

Characteristics of and Control over Resonance in the Electromotive Force of Electromagnetic Induction

Sang Don Bu, Jin Kyu Han, Jin Young Hyeon, Gi Gwan Kim

Department of Physics, Chonbuk National University, Jeonju, South Korea.
Email: sbu@chonbuk.ac.kr

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ABSTRACT

The principles of electromagnetic induction are applied in many devices and systems, including induction cookers, transformers and wireless energy transfer; however, few data are available on resonance in the electromotive force (EMF) of electromagnetic induction. We studied electromagnetic induction between two circular coils of wire: one is the source coil and the other is the pickup (or induction) coil. The measured EMF versus frequency graphs reveals the existence of a resonance/anti-resonance in the EMF of electromagnetic induction through free space. We found that it is possible to control the system's resonance and anti-resonance frequencies. In some devices, a desired resonance or anti-resonance frequency is achieved by varying the size of the resonator. Here, by contrast, our experimental results show that the system's resonance and anti-resonance frequencies can be adjusted by varying the distance between the two coils or the number of turns of the induction coil.

Keywords: Electromagnetic Induction; Electromotive Force; Resonator, Resonance/Anti-Resonance Frequency

1. Introduction

1.1. Resonance

Resonance occurs widely in nature and is exploited in many man-made devices. Electric resonance is used in many circuits [1-5]; for example, radio and TV sets use resonance circuits to tune in to stations. In these devices, many frequencies reach the circuit simultaneously through the antenna, but significant current flow is induced only by frequencies at or near the circuit's resonance frequency. By varying the inductance or capacitance, the device can be tuned to different stations. In physics, resonance is the tendency of a system to oscillate at a greater amplitude at the system's resonance frequencies than at others. At these frequencies, even small periodic driving forces can produce large amplitude vibrations because the system stores the vibrational energy. In this work, we studied electromagnetic induction between two circular coils of wire: one is the source coil and the other is the pickup (or induction) coil, and report the characteristics of and control over the resonance and anti-resonance in the electromotive force (EMF) of electromagnetic induction through free space.

1.2. How Does the Magnitude of Electromotive Force Behave as the Frequency Applied to the Source Coil Increases?

The experiment was performed based on Faraday's law:

$$\varepsilon = -\frac{d\Phi}{dt} \quad (1)$$

where ε and Φ are the EMF induced in the induction coil and the magnetic flux passing through the induction coil, respectively [1,6]. The experimental setup consists of two circular coils of wire composed of an electrically conductive copper wire of cross-sectional radius 0.35 mm tightly wound into a series of loops of 5 - 320 turns, radius 7 cm and height 2 cm, as shown in **Figure 1**. One coil (the source coil) is connected to a signal generator (AFG3021B, Tektronix), and the other (the pickup or induction coil) is connected to an oscilloscope (DSO 5012A, Agilent Technologies). The signal generator supplies a sinusoidal voltage to the source coil, creating a sinusoidal magnetic field. The AC magnetic field propagates through free space and reaches the pickup coil. According to Faraday's law, an electric field is induced in any region of space in which a magnetic field varies

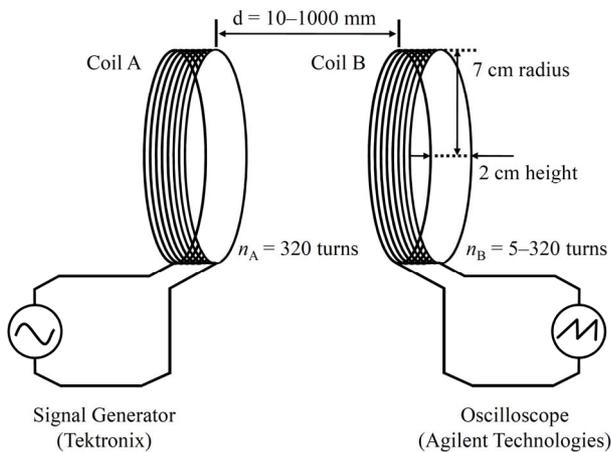


Figure 1. Schematic diagram of the experimental setup. The experimental setup consists of two coils. Coil A is the source coil, which transmits an electromagnetic wave through free space. Coil B is the pickup (or induction) coil, which detects an induced EMF. Coils A and B are aligned coaxially, such that the distance d between the two coils can be adjusted from 10 mm to 1000 mm. The coils are immobile during operation.

over time. Thus, an EMF is induced in the pickup coil. If the magnetic flux passing through the pickup coil is $\Phi = A \sin \omega t$, then the induced EMF is $\varepsilon = A \omega \cos \omega t$. As ω increases, ε increases, and the magnitude of ε is proportional to the rate at which the magnetic flux changes with time, so that faster changes give a stronger ε . Here our question is: *how does the magnitude of ε behave as ω increases, especially in the high frequency range of 10 kHz to a few MHz?*

2. Ease of Use Electromagnetic Induction Properties

2.1. Comparison of the Experimental and Predicted Electromotive Forces

Figures 2(a) and (b) show typical behaviors of 1) the root-mean-square value of ε (ε_{RMS}) of the pickup coil and 2) the phase difference φ between the applied voltage (to the source coil) and the generated ε (in the pickup coil), respectively, as a function of the applied frequency f_A (to the source coil). We expected that, according to Faraday's law and the radiation resistance of the coil, ε_{RMS} would increase with increasing f_A and then finally attain a new equilibrium, as shown by the black dashed line in Figure 2(a). Such a response could be modeled using the Langevin function [7],

$$L(f_A) = \coth f_A - \frac{1}{f_A}. \quad (2)$$

However, the experimental data exhibited a very different behavior, as shown by the red solid circles in Figure 2(a). A resonance behavior was observed at high

frequencies, and a relaxation behavior was observed at low frequencies. Resonance and anti-resonance peaks were clearly observed at 87 kHz and 285 kHz, respectively.

Figure 2(b) shows that at a frequency of 10 Hz, the ε of the pickup coil was 78° out-of-phase with respect to the wave applied to the source coil. As f_A increased, the phase difference decreased until the phases matched, where the phase-matching means that the phase difference is less than 5° . Meanwhile, the phase difference changed profoundly when the pickup coil became resonant. The ε of the pickup coil was 176° out-of-phase at frequencies between the resonance and anti-resonance peaks, that is, over a frequency range of 87 - 285 kHz.

The abrupt change in the phase difference may have been due to motions of the charges in the pickup coil. If the phase difference arose from the acceleration of conducting electrons induced by the electromagnetic radiation (generated from the source coil), the phase difference may be effectively independent of f_A at high frequencies. However, if the phase difference arose from the oscillations in the electric/magnetic dipoles or toroidal dipoles¹ [8-14] induced by the electromagnetic radiation, the phase difference would be expected to depend on f_A at high frequencies.

The radiation resistance in a coil with n turns composed of an electrically conductive copper wire may be modeled as follows. For a coil of $n = 320$, radius 7 cm and height 2 cm, as used here (see Figure 1), the electric dipole radiation term in the radiation resistance is smaller than the magnetic dipole radiation term, assuming that the distance from the coil is $r = c/f$, where c is the speed of light, for example, $r = 1000$ mm; the former is on the order of $(\omega/1000c)^2$, the latter is on the order of $(10\omega/c)^4$, where ω and c are the angular frequency and the speed of light, respectively. The resonance frequencies obtained in our experiments were much lower than the resonance frequencies of the valence electrons or electric/magnetic dipoles [6,7]. Therefore, in the particular geometry studied here, the abrupt changes in the phase difference may have predominantly arisen from oscillations in the toroidal dipoles [13,14] in the induction coil.

2.2. Adjusting of Resonance and Anti-Resonance Frequencies

To determine whether the resonance frequency f_R and

¹Toroidal moments were first considered by Zel'dovich in 1958. They are fundamental electromagnetic excitations that cannot be represented in terms of the standard multipole expansion. The toroidal dipole moment represents the lowest moment of the third independent electromagnetic multipole family, after the electric and magnetic moments. The properties of materials that possess toroidal moments and the classification of their interaction with external electromagnetic fields are topics of growing interest [8-14].

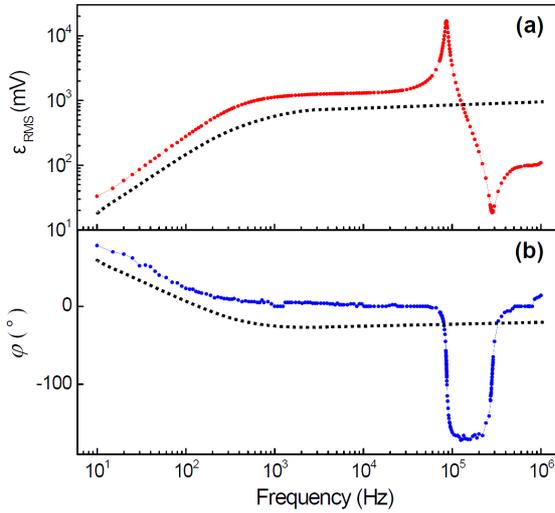


Figure 2. Comparison of the experimental and predicted values of ϵ_{RMS} and ϕ . The experimental measurements of ϵ_{RMS} and ϕ are shown in (a) and (b), respectively, indicated by red and blue solid circles. The amplitude of the signal applied to the source coil and the distance between the source and induction coils were then fixed at 14 V and 3 cm, respectively. The predicted values of ϵ_{RMS} and ϕ are shown in (a) and (b), respectively, with black dashed lines.

anti-resonance frequency f_{AR} could be adjusted, we examined the influence of three experimental parameters on the f_{R} and f_{AR} . **Figures 3(a)-(c)** show graphs of ϵ_{RMS} versus f for various 1) voltages applied to the source coil ($V_{\text{A}} = 2, 6, 10$ and 14 V), 2) distances between the two coils ($d = 10, 30, 100$ and 1000 mm), and 3) numbers of turns of the pickup coil ($n_{\text{B}} = 5, 50, 150, 250$ and 320), respectively. **Figures 4(a)-(c)** show variations of f_{R} and f_{AR} for V_{A} , d and n_{B} , respectively, obtained from **Figure 3**. The resonance and anti-resonance peaks are clearly evident in most of the curves in **Figure 3**. f_{R} was not influenced by V_{A} or d , as shown by the black squares in **Figures 4(a)-(b)**. The interval $\Delta f_{\text{R-AR}}$ between f_{R} and f_{AR} did not change significantly with increasing V_{A} (**Figure 4(a)**), whereas $\Delta f_{\text{R-AR}}$ decreased dramatically as d increased (**Figure 4(b)**). These results indicate that f_{AR} could be controlled by d independently of f_{R} . On the other hand, as n_{B} increased, f_{R} and f_{AR} decreased together, as shown in **Figure 4(c)**.

f_{R} and f_{AR} each show peculiar behavior. The f_{R} and f_{AR} curves shown in **Figure 4(b)** indicate that f_{R} does not depend on d and f_{AR} could be described by an equation involving the logarithm of d . A plot of f_{AR} as a function of $\log d$ yielded a straight line, as indicated by the red line in the inset of **Figure 4(b)**. Thus, f_{AR} can have the form " $m_1 \log d + c_1$ ", where $m_{i=1,2,3,4,5}$ and $c_{i=1,2,3,4,5}$ are the proportionality constants and the vertical axis intercepts (or meaningless constants), re-

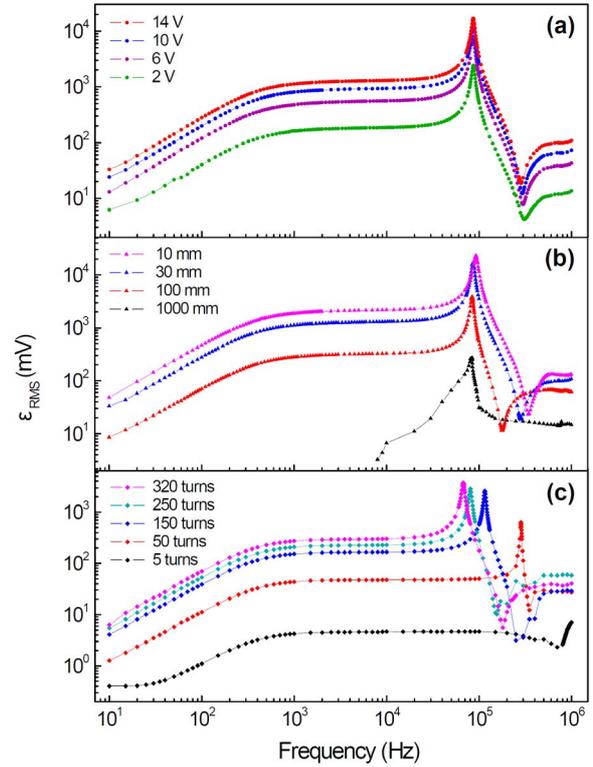


Figure 3. Plots of ϵ_{RMS} versus f_{A} . ϵ_{RMS} was measured as a function of the voltage applied to the source coil (V_{A}), the distance between the two coils (d) and the number of turns of the pickup coil (n_{B}), where the number of turns of the source coil (n_{A}) was fixed at 320. (a) Plots of ϵ_{RMS} versus f_{A} for V_{A} of 2, 6, 10 and 14 V, with d and n_{B} fixed at 30 mm and 320, respectively; (b) Plots of ϵ_{RMS} versus f_{A} for d of 10, 30, 100 and 1000 mm, with V_{A} and n_{B} fixed at 14 V and 320, respectively; (c) Plots of ϵ_{RMS} versus f_{A} for n_{B} of 5, 50, 150, 250 and 320, with V_{A} and d fixed at 14 V and 100 mm, respectively.

spectively. According to **Figure 4(c)**, f_{R} can have the form " $m_2 n_{\text{B}} + c_2$ ", rather than " $m_3 \log n_{\text{B}} + c_3$ ", and f_{AR} can also have the form " $m_4 n_{\text{B}} + c_4$ ". The experimental results obtained for various combinations of V_{A} , d and n_{B} show that f_{AR} could be described by an equation of the form " $m_1 \log d + m_4 n_{\text{B}} + m_5 V_{\text{A}} + c_5$ " with the condition $|m_1| \gg |m_4| \gg |m_5|$, and f_{R} could be described by an equation of the form " $m_2 n_{\text{B}} + c_2$ " with the condition $|m_2| \cong |m_4|$. It is noted that under the conditions of various n_{B} turns, all the behaviors of f_{R} and f_{AR} with d were like the results shown in **Figure 4(b)**, indicating that the cross term " dn_{B} " might be negligible.

We showed experimentally that pickup coils with different resonance/anti-resonance frequencies could be obtained by selecting the appropriate d and n_{B} for a given coil size. These results suggest the possibility of "a wireless power transfer station" that transmits power from a source coil to a large number of pickup coils

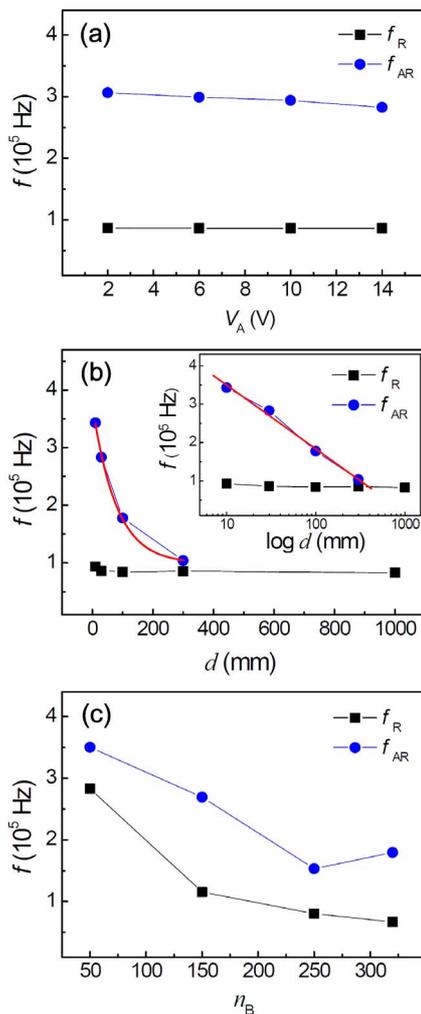


Figure 4. f_R and f_{AR} as a function of (a) V_A , (b) d and (c) n_B .

through free space. If several pickup coils (with different resonance/anti-resonance frequencies) were positioned around a source coil (*i.e.*, a short-range power networking system), power could be transferred from the source coil to the pickup coils by modulating the rate of change (over time) of the magnetic flux passing through each pickup coil. Wireless power transfer stations that are analogous to radio stations may be realized in the near future to permit everyone to use power anywhere without the need for wired power transmission.

3. Conclusion

In summary, we have studied the electromagnetic induction between two circular coils of wire and showed clearly the existence of resonance/anti-resonance in EMF of electromagnetic induction through free space. We believe that our results might provide a competitive approach toward the development of high-efficiency systems in devices of induction cookers, electric power transformers, and wireless energy transfer.

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