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Calculation of Start-Oscillation-Current for Lossy Gyrotron Traveling-Wave Tube (Gyro-TWT) Using Linear Traveling-Wave Tube (TWT) Parameter Conversions

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ABSTRACT

The start-oscillation-current of a gyro-TWT (gyrotron traveling-wave tube) determines the stable operating current level of the device. The amplifier is susceptible to oscillations when the operating current level is higher than the start-oscillation current. There are several ways of calculating the start-oscillation current, including using the linear and nonlinear theory of a gyro-TWT. In this paper, a simple way of determining the start-oscillation current of lossy gyro-TWT is introduced. The linear TWT parameters that include the effects of synchronism, loss, and gain, were converted to gyro-TWT parameters to calculate the start-oscillation-current. The dependence on magnetic field, loss, and beam alpha was investigated. Calculations were carried out for a V-band gyro-TWT for both operating and competing modes. The proposed method of calculating the start-oscillation current provides a simple and fast way to estimate the oscillation conditions and can be used for the design process of a gyro-TWT.

Keywords: Start-Oscillation-Current; Gyro-TWT; TWT; Lossy; Stable; Competing Mode

1. Introduction

The gyro-TWT (gyrotron traveling-wave tube) has long been viewed as an extremely promising device due to its high-power and broadband capabilities. Potential applications include radar, communication, surveillance, and scientific research [1]. However, in order for gyro-TWT to work properly, the interaction with competing mode must be suppressed. The beam current level where the unwanted oscillation takes place is called the "start-oscillation-current" $\left(I_s\right)'$ for gyro-TWT. Therefore in gyro-TWT, it is critical to operate the amplifier below I_s to ensure stability of the device. One way to increase I_s is to apply loss to the gyro-TWT circuit. Calculation results of I_s employing linear theory [2-4] and nonlinear theory [5] were reported for lossy gyro-TWT. However, using these methods require in-depth analysis on linear and nonlinear theories of gyro-TWT. In this paper, a simple method of obtaining I_s for lossy gyro-TWT by using the linear-TWT parameters is introduced. By using the linear TWT parameter conversions, the expression for I_s for the gyro-TWT was obtained. The parameter conversion process and the calculation results for I_s for gyro-TWT are presented.

2. Conversion of Linear TWT Parameters to Gyro-TWT

The gain parameter of a linear lossy TWT that corresponds to the start-oscillation condition can be expressed by Equation (1) [6]. For gyro-TWT, the gain parameter is described as Equation (2) [7]. By combining Equation (1) used in a linear TWT and Equation (2) used in a gyro-TWT (by setting $C_{st} = C_g$), the start oscillation current for lossy gyro-TWT can be expressed by Equation (3).

$$C_{st} = 0.0112\frac{L_{dB}}{N}\left(1+\frac{1013}{L_{dB}^2}\right) \tag{1}$$

$$C_g = \left(\frac{k_c^4}{2k_b^4} I_b F_{mn}\varepsilon_v\right)^{\frac{1}{3}} \tag{2}$$

$$I_s = \frac{2k_b^4}{F_{mn}\varepsilon_v k_c^4}\left(0.0112\frac{L_{dB}}{N}\right)^3\left(1+\frac{1013}{L_{dB}^2}\right)^3 \tag{3}$$

Here L_{dB} is the total loss of the circuit in dB, N is the circuit length in wavelength, k_c is the cutoff wavenumber $\left(\omega_c/c\right)$, k_b is the beam wavenumber $\left(\left(\omega-\Omega\right)/v_z\right)$, I_b is the beam current, F_{mn} is defined in Equation (4), and ε_v is defined in Equation (5).

$$F_{mn} = \frac{J_{1-m}^2\left(k_{mn}R_g/a\right)}{k_{mn}^2 J_m^2\left(k_{mn}\right)\left(1-m^2/k_{mn}^2\right)} \tag{4}$$

$$\varepsilon_v = \frac{2.348\times10^{-4}}{\gamma\beta_z}\left(\frac{\beta_\perp}{\beta_z}\right)^2\left(J_1'\left(\frac{\beta_\perp\omega_c}{\Omega_c}\right)\right)^2 \tag{5}$$

Here J_m is the Bessel function of order m, k_{mn} is the m^{th} Bessel root defined by $J_m'\left(k_{mn}\right)=0$, n is the radial mode number, m is the azimuthal mode number, R_a is the guiding center radius, a is the waveguide radius, β_z is the axial velocity normalized by the speed of light, $\beta_\perp$ is the transverse velocity normalized by the speed of light, ω_c is the cutoff frequency of the waveguide, and Ω_c is the relativistic cyclotron frequency. The comparison between critical parameters of linear and gyro-TWT is shown in **Table 1**.

3. Calculated Results and Discussion

In order to validate the I_s calculation method proposed above, a V-band (60 GHz) TE$_{11}$ gyro-TWT was chosen to evaluate the I_s values. **Figure 1** shows the dispersion diagram of the V-band TE$_{11}$ gyro-TWT for $\alpha = 0.85$, $V_b = 100$ kV, and $B_o/B_g = 1.0$. The waveguide mode expressed as $\omega^2 = \omega_c^2 + k_z^2c^2$ is shown in parabolas and the beam mode which can be described as $\omega = s\Omega_c + k_zv_z$ is shown in straight lines up to fourth harmonic. Here, ω is the frequency, k_z is the axial wavenumber, c is the speed of light, s is the harmonic number, and v_z is the axial velocity of the beam. The operating point is where the TE$_{11}$ waveguide mode grazes with the $s = 1$ beam mode. The possible competing mode interactions occur when the waveguide mode intersects with the beam mode. These include TE$_{11}$ and TE$_{21}$ with $s = 2$, TE$_{01}$ with $s = 3$, and TE$_{02}$ with $s = 4$ beam modes. The specification of the V-band TE$_{11}$ gyro-TWT is described in **Table 2**. The calculated I_s using Equation (3) is shown in **Figures 2-5** under various conditions. **Figure 2** shows dependence of I_s on circuit loss, L_{dB}, for several values of beam velocity ratio, α, for the operating TE$_{11}$ mode with $B_o/B_g = 1.5$ and $V_b = 100$ kV. The B_o/B_g indicates operating magnetic field, B_o, normalized by the grazing magnetic field, B_g. As L_{dB} increases, I_s increases which indicates that with higher value of L_{dB}, the device becomes more stable. For fixed value of L_{dB}, I_s increases as α decreases. This indicates that the loss stabilizes the device and the gyro-TWT becomes unstable for higher values of α. **Figure 3** describes I_s change with B_o/B_g of the operating TE$_{11}$ mode for several values of beam voltage, V_b, for fixed values of $\alpha = 0.85$ and $L_{dB} = 100$ dB. For $B_o/B_g < 1.1$, I_s decreases as B_o/B_g increases. For fixed value of B_o/B_g, I_s is higher for higher V_b when $B_o/B_g < 1.1$. For $B_o/B_g > 1.1$, higher beam voltage makes the device unstable and as the operating magnetic field increases the device becomes more stable due to increasing $|k_z|$. **Figure 4** shows I_s as a function B_o/B_g for several values of α and fixed values of $V_b = 100$ kV and $L_{dB} = 100$ dB. For $B_o/B_g < 1.1$, I_s decreases as B_o/B_g increases. For fixed value of B_o/B_g, I_s in

Table 1. Conversion of linear TWT parameters to gyro-TWT [7-8].

Parameter	Linear TWT	Gyro-TWT
Gain	$C = \left(\frac{Z_0I_0}{4V_b}\right)^{\frac{1}{3}}$	$C_g = \left(\frac{k_c^4}{2k_b^4}I_bF_{mn}\varepsilon_v\right)^{\frac{1}{3}}$
Synchronism	$b = \frac{1}{C}\left(\frac{u_0}{v_p}-1\right)$	$g = \frac{k_g-k_b}{k_bC_g}$
Loss	$d = 0.0184\frac{L}{C}$	$d = 0.0184\frac{L}{C_g}$

Z_0: Circuit impedance; v_p: Phase velocity; V_b: Beam voltage; L: Loss per wavelength; u_0: Beam velocity; k_a: Waveguide wavenumber.

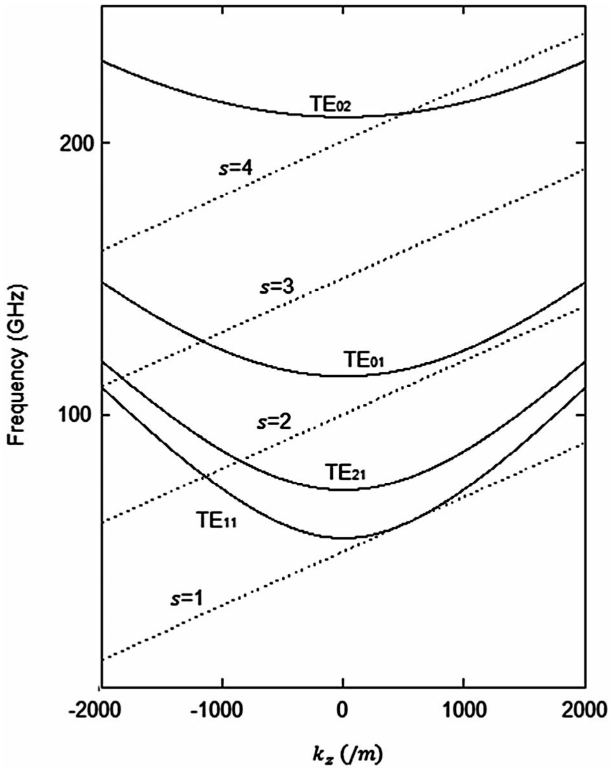

Figure 1. Dispersion diagram of a TE$_{11}$ 60 GHz gyro-TWT for α = 0.85, V_b = 100 kV, and $B_o/B_g = 1.0$. Waveguide modes (TE$_{11}$, TE$_{21}$, TE$_{01}$, TE$_{02}$) and beam modes (s = 1, 2, 3, 4) are shown.

Table 2. Specifications of the V-band TE_{11} gyro-TWT.

Parameter	Value
Beam voltage, V_b	100 kV
Beam current, I_b	3 A
Velocity pitch, α	0.85
Operating mode	TE_{11}
Cyclotron harmonic	$s = 1$
Magnetic field, B_o	21.4 kG
B_o/B_g	0.995
Circuit radius, r_w	0.16 cm
r_c/r_w	0.361
Circuit length	14 cm
Lamor radius, r_L	0.034 cm
Cutoff frequency	55 GHz
Efficiency	24.5%
Output power	78.2 W
Gain	41 dB
Bandwidth	5%

r_c: Guiding center radius.

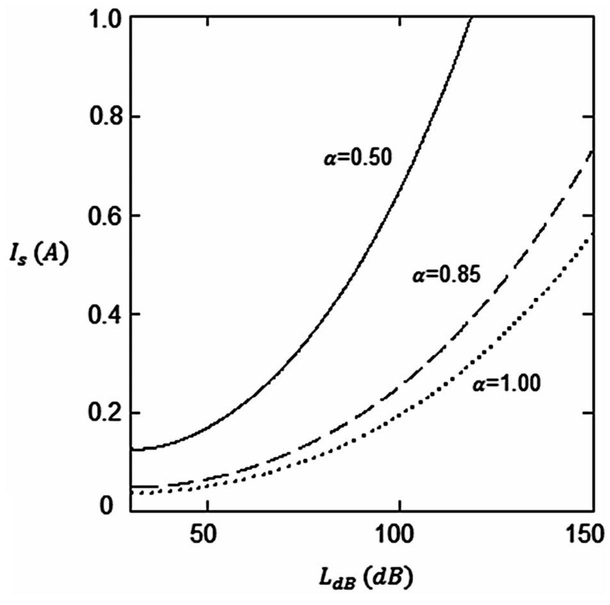

Figure 2. Start-oscillation-current, I_s, for the operating TE_{11} mode as a function of loss, L_{dB}, for different values of beam velocity ratio, α, and the fixed value of the operating magnetic field normalized by the grazing magnetic field, B_o/B_g = 1.5, and beam voltage, V_b = 100 kV.

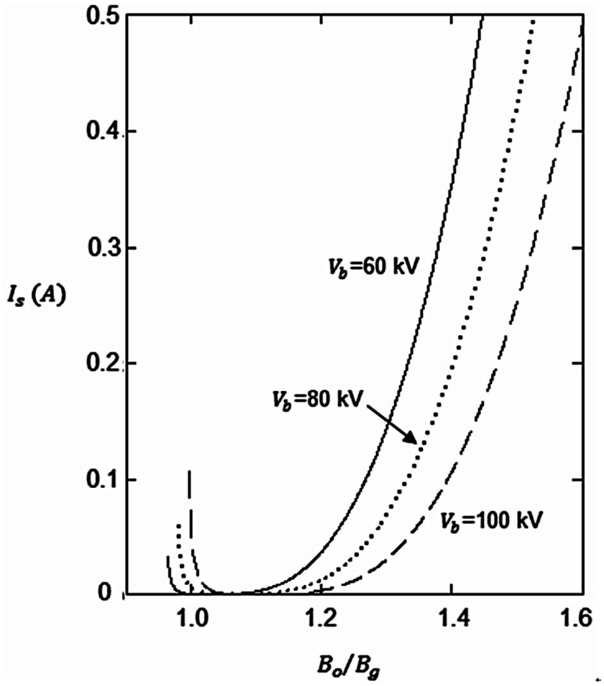

Figure 3. Start-oscillation-current, I_s, for the operating TE_{11} mode as a function of B_o/B_g for different values of beam voltage, V_b, and fixed values of α = 0.85 and L_{dB} = 100 dB.

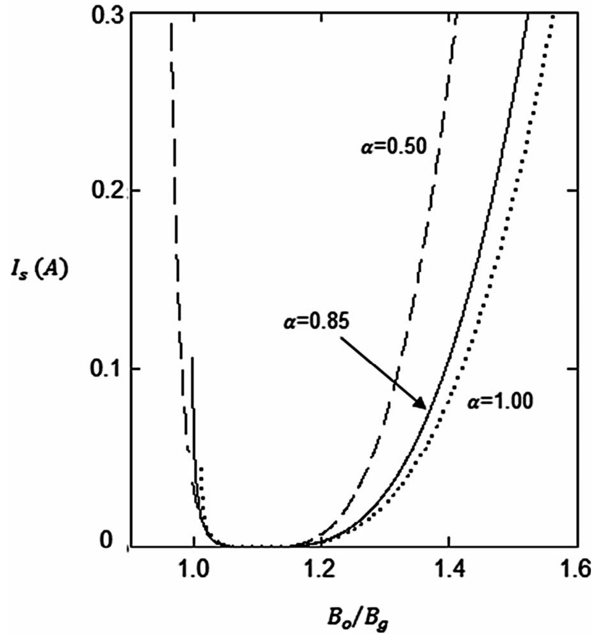

Figure 4. Start-oscillation-current, I_s, for the operating TE_{11} mode as a function of B_o/B_g for different values of α and fixed values of V_b = 100 kV and L_{dB} = 100 dB.

creases as α increases for $B_o/B_g < 1.1$. For $B_o/B_g > 1.1$, device is more stable for lower alpha because when the perpendicular component of the velocity decreases, the gyro-TWT beam-wave interaction becomes

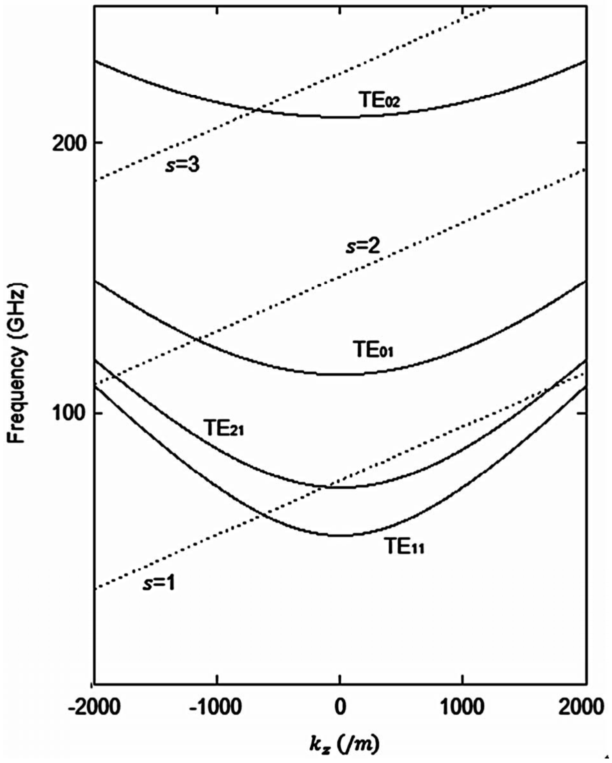

Figure 5. Dispersion diagram of a TE$_{11}$ 60 GHz gyro-TWT for α = 0.85, V_b = 100 kV, and B_o/B_g = 1.5. Waveguide modes (TE$_{11}$, TE$_{21}$, TE$_{01}$, TE$_{02}$) and beam modes (s = 1, 2, 3) are shown.

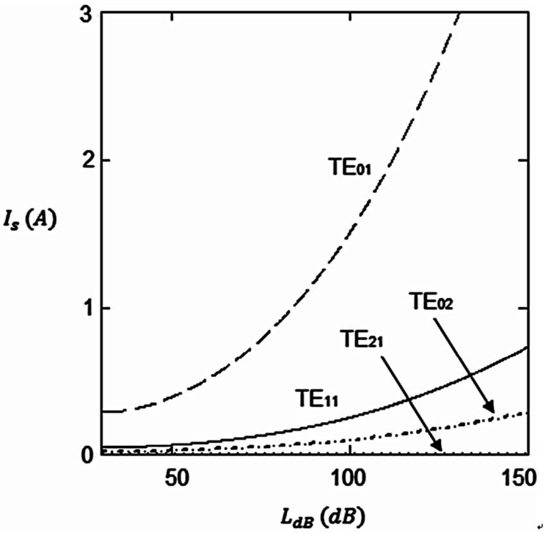

Figure 6. Start-oscillation-current, I_s, as a function of loss, L_{dB}, for different modes. Fixed values of α = 0.85, V_b = 100 kV, and B_o/B_g = 1.5 were assumed.

weaker. **Figure 5** shows the dispersion diagram of the device for α = 0.85, V_b = 100 kV, and $B_o/B_g = 1.5$. **Figure 6** describes I_s as a function of L_{dB} for four different modes: TE$_{11}$, TE$_{21}$, TE$_{01}$, and TE$_{02}$. Fixed values of α = 0.85, V_b = 100 kV, and $B_o/B_g = 1.5$ were assumed. The lowest I_s occurs for the TE$_{21}$ and the TE$_{01}$ mode exhibits the highest I_s value. As can be seen in **Figure 5**, this is due to TE$_{21}$ mode having the lowest $|k_z|$ value and the TE$_{01}$ mode having the highest $|k_z|$ value at the intersection of the beam-wave dispersion diagram. The I_s of the TE$_{01}$ mode is the most sensitive to L_{dB} variation.

4. Summary and Conclusion

In this paper, an expression for I_s for lossy gyro-TWT was derived using linear TWT parameter conversions. For V-band TE$_{11}$ gyro-TWT, I_s was calculated for various parameters including loss, beam voltage, magnetic field, and beam velocity ratio. The method introduced in this paper can be used to quickly estimate the I_s values of a lossy gyro-TWT.

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