful gravitational force that no particle or electromagnetic radiation can escape. The classic Schwarzschild radius equation states that a BH arises when the annihilation energy of the total mass with a Schwarzschild radius of r_s, $(\text{mass}_{BH\_rs})c^2$ in joules, equals two times *GBE* in joules for any BH horizon measuring r_s (see Equation (1) and Equation (2)). From a quantum perspective of a pure binary NS system transformation to a BH, Equation (2) is a mathematical imperative expressing the equivalence between the integer number of neutrons defining the annihilation mass and the paired masses defining the *GBE* [22] [23].

$$r_s = \frac{2G\left(\text{mass}_{BH\_r_s}\right)}{c^2} \tag{1a}$$

or

$$\left(\text{mass}_{BH\_r_s}\right) = \frac{r_s c^2}{2G} \tag{1b}$$

$$\begin{aligned} \text{mass}_{BH\_r_s} \times c^2 &= \frac{r_s c^4}{2G} = \text{annihilation\_}E_{BH_{rs}} J = 2\left(GBE_{BH_{rs}}\right) \\ &= 2\frac{G\times\left(\text{mass}_{BH\_r_s}\right)^2}{r_s} = 2\frac{G\times\left(\frac{r_s c^2}{2G}\right)^2}{r_s} \end{aligned} \tag{2}$$

This is a specialized physical/mathematical setting where two entities, annihilation energy of a mass and a different form of energy, two times of the binding energy in this case, must be identically scaled in a paired relationship. It leads to a dimensionless quantum number of neutrons relationship defining

both simultaneously. Another similar physical single and paired example would be related to matter/anti-matter pair production which is also important in the properties of BHs [24]. In pair product there are photon and two anti-particles in which the total energies are scaled equally. In anti-matter matter annihilation there are two matter entities and two photons. The number 2 reflects the paired physical state relationship. Both are associated with physical transformation settings where the velocity of matter is related to the speed of light, c, and the annihilation energy of the rest mass, mc^2. In the transformation of a binary system of NSs to a BH, the *GBE* scaling can be defined equivalently and solely in integer multiples of the energy of n^0, equaling 1.5053×10^{-10} J, which is a natural energy unit. This energy unit could be also expressed in term of its equivalent in frequency, ν_{n^0}, (by dividing the energy by Planck's constant h) or in terms of n^0 Compton wavelength, c/ν_{n^0}. Therefore gravity can be scaled as a single natural unit system that is inter-related by integer multiples or integer fractions of n^0 units.

The goal of this paper is to evaluate the specific units from a quantum integer perspective of a natural scaling based solely on the neutron, which define a system of gravity that is conceptually and computationally consistent with standard gravitation models and methods for NSs and BHs.

2. Virtual Neutron Gravity Energy Unit, GBE_{n0_unit}

We define the simplest virtual n^0 based natural unit gravitational system, GBE_{n0_unit}, as the *GBE* of virtual paired n^0 masses separated by λ_{n^0}, a distance that is equal to the speed of light divided by the annihilation frequency of the neutron, ν_{n^0}. The *GBE* in Joules of this system, GBE_{n0_unit} J, 1.4188×10^{-49} J, is described in Equation (3).

$$GBE_{n^0\_unit}\ \mathrm{J} = \frac{G\left(m_{n^0}\right)^2}{\lambda_{n^0}}\ \mathrm{J} = \frac{G\left(\nu_{n^0}\right)\left(m_{n^0}\right)^2}{c}\ \mathrm{J} = 1.4188\times10^{-49}\ \mathrm{J} \qquad (3)$$

Here, m_{n^0} denotes the mass of the neutron, 1.6749×10^{-27} kg, G, the universal Newtonian gravitational constant, and c the speed of light. ν_{n0} represents the frequency equivalent of the neutron, 2.2719×10^{23} Hz, $\left(m_{n^0}c^2\right)/h$. λ_{n^0} is the n^0 Compton wavelength 1.3196×10^{-15} m. $GBE_{n^0\_unit}$ values in other physical unit equivalents include frequency 2.1413×10^{-16} Hz ($\nu_{GBE\_n^0\_unit}$), Compton wavelength 1.4001×10^{24} m, and mass 1.5787×10^{-66} kg.

3. Virtual Neutron Gravity Dimensionless Coupling Proportionality Constant Unit, $k_{n^0\_unit}$

GBE_{n0_unit} J is related to a proposed dimensionless coupling proportionality constant ratio with the n^0_unit in J, with $GBE_{n^0\_unit}\ \mathrm{J}/n^0\ \mathrm{J}$ equaling 9.4252×10^{-40} from Equation (4) in multiple physical units. Here n^0 J represents the Joule equivalent of n^0, 1.5053×10^{-10} J. This proportionality constant is referred to as $k_{n^0\_unit}$, where $k_{n^0\_unit}$ times any physical unit value of the neutron equals $GBE_{n^0\_unit}$ in the corresponding unit.

$$\left(k_{n^0\_\text{unit}}\right)=\left(\frac{GBE_{n^0\_\text{unit}}\ \text{J}}{n^0\ \text{J}}\right)=\left(\frac{GBE_{n^0\_\text{unit}}\ \text{J}}{h\left(\nu_{n^0}\right)}\right)$$
$$=\left(\frac{\nu_{GBE_{n^0\_\text{unit}}}\ \text{Hz}}{\nu_{n^0}\ \text{Hz}}\right)=\left(\frac{GBE_{n^0\_\text{unit}}\ \text{kg}}{m_{n^0}\ \text{kg}}\right)=9.4252\times10^{-40} \tag{4}$$

4. $k_{n^0\_\text{unit}}$, Planck Time, and Hydrogen

Our proportionality constant, k_{n0_unit} is also defined by several equivalent relationships involving GBE_{n0_unit}; Planck's constant, h; Planck time, t_P; and the frequency equivalents of the neutron, proton (p+), electron (e^-), Bohr radius (a_0), and GBE of the electron in hydrogen; respectively ν_{n0}, ν_{p+}, ν_{e-}, ν_{a_0}, ν_{GBE_e} in Equation (5). The ν_{GBE_e} is derived and valid identical to the Bohr radius derivation. These are also natural unit constants. One is defined by the degenerate neutron state and the other equivalently by the hydrogen state that is non-degenerate. Additionally, the negative n^0 beta decay products represent the closest natural gravitational system to the virtual pure n^0 system. Thus we find that the n^0 serves as a fundamental unit for the origin of all of the hydrogen products as well. The product of t_P^2, the frequency of one mass, the frequency of the other mass, the frequency of the distance equals the GBE in Hz. In the neutron unit setting the k_{n0_unit} is related to the product of t_P^2 and $\nu_{n^0}^3$, divided by ν_{n0}, therefore the product of t_P^2 and $\nu_{n^0}^2$, Equation (5).

$$k_{n^0\_\text{unit}}=\left(\frac{\nu_{GBE_{n^0\_\text{unit}}}\ \text{Hz}}{\nu_{n^0}\ \text{Hz}}\right)=\left(\frac{\left(\nu_{n^0}\right)^3}{\nu_{n^0}}\right)\left(\frac{\nu_{GBE\_e^-}}{\nu_{p+}\nu_{e^-}\nu_{a_o}}\right)$$
$$=\left(\frac{\left(\nu_{n^0}\right)^2\nu_{GBE\_e^-}}{\nu_{p+}\nu_{e^-}\nu_{a_o}}\right)=\left(\nu_{n^0}t_P\right)^2=9.4252\times10^{-40} \tag{5}$$

From Equation (5) the ratio of $\nu_{GBE\_n0\_\text{unit}}$ divided by $\nu_{GBE\_e^-}$ is related to the ratio of quantum entities of ν_{n^0} cubed divided by the product of ν_{p+}, ν_{e-}, and ν_{a_0}, 7.3836×10^7, Equation (6).

$$\frac{\nu_{GBE_{n^0\_\text{unit}}}}{\nu_{GBE\_e^-}}=\left(\frac{\left(\nu_{n^0}\right)^3}{\nu_{p+}\nu_{e^-}\nu_{a_o}}\right)=7.3836\times10^7 \tag{6}$$

5. Proposal of a Gravitational Constant Analogous of the Fine Structure Constant, α_G

We also find interest in computing the square root of $2k_{n^0\_\text{unit}}$, 4.3417×10^{-20}, a dimensionless constant similar to the fine structure constant, α, in both concept and computation, but for gravitational, non-electromagnetic physical systems. It is described herein as α_G, Equation (7). The fine structure constant α is related to the electromagnetic binding energy of the electron in H, and involves p+, e^-, and a_0. Additionally α_G represents the ratio of a velocity divided by c that is also a β value from Special Relativity. This is described as β_G for this specific setting, just

as α represents a specific β for hydrogen. The mass of the electron times the product of $\alpha \times c$ squared divided by two is equal to the ionization binding energy of hydrogen, which is the Rydberg constant equivalent. The kinetic energy and the binding energy are similar in concept to the transformation point to a BH in Equation (2). An identical gravitational mathematical property will be shown later in Equation (8).

$$\begin{aligned}\alpha_G = \beta_G &= \sqrt{2k_{n^0\_\text{unit}}} = \sqrt{2}\nu_{n^0} t_P = \sqrt{\left(\frac{2\nu_{GBE_{n^0\_\text{unit}}}\ \text{Hz}}{\nu_{n^0}\ \text{Hz}}\right)} \\ &= \nu_{n^0}\sqrt{\left(\frac{2\nu_{GBE\_e^-}}{\nu_{p^+}\nu_{e^-}\nu_{a_o}}\right)} = \nu_{n^0}\sqrt{\left(\frac{2\nu_{GBE_{n^0\_\text{unit}}}}{(\nu_{n^0})^3}\right)} = 4.3417 \times 10^{-20}\end{aligned} \tag{7}$$

The kinetic energy of this virtual n^0-system is the mass of n^0 times the product of $(\alpha_G \times c)$ squared divided by 2. This equals $GBE_{n^0\_\text{unit}}$, Equation (8). Thus we find the Gravitational Binding energy neutron unit exactly equals the kinetic energy of the binary neutron pair identical to the pattern seen with one definition of α. This pure n^0-based unit gravitational system is a virtual system because it does not have a significantly long lifetime. Our derived constant α_G equals the square root of the product of 2 and ν_{n^0} squared times the frequency equivalent of the *GBE* of the electron in hydrogen, $\nu_{GBE\_e^-}$ divided by the product of ν_{p^+}, ν_{e^-}, and ν_{a_0} from Equation (7). This unites gravitational scaling and multiple other fundamental quantum entities.

$$GBE_{n^0\_\text{unit}}\ \text{J} = \frac{G\left(m_{n^0}\right)^2}{\lambda_{n^0}}\ \text{J} = \frac{G\left(m_{n^0}\right)^2 \nu_{n^0}}{c}\ \text{J} = m_{n^0}\left(\frac{\left(c\alpha_G\right)^2}{2}\right)\ \text{J} \tag{8}$$

The mathematical connection with the closest natural atomic system, which represents hydrogen, shows that ν_{GBE_e} is a fundamental constant with similar importance to quantum gravity, and is analogous to the significance of the electromagnetic binding energy of the electron in H to photon and electromagnetic systems. This constant cannot be measured, but it does not diminish its physical significant. In fact, the Bohr radius, rest mass of the electron, and the Rydberg constant that define the hydrogen quanta cannot be experimentally measured either. This fundamental frequency $\nu_{GBE\_e^-}$ equals, 2.9000×10^{-24} Hz; equivalently 2.1381×10^{-74} kg; 1.0337×10^{32} m; or 1.9216×10^{-57} J. The product of ν_{n^0} and $\nu_{GBE\_e^-}$ equals 0.65885 Hz^2 so $\nu_{GBE\_e^-}$ is closely scaled by the mathematical scalar reciprocal of ν_{n^0}. Whence $\nu_{GBE\_e^-}$ divided by the product of ν_{p^+}, ν_{e^-}, and ν_{a_0} also equals the fundamental gravitational, quantum, and relativistic constant Planck time squared, t_P^2. Planck time t_P ($\hbar$ not the reduced Planck's constant, $\hbar$) equals $1.35141(08) \times 10^{-43}$ s. All frequency equivalent gravitational systems fulfill the relationship of the t_P^2 ratio including that of hydrogen and $\nu_{GBE\_n0\_\text{unit}}$, Equation (7).

The value of α_G equals the product of square root of 2, ν_{n^0}, and t_P as derived in Equation (7). Thus α_G is inter-related with the quantum gravitational property of H in an analogous fashion as α is inter-related to both the quantum and elec-

tromagnetic properties of H. Now GBE_{n0_unit} can be calculated from ratio/product relationships of G, c, m_{n^0}, v_{n^0}, α_G, λ_{n^0} as seen in Equation (8).

6. Definition of the Newtonian Gravitation Constant in Neutron Units

As an added and quite surprising result the Newtonian gravitational constant G can be completely defined in the defined units of the n^0 mass, its Compton wavelength, its unit GBE; or with c and neutron frequency equivalent as in Equation (9).

$$G = \frac{\lambda_{n^0} GBE_{n^0\_\text{unit}}}{\left(m_{n^0}\right)^2} = \frac{cGBE_{n^0\_\text{unit}}}{v_{n^0}\left(m_{n^0}\right)^2} \tag{9}$$

Equivalently, the Newtonian gravitational constant G can also be defined in units of n^0, c, h, t_P, v_{P+}, v_{e-}, v_{a0}, v_{GBE_e-},and neutron beta decay products in H as shown in Equation (10). This is analogous to the inter-relationship of multiple fundamental quantum constants that can define a_0 or α.

$$\begin{aligned} G &= \frac{c^3\left(\alpha_G\right)^2}{2\left(m_{n^0}\right)\left(v_{n^0}\right)} = \frac{\left(c\alpha_G\right)^2 \lambda_{n^0}}{2\left(m_{n^0}\right)} = \frac{c^3\left(k_{n^0\_\text{unit}}\right)}{\left(m_{n^0}\right)\left(v_{n^0}\right)} \\ &= \frac{c^3 v_{n^0}\left(t_P\right)^2}{m_{n^0}} = \left(\frac{v_{GBE\_e-}}{v_{p+} v_e v_{a_0}}\right)\left(\frac{c^3 v_{n^0}}{m_{n^0}}\right) = t_P^2\left(\frac{c^5}{h}\right) \end{aligned} \tag{10}$$

7. Gravitational Binding Energy of Any System from an Integer Neutron Perspective

The GBE can be calculated for any system using the integer unit n^0 perspective by Equation (11). mass_1/m_{n0} is the integer number of equivalent neutrons of one mass, $\#1n^0$. Also mass_2/m_{n^0} equals the integer number of neutron equivalences of the second mass, $\#2n^0$. We define $\#\lambda_{n0}$ as the integer number of n^0 Compton wavelengths of the masses separation distance, λ. All of the # values are dimensionless integers.

$$GBE = GBE_{n^0\_\text{unit}}\left(\frac{\lambda_{n^0}}{\lambda}\right)\left(\frac{\text{mass}_1}{m_{n^0}}\right)\left(\frac{\text{mass}_2}{m_{n^0}}\right) = GBE_{n^0\_\text{unit}}\left(\frac{\left(\#1n^0\right)\left(\#2n^0\right)}{\#\lambda_{n^0}}\right) \text{ J} \tag{11}$$

8. An Example of the Transition of a Symmetric Binary Pure Neutron NS System to a BH

Consequently we have just established the transformation of binary symmetric NSs of identical mass, merging into a BH as in Figure 1. r_s is arbitrarily chosen to be $c \times s$ for mathematical simplicity, and it is a fundamental unit distance linked to all of the other constants. A Schwarzschild radius, r_s, of $c \times s$ equals 299,792,458 m is utilized to demonstrate a relevant cosmological dimension in the Schwarzschild ratios shown in Equation (12). The mass of such a BH equals the product of c^2 times $c \times s$ divided by the product of 2 times the gravitational

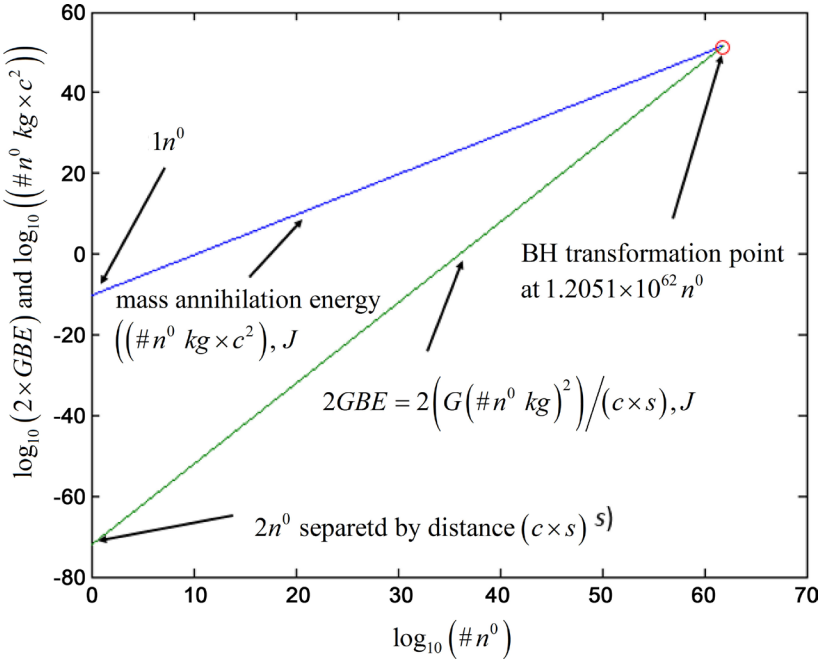

Figure 1. Transformation of a symmetric binary neutron star system to a black hole. **Figure 1** is a plot showing the transformation of the binary symmetric neutron system to a black hole with a Schwarzschild radius of $c \times s$. The *X*-axis is $\log_{10}$ # of n^0. The *Y*-axis is in $\log_{10}$ Joules of the annihilation energy of the # of n^0, the upper blue line; and 2 times the gravitational binding energy, *GBE*, of a system with # of n^0 separated at a distance of $c \times s$, the lower green line, Equation (2). The far left side of the plot begins at a binary single n^0 system. To the right the integer number of n^0 of each component of the pair increases towards 1.2051×10^{62} neutrons. At that specific # of n^0, red circle, 2 times the *GBE* and the annihilation energy of the mass are equal leading to the transformation to a BH.

constant *G*, which computes to a total mass of 2.0186×10^{35} kg from Equation (12). This is described as $\text{mass}_{BH\_c\times s}$. Other unit equivalents of $\text{mass}_{BH\_c\times s}$ are: 1.8141×10^{52} J; frequency of 2.7379×10^{85} Hz; Compton wavelength of 1.0950×10^{-77} m; and integer number of neutrons $1.2051 \times 10^{62} n^0$. We evaluate the *GBE* of a paired neutron system starting with 1 n^0 to 1.2051×10^{62} n^0, or 2.0186×10^{35} kg for the mass. The product of 2, the mass squared and *G* divided by $c \times s$ equals twice the *GBE* of each individual incremental pair neutron systems, Equation (13).

$$\frac{c^3 \times s}{2G} = \text{mass}_{BH_{c\times s}} = 2.0185\times10^{35}\ \text{kg} \tag{12}$$

$$\text{mass}_{BH_{c\times s}} \times c^2 = \frac{2\times G\times\left(\text{mass}_{BH_{c\times s}}\right)^2}{c\times s} \tag{13}$$

The factor 2 is taken into account in these calculations to define the point of transition of the binary n^0 masses to a BH. Since the *GBE* increases with the square of the number of n^0 there is a point where the two lines and energy values converge, **Figure 1**. This is the transformation point to a *BH*. At neutron star pairing with a mass of 2.0185×10^{35} kg 2 times the *GBE* equals the annihilation energy of the mass fulfilling the mathematical definition of a BH, Equations ((2),

(13)). This demonstrates that binary gravitational systems from any pairing of two neutrons, two stars, and two NS, and one BH can be described by a quantum n^0-based system.

9. Discussion

These results show how gravitation systems, indeed the Newtonian Gravitational constant itself, can be completely scaled by a quantum or n^0-unit system (see Equations ((9)-(11)).) This is an imperative of the mathematical definitions of pure NSs transforming to simplest non-rotating BHs. The *GBE* of any system represents dimensionless integer fractional values of the neutron as the base unit, which provides dimensionless ratio coupling constants defined by k_{n0_unit} and α_G. Surprisingly, these are related to quantum subatomic constants derived from the frequency equivalents of the neutron, hydrogen, and t_P, (Equation (10)). This demonstrates a system that bridges from quantum to gravitational. Similarly both α and α_G represent conceptually and computational parallel β values, but are relegated to widely divergent physical settings. The fine structure constant α relates to electromagnetic relationships of hydrogen, whereas α_G is for neutron gravitational relationships. The electromagnetic binding energy of the electron in hydrogen, related to the Rydberg constant, is a fundamental scaling factor for all electromagnetic relationships including α. The *GBE* of the electron in hydrogen plays an identical role for neutron quantum gravity. Though our derived *GBE_e*-value is not directly observable it still represents an important fundamental inter-relational constant identical in concept to the other hydrogen quanta.

The time and distance units of gravity may also be defined by n^0-equivalents. The time unit is one second divided by $v_{n^0}s$, 4.4017×10^{-24} s. The distance unit is λ_{n0}, c/v_{n^0} equals 1.3196×10^{-15} m. Any gravitational system can be interrogated where time, distance, matter, and energy can all be evaluated as one of these quantum states. Likewise in quantum physics, where there are only specific quantum states possible, the only possible gravitational states are related to integral numbers of neutrons for the masses and λ_{n^0} for the distances in a purely n^0-unit gravitational system, Equation (11). Clearly, all possible GBEs are also only integer or integer fraction values of $GBE_{n^0\_\text{unit}}$. Perhaps the quantization of the gravitational field may be discovered in our approach.

10. Conclusions

In conclusion, mathematical definitions of pure Neutron Stars and Black Holes define an imperative that the scaling of the gravitational binding energy must be defined in quantum unit equivalents of the neutron. The dimensionless ratios, k_{n0_unit} and α_G, of a pure neutron gravitational system, are defined by four quantum entities, and the n^0, the speed of light, the products of negative n^0 beta decay, p^+, e^-, a_0, $v_{GBE\_e^-}$, as in Equations ((5), (7)). Newton's Gravitational constant G, can be defined solely in n^0-units and/or the speed of light, t_P, and hydrogen quantum values as in Equations ((9), (10)). Therefore gravity can be viewed as

being scaled by the neutron which is an imperative in the NS BH transition. Time (chronon), matter (second quantization), distance (Planck scales), and energy (Rydberg spacing) also must be quantum integer based on n^0. Since all of the dimensions of a gravitational system are scaled by a single natural unit, the neutron, the system can be computationally analyzed as dimensionless inter-relationships just as electromagnetic systems are with the fine structure constant as in Equation (11).

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