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Analysis of Complex Electromagnetic Structures by Hybrid FDTD/WCIP Method

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ABSTRACT

This paper proposes a hybrid full-wave analysis using Finite-Difference Time-Domain (FDTD) and Wave Concept Iterative Process (WCIP) methods, developed to analyze locally arbitrarily shaped microwave structures and Multilayer Planar structure. Using the equivalence principle, the original problem can be decomposed into two sub regions and solve each sub region separately. An interpolation scheme is proposed for communicating between the FDTD fields and WCIP wave, which will not require the effort of fitting the WCIP mesh to the FDTD cells in the interface region. This method is applied to calculate the scattering parameters of arbitrary (3-D) microwave structures. Applying FDTD to 3D discontinuity and WCIP to the remaining region preserves the advantages of both WCIP flexibility and FDTD efficiency. A comparison of the results with the FDTD staircasing data verifies the accuracy of the proposed method.

Keywords: FDTD; Hybrid Finite-Difference Time-Domain (FDTD); Hybrid Techniques; WCIP Method

1. Introduction

The WCIP method is a widely used numerical technique for characterizing various electromagnetic problems.

It has long been recognized that the accuracy and efficiency of the method can be dramatically improved through the use inhomogeneous layers and for planar structures with a curved boundary due to the staircasing approximation.

Several researches have been developed to overcome these difficulties in the WCIP method. The effort spent by the researches to develop the WCIP method is restricted to study the homogeneous layers [1].

One attempt to study the inhomogeneous layers is presented in [2,3]. This consists in hybridizing the WCIP method with a differential formulation using the differential operator in conjunction with the Maxwell equation in their local form to link the waves on the two sides of the inhomogeneous layer [4].

Hybrid methods, which combine the desirable features of two or more different techniques, are developed to analyze complex electromagnetic problems that cannot be resolved conveniently, and/or accurately, by using them individually [5].

This paper proposes a new hybrid approach by introducing an interpolation scheme for communication between the FDTD field and the WCIP wave in the interface region. This approach employs a combination of Fourier transformation and iteration process, and exchanges information on the field values, back and forth, between the FDTD sub problem and the WCIP region. The iterative technique is rapidly convergent only when the mutual interaction between the two sub domains is relatively weak. The comparison between the scattering parameter results obtained with the proposed hybrid method, and FDTD stair casing data verifies the accuracy of this analysis.

2. Formulation

The idea of the hybrid approach consists to combines the above two methods in a manner that retains the advantages of both. Then, the hybrid method is implemented as follows:

First, the partition of the global domain in two sub domains:

- Interior model or FDTD domain (3D discontinuity domain);
- Exterior model or spectral domain (homogeneous domain).

Second, for the physical behavior, it is necessary to develop accurate procedures to support the Interaction between these two models, is fulfilled by enforcing boundary conditions, *i.e.*, the continuity of the tangential fields on the equivalent surfaces S.

The processes started, by dividing D the computational domain, into two sub domains DWCIP and DFDTD corresponding to the WCIP and FDTD regions, respectively, such that D = DWCIP ∪ DFDTD.

Next, mesh these two regions using structured meshes, respectively, with common nodes shared at the interface but with no overlapping (DWCIP ∩ DFDTD = Ø).

The structured grid of the WCIP is well suited to conforms the FDTD mesh at the interface, and this allows us to limit the number of unknowns in the FDTD region.

The FDTD and WCIP models describing the corresponding region are coupled together through the boundary conditions on the surfaces S:

$$\frac{\hat{n}\times\boldsymbol{E}_1}{Z_1}=\frac{\hat{n}\times\boldsymbol{E}_2}{Z_2} \tag{1}$$

$$\hat{n}\times\boldsymbol{H}_1=\hat{n}\times\boldsymbol{H}_2 \tag{2}$$

Now, it is important to turn to the marching scheme that has been implemented in our algorithm. The FDTD technique a well-known and has been used widely, and hence, we do not need to quote his formulation in this work [6,7].

2.1. Formulation of the Wave Concept Iterative Method

The Wave Concept Iterative Process (WCIP) counts among the most recent and the most efficient iterative methods.

It was developed as an instrument for the study of guide wave and planar circuits, its applicability extends to all range of geometrical dimensions of the scattering obstacle. The WCIP approach consists in separating the structure under study into interfaces with upper and lower homogeneous media.

The boundary conditions on the interface are represented by the diffraction operator, S, and in the homogeneous media by Γ the reflection operator. They are defined in spatial and modal domains, respectively.

The two conditions for existence of the Wave Iterative formulation are: first, the partition of the global domain in two subdomains:

- Spatial domain (interfaces or lumped elements);
- Spectral domain (medium or propagation).

The Wave Concept method described here is based on full wave transverse formulation, where the dual quantities current density and electric field are considered.

The incident ***A*** and reflected ***B*** waves are calculated from the tangential electric ***E*** and magnetic ***H*** fields, on the interface

$$\begin{cases}\boldsymbol{A}_i=\dfrac{1}{2\sqrt{Z_{0i}}}\left(\boldsymbol{E}_i+Z_{0i}\boldsymbol{J}_i\right)\\[2ex] \boldsymbol{B}_i=\dfrac{1}{2\sqrt{Z_{0i}}}\left(\boldsymbol{E}_i-Z_{0i}\boldsymbol{J}_i\right)\end{cases} \tag{3}$$

where indicates the medium 1 or 2 corresponding to a given interface Ω. Z0i is the characteristic impedance of the same medium (*i*) and $\boldsymbol{J}_i$ is the surface current density vector given as

$$\boldsymbol{J}_i=\boldsymbol{H}_i\wedge\boldsymbol{n}_i \tag{4}$$

where $\boldsymbol{n}_i$ is the outward vector normal to the interface.

In general case the planar structures are modeled in the WCIP method by a thin metallic plate at the interface Ω between two medium, enclosed by a rectangular waveguide with transversal cut. The Γ operator is described in the modal domain and assigns the propagation boundary conditions at upper and lower interface.

The S operator expressed in the spatial domain assigns boundary conditions at the interface plan and represents the different possible sub domains (dielectric, metal and source). Then, the air-dielectric interface plan Ω is divided into cells (named pixels), forming a uniform grid, used to discretize each sub domain.

The spatial and modal waves are directly deduced from each other with the help of Fast Modal Transformation (FMT) and its inverse transform (IFMT).

The decomposition of the electromagnetic wave in guided modes propagating in waveguide with electric or magnetic wall (TE and TM modes) takes place by the use of this Fast Modal Transformation.

The procedure is repeated until convergence of the input admittance Yin and the frequency parameter of the structure is obtained (The convergence is obtained once in Yin does not vary as the number of iterations increases) [8].

2.2. Hybrid WCIP-FDTD Algorithm

The hybrid method combines the method of WCIP in the frequency domain to solve the homogeneous problem and the FDTD method to handle the inhomogeneous dielectric object [9,10]. This approach employs a combination of Fourier transformation and iteration, and exchanges information on the field values, back and forth, between the homogeneous dielectric sub problem and the dielectric inhomogeneous region. The iterative technique is rapidly convergent only when the mutual interaction between the two sub regions relatively weak.

The hybridization technique is based upon the use of the concept of surface impedance boundary conditions it begins by dividing the original problem into two separate ones. The first one of these, which contains the inhomogeneous or 3D discontinuity region, is solved by using the FDTD scheme, while the second, which deals with the homogeneous sub region, is handled via the WCIP scheme.

A time-stepping solution procedure is implemented as follows.

The initial condition of the hybrid approach:

Initial incident wave in the WCIP region can be expressed as:

$$\boldsymbol{A}_1^{(0)} = 0 \tag{5}$$

The situation is equivalent to an ABC with free space impedance.

Initial excitation in the FDTD region can be expressed as:

$$\boldsymbol{B}_1^{(0)} = 0 \tag{6}$$

Begin the iteration process of the hybrid method by using the excitation source in the FDTD region and running the conventional sub region. The FDTD region is surrounded by an impedance bounding surface (an equivalence-principle surface) which is used to relaunch the inward and outward traveling fields between the two domains.

Next, the FDTD algorithm is applied in the relatively subregion, the equivalence principle is implemented in the 3D-FDTD computer code and all fields in this subregion is obtained.

Then updating the fields at the interface surface between the two subregions, and will be used in the reconstructing the incident wave in the WCIP region.

$$\boldsymbol{A}_i = \frac{1}{2\sqrt{Z_{0i}}}\left(\boldsymbol{E}_i + Z_{0i}\boldsymbol{J}_i\right) \tag{7}$$

At the interface S of the FDTD domain the field relation defined as flows:

$$\boldsymbol{E} = Z_0 \boldsymbol{J} \tag{8}$$

The surface current density vector given as:

$$\boldsymbol{J} = \hat{n} \times \boldsymbol{h} \tag{9}$$

$\hat{n}$ is the outward vector normal to the interface. Z_0 a characteristic impedance.

Then

$$\boldsymbol{A}_{1t}^{(i)} = \frac{a}{\sqrt{Z_0}} \cdot \boldsymbol{E}_t\left(x, y\right) \tag{10}$$

$\boldsymbol{e}_t\left(x, y\right)$ = the electric fields on interface surface S of the FDTD model.

Apply the FFT on the expression of the incident wave to pass the time domain to frequency domain, and can be used in the initial condition in the WCIP processes.

Starting the WCIP process, it has already been seen that ***B*** waves can be determined if ***A*** waves are known and vice versa. The reflected wave B can be computed by applying the basically boundary conditions of the WCIP technique in the homogeneous media represented by the reflection operator.

$$\boldsymbol{B} = \hat{\Gamma}\boldsymbol{A} \tag{11}$$

After convergence of the iterative process of the WCIP approach. The reflected waves B is modeled by equivalent source of the internal impedance Z_0 and are defined as follows:

$$\boldsymbol{B} = \frac{1}{2\sqrt{Z_0}}\left(\boldsymbol{E} - Z_0\boldsymbol{J}\right) = \frac{\boldsymbol{E}_0}{2\sqrt{Z_0}} \tag{12}$$

$$\boldsymbol{E}_0 = 2\sqrt{Z_0} \cdot \boldsymbol{B} \tag{13}$$

The continuity boundary conditions at the equivalent surface according to the two sub regions are written as [12,13]:

Metallic domain

$$\boldsymbol{E}_x = \boldsymbol{E}_y = 0 \tag{14}$$

$$\boldsymbol{J} = -\frac{\boldsymbol{E}_0}{Z_0} \tag{15}$$

Dielectric domain

$$\boldsymbol{J} = -\frac{\boldsymbol{E}_0}{Z_{0i} + Z_{si}} \tag{16}$$

$$\boldsymbol{E} = -Z_{si}\frac{\boldsymbol{E}_0}{Z_{0i} + Z_{si}} \tag{17}$$

$$\boldsymbol{J}_i = \boldsymbol{H}_i \wedge \boldsymbol{n}_i$$

$\boldsymbol{H}_i$ et $\boldsymbol{E}_i$ magnetic and electric field.

Z_{0i} a characteristic impedance of the medium (*i*).

Applying the IFFT of the electric fields at the interface S according to the WCIP scheme, the magnetic fields can then be evaluated from the electric field along the interface between WCIP region and FDTD region and are used as the excitation source for the sub region solved by the FDTD procedure. The procedure can now be repeated to continue the iteration process shown in **Figure 1**. The solution is checked at each iteration step until a steady state solution is obtained and illustrated by **Figure 2**.

Accurate solution to the hybrid problem can be achieved by a minimal cost of iterations verified in **Figure 3**.

3. Numerical Results

Wave concept iterative process and Finite Difference-Time Domain (FDTD) techniques have been used to characterize essentially commonly found discontinuities.

Specifically, a microstrip short circuit, a Short-Circuited Stubs microstrip Filter, and a rectangular dielectric resonant antenna were analyzed and their electrical performance was studied.

This hybrid method was applied in the first steps to characterize the cylindrical via-hole grounds in microstrip. The computational domain is split into two regions, the homogeneous dielectric and no metallic discontinuity region is replaced by WCIP region and the discontinuity

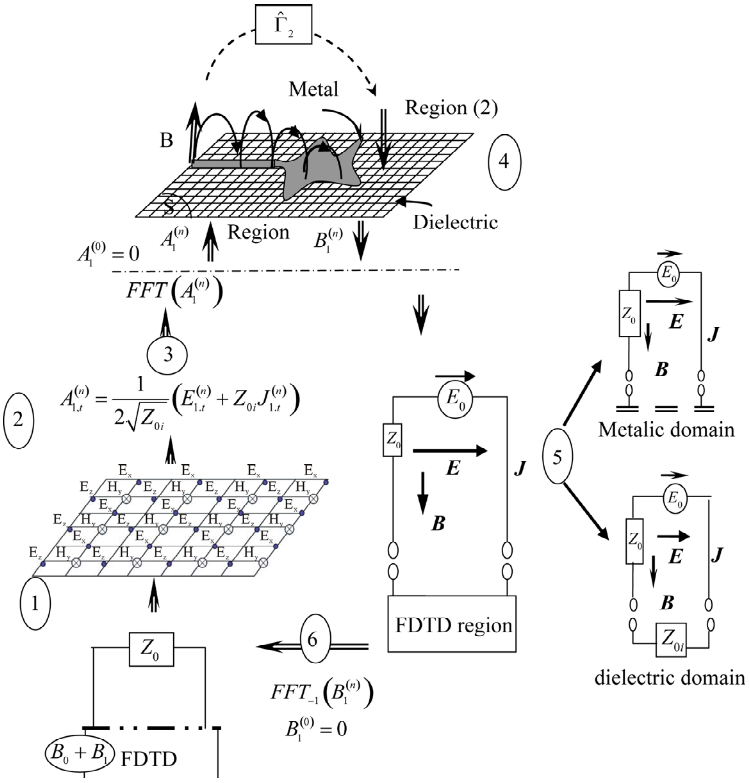

Figure 1. Schematic processes of the hybrid FDTD-WCIP approach.

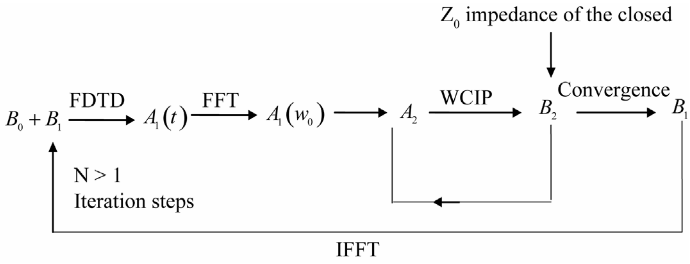

Figure 2. Hybrid method algorithm.

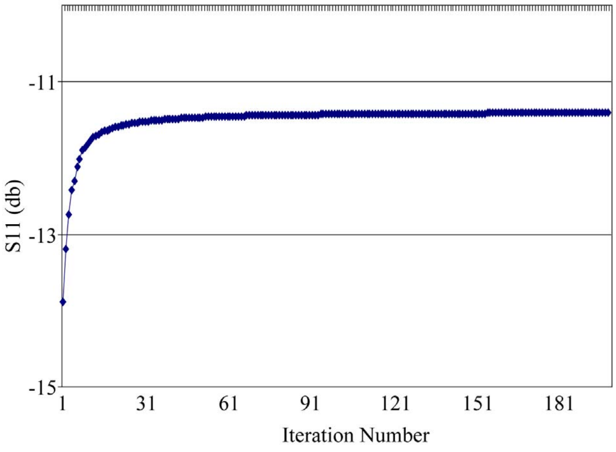

Figure 3. Convergence of the S11 on the via-hole grounded microstrip. During the iteration process of the hybrid method at 10 Ghz.

region is replaced by the FDTD region, as shown in **Figure 1**. Since the microstrip and ground plane coincide with the top and the bottom boundaries of the FDTD region and, the Dirichlet boundary conditions are applied to the top and the bottom as well as the via-hole cylinder wall [11]. The hybrid technique reduces the computational cost in the FDTD analysis and increases the computational efficiency.

The parameters of the first analyzed via-hole grounded microstrip structure presented in **Figure 4** are as follows: via-hole diameter is 0.6 mm, microstrip width is 2.3 mm, and substrate thickness is 0.794 mm. lastly, the substrate has a low dielectric constant ($\varepsilon r = 2.3$).

The derived results from the present method agreed very well. It has been found that a via at the center of a microstrip line provide a good dc connection at higher frequencies resulting in substantial coupling between the

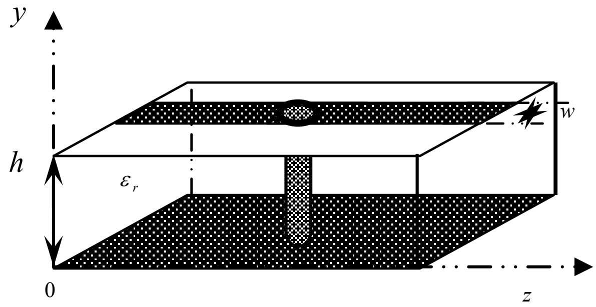

Figure 4. The via-hole grounded microstrip.

two sections of the line shown in **Figure 5**.

This hybrid method can be applied toward analyzing 3-D locally arbitrarily shaped structures accurately and efficiently. The next example is a three short-circuited stubs distributed highpass filter presented in **Figure 6**, designed using the conventional technique [12-14]. For which the design parameters are: Cut-off frequency, fc = 1.3 GHz, the dielectric Constant is $\varepsilon r = 2.2$ and the height of substrate is $h = 1.57$ mm, Characteristic impedance of terminating microstrip line, $Z_0 = 50\ \Omega$, Guided wavelength $\lambda gc = 167.24$ mm, corresponding width of the microstrip = 4.83 mm, Number of stub elements, $n = 3$ [14].

The electrical characteristics of the ground connection vs. frequency as evaluated by FDTD and WCIP are shown in **Figure 7** and demonstrate very good agreement between the two methods. The slight discrepancy between the values can be attributed to numerical errors associated with both techniques.

We will now present inhomogeneous dielectric structure to validate the hybrid technique to modeling a multilayer structure.

The present method was applied to characterize the rectangular dielectric resonant antenna structure [15-20] shown in **Figure 8**.

The parameters of the final example: the high permittivity ($\varepsilon r_d = 48$) rectangular Dielectric Resonator. The microstrip line is fabricated on a substrate of dielectric constant $\varepsilon_{rs} = 4.28$ and thickness $h_s = 1.6$ mm. The length of the microstrip line is chosen to be twice the resonator length ($l_f = 2l_d = 6.8$ cm), the width of the microstrip line $wf = 0.3$ cm, and the Ground plane dimensions: $lg = 9$ cm, $wg = 4$ cm.

A firstly we simulate the FDTD cell size is $\Delta x = 0.2$ mm, $\Delta y = 0.625$ mm, $\Delta z = 0.703$ mm and grid size is 60 × 64 × 128.

In the WCIP the cell size is $\Delta y = 0.625$ mm, $\Delta z = 0.703$ mm and the grid size is 64 × 128.

In order to evaluate the precision factor of the hybrid method, we use the cell with very fine sizes, to explain the influence of the discretization of the divergence of the method at high frequencies. So we must assign a simulation with very small cell size and compare it with

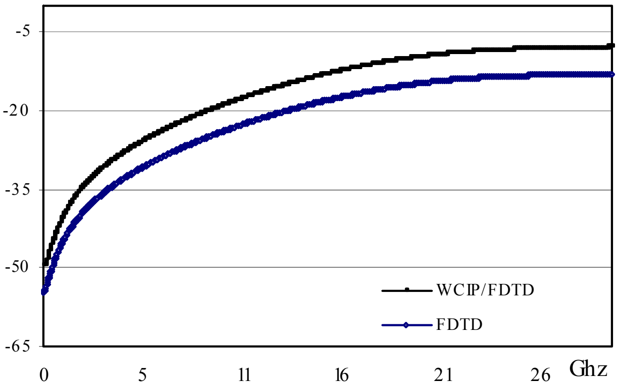

Figure 5. Comparison of the S21 response for the proposed the via-hole grounded microstrip obtained by the hybrid method and the FDTD simulation results.

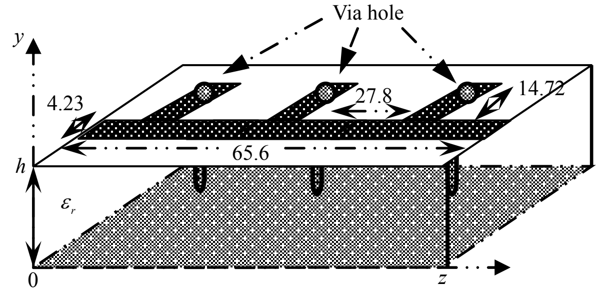

Figure 6. Three short-circuited stubs distributed highpass filter.

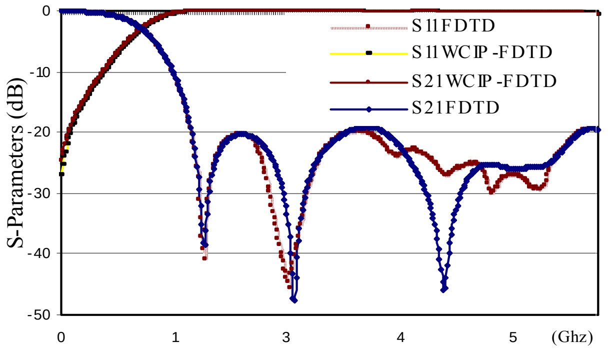

Figure 7. Comparison of the S11 response for the proposed filter obtained by the hybrid method and the FDTD simulation results.

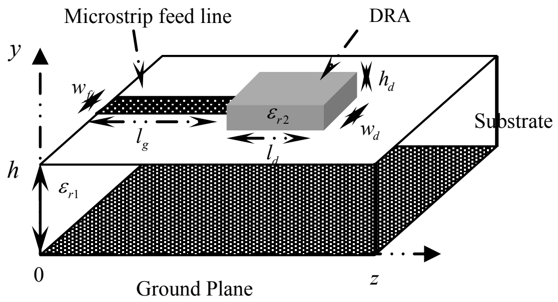

Figure 8. Lay out of the rectangular DRA antenna.

Table 1. Comparison between the FDTD solution and the Hybrid FDTD-WCIP technique regarding the computational time for the different structure illustrated in this work.

	Computer used for the simulation Pentium(R)4 CPU 3.20 Ghz				
	FDTD Method		Hybrid FDTD-WCIP		$\frac{T\_FDTD}{T\_HYBRID}$
	Number of cells	Time of the simulation	Number of cells	Time of the simulation	
HP Filter	491,520	58 min 52 s	114688	22 min 20 s	2.6358
DRA antenna	1,966,080	243 min 03 s	90112	89 min 20 s	2.714

the first simulation with a regular discretization, The FDTD domain is 60 × 128 × 256 and the WCIP grid size is 128 × 256, and the cell size is Δx = 0.2 mm, Δy = 0.3125 mm, Δz = 0.35 mm.

Good agreements are observed in **Figure 9** for the results of the reflection parameters between the hybrid method and the FDTD method [21], which verified the accuracy of the proposed hybrid method.

Let us now consider the computational cost of the Hybrid FDTD-WCIP method, comparing it with the conventional FDTD method in **Table 1**.

Even for more general structures with several different kinds of layers, the memory requirement for the present method is much smaller than that for the other hybrid FDTD methods [22].

It should be noted that, although the hybrid algorithm (FDTD-WCIP) is totally stable, but this approach suffer from the reduced of accuracy and computational efficiency when increasing the frequency, due to the inhererent discretization imposed by the WCIP Method.

We show that the accuracy can be greatly improved by using a higher number of cells in the contact surface S between the two domains, complying with condition imposed by the WCIP method. So to increase the number of cell must satisfy both conditions:

The accuracy condition in the WCIP method requires that the cell size must be less than $\lambda/50$, which increases the number of cells in three dimensional domain and gradually increases the computation time and cost memory.

Second is there must be a number of cell in the WCIP domain(in the *xy* plane) of the order of P = 2 ^ n, which is forcing us to round each time the number of cell in the *xy* plane by a value above or below the value that satisfy the discretization developed by the FDTD method.

These two conditions that promote the divergence of this method in the high frequency. To solve this problem we propose to develop a hybrid method with an irregular mesh, which can limit the divergence constraints on the conditions of the discretization in the WCIP method.

4. Conclusions

It is estimated whereas we will solve many problems by using the hybrid method which will allow us to have much more facility in modelling of the structures and especially to make simulation with a minimum of time. We succeeded in formulating a three-dimensional and fast numerical method.

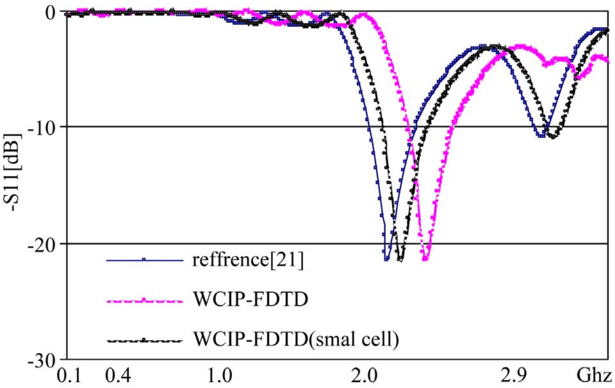

Figure 9. Comparison of the results obtained by the proposed hybrid method with the FDTD simulation results.

A hybrid FDTD-WCIP method is implemented in this letter using an iterative solution approach. Numerical results show that the hybrid method is accurate and efficient especially in terms of memory usage.

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