

# On the Walker' Model for the Carbon Dioxide in the Earth's Atmosphere

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## Abstract

Climate change is a wicked problem because it is hard to say what the problem is, and to define it clearly. However, we know that global temperature rise correlates with increasing levels of atmospheric carbon dioxide [1] and [2]. In this paper, we analyze a model for the carbon dioxide developed by Walker in [3] with several source terms. Our numerical results show that the burning fossil fuels have an effect on the carbon dioxide in the earth's atmosphere and the climate change problem, one of the major global challenger of our time.

## Keywords

Carbon Dioxide, Earth's Atmosphere

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*The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble.*

P. A. M. Dirac (1929)

## 1. Introduction

All of us know that Mathematics and Physics have traditionally shared their interests and developments; many are the historical examples in which physical phenomena have motivated original mathematical theories and vice versa. On the contrary, the connection between Mathematics and Life Sciences (Ecology, Zoology, Physiology...) have been rather scarce, perhaps because they have kept themselves in a prudently descriptive. This does not mind that biologists have ignored the mathematical progresses (Biostatistics is a common subject in all the curricula of the university degrees in Biology), but if we do not consider this tool

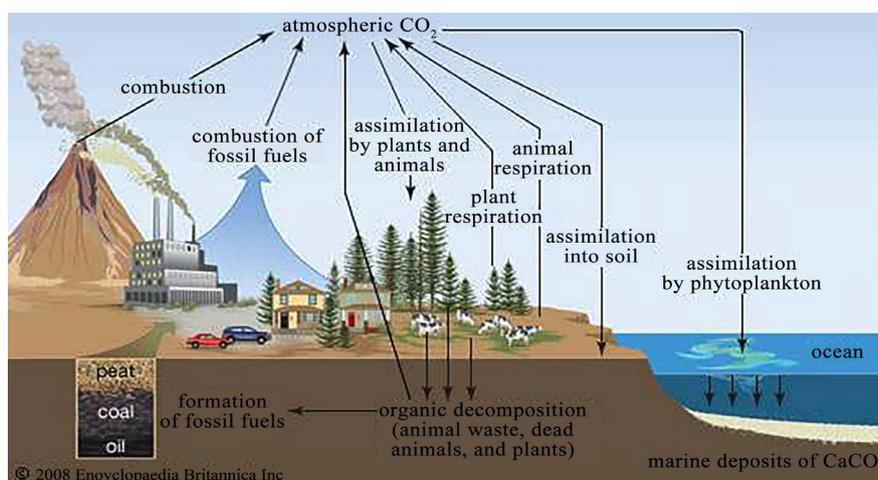
to arrange laboratory data or samples, it was very scarce connection between Mathematics and Biology until a very few years ago. A proof of this affirmation is that the main scientific paper which deals with this topic, Journal of Mathematical Biology appeared in 1974, and the development of Computation Sciences allows to study the planet Earth using the *universal trilogy: modeling, analysis and control* proposed by Prof. J.L. Lions in [4]. In fact, the application of Mathematics in Life Sciences is a speciality which is growing at a spectacular pace. Besides the classical [5] and [6], more recent texts, for example [7]-[21] explain other models in Physiology, Genetic, Cellular Biology, Climatology ... or [22], [23] and [24] develop stochastic differential equation models applied in Epidemiology, Predator-Prey Problems, Chemical Reactions ... and even [25] uses Variational Methods in Molecular Modeling.

The carbon dioxide ( $\text{CO}_2$ ) is a colorless and odorless greenhouse effect gas which affects climate because it traps the heat that otherwise would dissipate to the space; thus, it is directly associated with the Earth's temperature. In other dead planets, the whole atmosphere is almost exclusively composed by  $\text{CO}_2$ ; for instance, Venus' atmosphere consists in a 98% of that gas and the temperature on its surface reaches  $477^\circ\text{C}$ .

At the beginning of the XX century, three of every 10,000 earth's atmospheric molecules contain  $\text{CO}_2$ ; so one can say that, although it was not a very abundant gas, it was important because it did prevent our congelation. If the  $\text{CO}_2$  percentage in the atmosphere became twice as much, some scientists consider that the average temperature of our planet would go up by between  $3^\circ\text{C}$  and  $6^\circ\text{C}$ , with consequences not difficult to imagine.

The carbon dioxide cycle can be summarized in the following scheme call the biogeochemical cycle in **Figure 1**.

It is therefore evident that the burning fossil fuels has a effect on the carbon dioxide in the earth's atmosphere. Nowadays, the coal combustion power stations are the ones which produce the highest rate of  $\text{CO}_2$  by energy unity. In



**Figure 1.** Carbon cycle from Encyclopedia Britannica.

the data gathered in the March and April 2006 issues of the **National Geographic** magazine, the emissions of this greenhouse effect gas produced from other fuels as petroleum, natural gas, biomass ... are much smaller; in the USA, for instance, an 83% of the CO<sub>2</sub> emissions is calculated to come from the coal combustion power stations.

The situation becomes worrying when we know that, nowadays, the biggest electricity generation source is the coal combustion. The USA consume nowadays some 1094 million tons of coal which generate approximately the 50% of the electric energy they consume. Which happens in other parts of the world? Here the Asiatic giant does appear: China consumes about 1531 million tons of coal and its demand will grow more than twice as much towards the year 2025 in order to satisfy the industry and consumers. According to the sources commented above, worldwide coal consumption will increase by a 56% by the year 2025, with the resulting carbon dioxide emissions.

This paper is organized as follows. In Section 2 we describe the Walker' Model for Carbon Dioxide in the Earth's Atmosphere, in Section 3 we computed its numerical solutions and finally in Section 4 we draw the main conclusions.

Our numerical methods were implemented in Matlab, the codes that we have used are available on request. The experiments were carried out in an Intel(R) Xeon(R) CPU ES-2620 0 @ 2.00 GHz 16 GB of RAM.

## 2. The Walker' Model for Carbon Dioxide in the Earth's Atmosphere

The seawater has a much greater storage capacity for carbon dioxide than the atmosphere because this gas is contained in seawater in three different forms: as solute carbon dioxide gas (CO<sub>2</sub>), as hydrogen carbonate (HCO<sub>3</sub>) and as carbonate (CO<sub>3</sub>). When part of the CO<sub>2</sub> in the water is converted into HCO<sub>3</sub> and CO<sub>3</sub>, seawater has a much greater storage capacity for carbon dioxide than the atmosphere.

The model developed by J. C. G. Walker in [24] simulates the interaction between the exchange of the carbon dioxide stored in three media: atmosphere, the shallow ocean and the deep ocean. The model is also commented in reference [26].

The five variables of the model are time functions:

$p$  partial pressure of carbon dioxide in the atmosphere;

$\sigma_s$  total dissolved carbon concentration in the shallow ocean;

$\sigma_d$  total dissolved carbon concentration in the deep ocean;

$\alpha_s$  alkaline degree in the shallow ocean;

$\alpha_d$  alkaline degree in the deep ocean.

Other three quantities appear in the equilibrium equations in the the shallow ocean:

$h_s$  hydrogen carbonate concentration;

$c_s$  carbonate concentration;

$p_s$  partial pressure of gaseous carbon dioxide.

The rate of change of the five principal variables is given by five ordinary differential equations. The first one is the exchange equation of carbon between atmosphere and shallow ocean is

$$\frac{dp}{dt} = \frac{p_s - p}{d} + \frac{f(t)}{\mu_1}, \quad (1)$$

where  $d$  and  $\mu_1$  are two constants and  $f(t)$  is a source term which represents the combustion and emission of the gases to the atmosphere.

The other four differential equations which describe the exchanges between shallow and deep oceans are

$$\begin{aligned} \frac{d}{dt} \sigma_s &= \frac{1}{v_s} \left( (\sigma_d - \sigma_s) \omega - k_1 - \frac{p_s - p}{d} \mu_2 \right), \\ \frac{d}{dt} \sigma_d &= \frac{1}{v_d} (k_1 - (\sigma_d - \sigma_s) \omega), \\ \frac{d}{dt} \alpha_s &= \frac{1}{v_s} ((\alpha_d - \alpha_s) \omega - k_2), \\ \frac{d}{dt} \alpha_d &= \frac{1}{v_d} (k_2 - (\alpha_d - \alpha_s) \omega) \end{aligned} \quad (2)$$

and the algebraic equations for the equilibrium of  $\text{CO}_2$  between the atmosphere and the shallow ocean

$$\begin{aligned} h_s &= \frac{\sigma_s - \sqrt{\sigma_s^2 - k_3 \alpha_s (2\sigma_s - \alpha_s)}}{k_3}, \\ c_s &= \frac{\alpha_s - h_s}{2}, \\ p_s &= k_4 \frac{h_s^2}{c_s}. \end{aligned}$$

The numerical values of the model constants are

$$\begin{aligned} d &= 8.64, \\ \mu_1 &= 4.95 \times 10^2, \\ \mu_2 &= 4.95 \times 10^{-2}, \\ v_s &= 0.12, \\ v_d &= 1.23, \\ \omega &= 10^{-3}, \\ k_1 &= 2.19 \times 10^{-4}, \\ k_2 &= 6.12 \times 10^{-5}, \\ k_3 &= 0.997148, \\ k_4 &= 6.79 \times 10^{-2}. \end{aligned}$$

### 3. Numerical Results

The five differential Equations (1)-(2) cannot be solved in an exact way, so

approximate solutions are looked for. The numerical methods to solve ordinary differential equation systems have been greatly developed during the last years and nowadays there are very efficient implementations. In this paper, we analyze the commands of the MATLAB programming language which can be consulted in references [26] and [27]; for deeper details about these methods and their implementations, we recommend reference [28].

The initial conditions will represent the preindustrial equilibria; bearing in mind that the first relevant coal mines in Great Britain correspond to the 1830 decade, we will suppose that the values that the variable took in that year were

$$\begin{aligned} p &= 1.00, \\ \sigma_s &= 2.01, \\ \sigma_d &= 2.23, \\ \alpha_s &= 2.20, \\ \alpha_d &= 2.26. \end{aligned}$$

As far as the source  $f(t)$  is concerned, we will use the data appeared in the April 2006 issue of the *National Geographic* magazine which indicate the millions of tons of CO<sub>2</sub> from the electricity production, summarized in the following table:

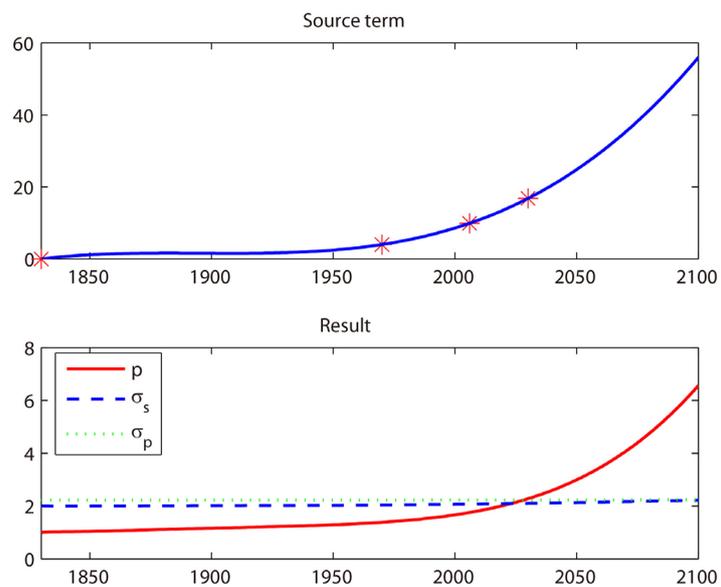
Year	1830	1970	2006	2030
Millions of tons	0	4.000	9.900	16.800

The first situation studied corresponds to a source term represented in the upper part of **Figure 2**, where we have calculated the third degree polynomial interpolating the four points of the former table. The numerical results obtained, which are represented on the lower part of the same in **Figure 2**, say that, by the end of this century, the variable  $p$  associated to the concentration of CO<sub>2</sub> in the atmosphere will have increased considerably, while the concentrations in the oceans would grow very little. The other two variables  $\alpha_s$  and  $\alpha_p$  are not represented because they have scarcely moved from their initial value. The evident conclusion of this experiment is that with the current growth model, the planet Earth would suffer a rise in temperature on the Earth surface difficult to specify but with a high risk.

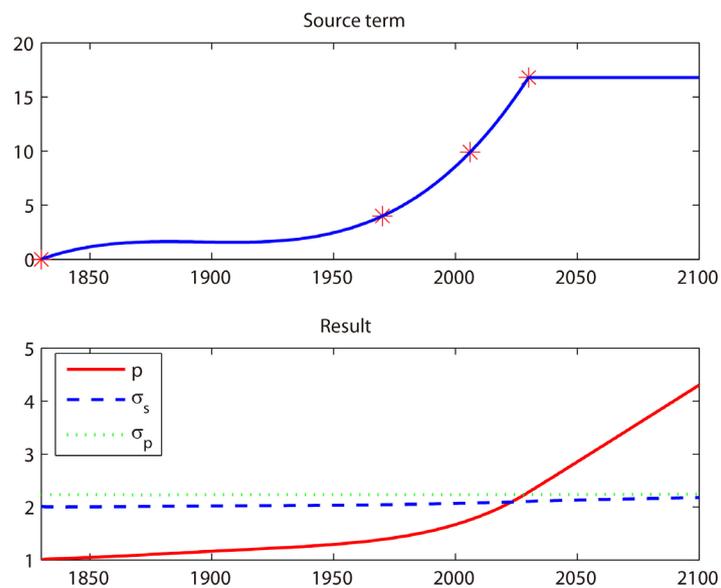
According to the sources commented above, it is foreseen that in the following 25 years the electricity consumption become twice as big and, unless the governments begin right now to build thousands of expensive nuclear reactors, this implies that coal and natural gas will satisfy almost the whole demand. In the second example, we will suppose that the emissions will remain constant from year 2030 onwards; it corresponds to the situation represented on the upper part of figure. The solutions represented in the lower part ameliorate the bad predictions of **Figure 3**, but we cannot describe the situation as optimistic either. CO<sub>2</sub> proportion in the atmosphere goes on growing, although the growth looks rather lineal; in the former example, the growth curve had a more

exponential shape.

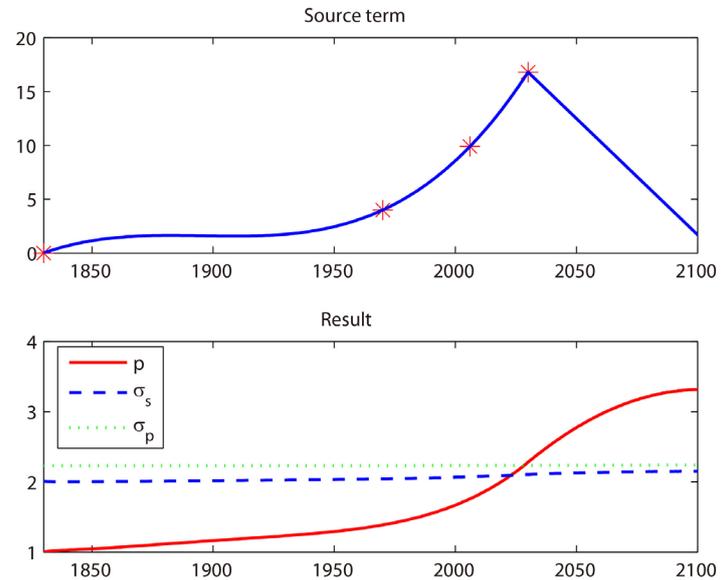
In the third example considered, we will suppose a more optimistic situation in which an international agreement is reached in order that by the end of this century the CO<sub>2</sub> emissions may be a 90% smaller than the unavoidable emissions of the year 2030. The situation is represented in **Figure 4**. Evidently, the situation gets better, since the gas quantity is stabilized. It should cause no surprise to observe the plain in the figure, because carbon dioxide is a very stable gas that remains long time in the Earth's atmosphere; as somebody said, the CO<sub>2</sub> our grandmothers caused when they turned on their kitchens is probably still going around.



**Figure 2.** Results with the same growth model.



**Figure 3.** Results without increasing the emissions.



**Figure 4.** Results with linear elimination of the emissions.

## 4. Conclusions

In the light of these results, it might be opportune to make several brief comments:

1) Since the mathematical models always take what is regarded as the most relevant information and does ignore many other details, the conclusions ought to be always prudent; as Lord May of Oxford says in [29], we are not vaccinated against the excesses. In any case, these numerical results do not contradict the observations, which would give a certain warranty to the model. The specialists are those who will have to discuss the details and extract the conclusions. However, slow and steady change simply leads to habituation, not action.

2) This model describes a serious and very worrying situation which remind the polemic speech of the mathematician James Lovelock in [30] in the year 2004, defending the necessity to produce electricity in nuclear centrals; it should not be forgotten that Lovelock is the writer of the famous book [31], where he arguments that Earth is only an organism which he called Gaia, and that he is a known ecologist militant as well. Obviously, in the decisions other kinds of considerations take part: economical, demographical, political ... to cope with a change in climate.

Studying the world that surrounds us, the Nature, has occupied the best thinkers from the most remote times, as Voltaire says in his philosophical dictionary: Tous les raisonneurs depuis Thalès, et probablement longtemps avant lui, ont joué colin-maillard avec toi; ils ont dit: “Je te tiens”! et ils ne tenaient rien. Nous ressemblons tous à Ixion; il croyait embrasser Junon, et il jouissait que d’une nuée (All reasoners since Thales, and probably long before him, have played at blind man’s buff with you (Nature); they have said: “I have you”! and they had nothing. We all resemble Ixion; he thought he was kissing Juno, and all that he possessed was a cloud).

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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