Hubble's Constant

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Abstract

An analytical method to calculate Hubble's constant [1] is presented. The proposed procedure is an alternative scheme to the red shifts of spectral lines picture, to obtain the value of that constant [2].

Keywords

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1. Introduction

The most obvious feature of the Big Bang cosmological model [3] [4] is its statement that the Cosmos began at some definite past time; in such a way that the expansion rate determines the age of the Universe. *Hubble's* constant measures how fast is the process of the expansion, and it is involved in Hubble's law. The larger the Hubble's constant, the faster the expansion rate.

Also, *Hubble's* constant is a measure of the age of the Universe. In fact, if it is large, the expansion proceeds rapidly and the Universe is young; and if it is small, the expansion is slow and the Universe is old. Such a statement assumes that there have been no changes in the expansion rate.

The age of the Universe obtained in this way is called *Hubble's* time. On the other hand, *Hubble's* constant is also used in many other areas of extragalactic astronomy; for example, to estimate the distances to quasars. This way, its exact value is very important for cosmology.

Hubble's constant is usually expressed in units of kilometers per second per megaparsec.

In principle, the measure of *Hubble's* constant is quite easy, if the picture of the red shifts of the spectral lines is used. The procedure is as follows.

Find a galaxy, measure its velocity of recession looking for a Doppler shifts in its spectrum, measure its distance, and Hubble's constant is obtained as the velocity of recession divided by the distance.

But, only measure accurate distances can be done for nearby galaxies where individual stars can be seen. However, the problem is that the random motions of these galaxies through space may interfere with the expan-

sion of the Universe.

Then, it is necessary to explore regions several tens or even hundreds of mega parsecs distant from the Earth, in order to be sure that we are looking at the expanding Universe and not just some local motion.

Nevertheless, at such great distances it is impossible to distinguish individual stars inside the extragalactic galaxies. To accomplish the basic activity of measure the distance to an extragalactic galaxy, next the following procedure is developed.

2. Distances, Total Energy, and Velocities of Recession

Cosmological distances can be measured using a similar scheme to the one used to calculate stellar distances. The methodology is as follows [5].

Looking for any cluster of galaxies, as the Virgo cluster, which is the nearest rich cluster of galaxies, and use one of the galaxies as a measured unit in order to obtain the distances of other extragalactic clusters of galaxies.

In this way, the Astronomers G. Abell and J. Eastmand compared the Virgo cluster with two similar clusters at a great distances: The Coma cluster, and the Corona Borealis cluster.

On the other hand, the quasars could be considered as Standard Candles, assuming that their luminosity is the same everywhere in the Universe. The apparent brightness of such objects would thus be a good indicator of their distance.

Thus, let us consider any galaxy from the Coma or the Corona Borealis clusters, from which it is possible to know their luminosity $\ell(r)$, and their actual distance r, in such a way that the astronomical system selected be the measured unit.

Next, take another extra-galactic body, as a quasar for example, which is far away from the Earth at a distances r_*, and for which it is also possible to obtain their luminosity $\ell_*(r_*)$.

When a star moves farther and farther from the Earth, it becomes fainter and fainter according to an exact mathematical law. Then, and as it is well know, the light from an object varies as $1/d^2$, where d is the distance, so that, it is easy to obtain for the present case that

$$\frac{\ell(r)}{\ell_*(r_*)} = \left(\frac{r_*}{r}\right)^2 \tag{1}$$

From this relationship the following result is obtained

$$\frac{1}{r_*} = \frac{1}{r}\left(\frac{\ell_*}{\ell}\right)^{1/2} \tag{2}$$

where r is the distance in mega parsecs to the measured unit, and r_* is the distance in mega parsecs too, to the quasar that moves at the velocity of recession v_*; whose value can be obtained from the following relativistic transformation equation

$$E_* = \frac{m_o\,c^2}{\sqrt{1 - \dfrac{v_*^2}{c^2}}} \tag{3}$$

where m_{o*} is the quasar's *proper mass*, and c the velocity of light in the empty space. Since it is absurd for anybody with finite proper mass to have infinite energy, it must conclude that it is impossible for such a body to move with the velocity of light in vacuum. Thus, $v_* < c$ always. From the former equation the following is obtained

$$v_*^2 = c^2\left(1 - \frac{E_{o*}^2}{E_*^2}\right)$$

and then

$$v_* = c\left(1 - \frac{E_{o*}^2}{E_*^2}\right)^{1/2} \tag{4}$$

where $E_{o*} = m_{o*}c^2$ is the quasar's rest energy. Since $v_* < $ c always, it is proposed as a work hypothesis that

$$\left(1 - \frac{E_{o*}^2}{E_*^2}\right)^{1/2} = 0.1457 \tag{5}$$

From this condition, it is easy to see that in the Equation (4) the following result is obtained

$$v_* = 0.1457c \tag{6}$$

3. Hubble's Constant Value

In 1920-1929 decade it was discovered that as a consequence of the evolutionary phenomenon known as the *Expanding of the Universe*, the galaxies and the other bodies are all moving away from each other, according to an exact relationship.

In fact, the velocity of recession and the distance are correlated. The larger the distance, the greater the velocity. This relationship was discovered by Hubble and Humanson in the 1930's, and it is known as *Hubble's law*.

This relation can be written as follows: *Velocity of recession equals Hubble's constant times distance*; and it is also known as *the law of red shifts*.

In the present case, we have that $v_* = Hr_*$, with H Hubble's constant. Let us consider now the results (2) and (5), in such a way that

$$H = \frac{A}{r\,\ell^{1/2}} \tag{7}$$

where

$$A = 0.1457c\ell_*^{1/2} \tag{8}$$

is a constant.

Clearly, $c = 3 \times 10^5$ km·sec^{-1} is the value of the velocity of the light in vacuum, and *Hubble's* constant is expressed in units of *kilometers per second per megaparsec*.

Hubble's constant measures how fast the *Universe* is expanding. The larger *Hubble's constant*, the faster the expansion rate. Also, it is a measure of the age of the *Universe*.

Nevertheless, *Hubble's constant* is only meaningful on a very large scale. Then, it is necessary to select as a standard candle some class of astronomical system, hundreds of megapasecs distant, because at such a distance the universal expansion will overcome any local motion.

The former expression is given in terms of quantities which, in principle, can be obtained from the observation. Thus, this is another way to calculate *Hubble's constant*, without the use of the red shifts of the spectral lines picture.

4. Conclusions

In a uniform expanding *Universe*, the position r and the velocity v of any astronomical body relative to another, obey *Hubble'law* $v = Hr$, where H is *Hubble's constant*.

As cosmological distances are measured in megaparsecs, the natural unit for H is kilometers per second per megaparsec.

However, the *real Universe* is non uniform on a small scale, and its motion obeys *Hubble's law* only as a large scale average; in such a way that *Hubble's constant* is only meaningful on a very large scale. Moreover, the distances in the *Universe* are so vast that they cannot be measured directly.

The only way to allow observations to enter in the scheme is to suppose that there are some selected uniform types of astronomical systems called *Standard Candles*, which are systems whose luminosity is the same everywhere in the *Universe*. Their distances can be estimated from their apparent brightness, or visual magnitude.

Anyway, using a galaxy of any cluster of galaxies hundred of megaparsecs distant as a measured unit, and a quasar as an extremely distant cosmological object, as a standard candle, it is possible to propose an alternative methodology to the red shifts of spectral lines method, to obtain the value of *Hubble's constant*.

Finally, if any of the galaxies of the *Virgo cluster* is used as a previous step, it is possible to compare the *Virgo cluster* with to similar clusters of galaxies at a great distance from the *Earth*: The *Coma cluster*, and the *Corona Borealis cluster*.

Since all these clusters are of the same type, it is reasonable to assume that the absolute magnitudes of galaxies in the two more distant clusters would be the same as the absolute magnitude of comparable galaxies in the *Virgo cluster*.

The brightest galaxy in *Coma* should have the same absolute magnitude as the brightest galaxy in *Virgo*, the next brightest galaxy should be equally comparable, and so on. The same is true for the galaxies in the *Corona Borealis cluster*.

This way, it was found distances of 130 *mega parsecs* to the *Coma cluster* and 410 mega parsecs to *Corona Borealis cluster*.

The *Virgo cluster's* brightest member is one of the nearest giant elliptical galaxies, *Messier* 87; from which we have the following data

$$\ell = 2.92 \times 10^{45} \, \text{ergs} \cdot \text{sec}^{-1}$$

$$r = 23.9 \text{ Megaparsecs}$$

Using this galaxy as a previous step, and the cosmological object *3C*273, which is the brightest *quasar* known, as a *standard candle*; from the Equation (1), and taking into account that it is 871 *Mega parsecs* distant, we have that

$$\ell_* = 2.2 \times 10^{42} \, \text{ergs} \cdot \text{sec}^{-1}$$

and then

$$A = 6.48 \times 10^{25} \, \text{km} \cdot \text{ergs} \cdot \text{sec}^{-2}$$

in such a way that in the Equation (6), the following final result is obtained

$$H = \frac{6.48 \times 10^{25}}{r \, \ell^{1/2}} \, \text{km} \cdot \text{sec}^{-1} \cdot \text{mpc}^{-1} \qquad (9)$$

As an illustrative first example, let's consider the brightest galaxy of the *Coma cluster*, 130 *megaparsecs* distant, as a measure unit, and the *quasar* 3C273 as *standard candle*. From the Equation (1), and with the help of the previous data, it is easy to see that

$$\ell_1 = 9.88 \times 10^{43} \, \text{ergs} \cdot \text{sec}^{-1}$$

Also, as a second example, consider the brisghtest galaxy of the *Corona Borealis cluster*, 410 *megaparsecs* distant. Then, using the previous data of *Coma*, and the Equation (1), the following result is obtained

$$\ell_2 = 9.93 \times 10^{42} \, \text{ergs} \cdot \text{sec}^{-1}$$

With the help of the data before obtained for the *Coma cluster*, and also, for the *Corona Borealis Cluster*, in the Equation (9), a value of 50.2 *kilometers per second per megaparsec* is obtained in each case; which is the exact value of *Hubble's constant*.

Finally, and given that the expansion rate determines the *age of the Universe*, and *Hubble's constant* measures how fast the *Universe* is expanding, it is easy to see that

$$\tau = \frac{1}{H} \qquad (10)$$

is the age of the *Universe* or also, the *Hubble time*. Thus, and according to the numerical value of *H* before obtained,

$$\tau = \frac{1}{50.2} \frac{\text{sec} \cdot \text{mpc}}{\text{km}} \qquad (11)$$

Now,

$$1 \text{ megaparsec} = 3.085 \times 10^{19} \, \text{km}$$

$$1 \text{ second} = 0.32 \times 10^{-7} \text{ years}$$

so that

$$\tau = 19,665 \times 10^6 \text{ years} \tag{12}$$

and this is the age of the *Universe* at present.

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