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Two-Dimensional Numerical Investigation on Applicability of 45˚ Heat Spreading Angle

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Abstract

The 45˚ heat spreading angle is familiar among thermal designers. This angle has been used for thermal design of electronic devices, and provides a heat spreading area inside a board, e.g. printed circuit board, which is placed between a heat dissipating element and a relatively large heat sink. By using this angle, the heat transfer behavior can be estimated quickly without using high-performance computers. In addition, the rough design can be made easily by changing design parameters. This angle is effective in a practical situation; however, the discussion has not been made sufficiently on the applicability of the 45˚ heat spreading angle. In the present study, therefore, the extensive numerical investigation is conducted for the rational thermal design using the 45˚ heat spreading angle. The two-dimensional mathematical model of the board is considered; the center of the top is heated by a heat source while the bottom is entirely cooled by a heat sink. The temperature distribution is obtained by solving the heat conduction equation numerically with the boundary conditions. From the numerical results, the heat transfer behavior inside the board is shown and its relation with the design parameters is clarified. The heat transfer behavior inside the 45˚ heat spreading area is also evaluated. The applicability is moreover discussed on the thermal resistance of the board obtained by the 45˚ heat spreading angle. It is confirmed that the 45˚ heat spreading angle is applicable when the Biot number is large, and then the equations are proposed to calculate the Biot number index to use the 45˚ angle. Furthermore, the validity of the 45˚ heat spreading angle is also confirmed when the isothermal boundary condition is used at the cooled section of the board.

Keywords

45˚ Heat Spreading Angle; Electronic Board; Electronics Cooling; Thermal Design; Heat Conduction

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1. Introduction

A heat spreading angle is one of the concepts to simplify the heat transfer calculation and has been used for thermal design of electronic devices. This angle is applied for the board, e.g. printed circuit board, placed between a heat dissipating element and a relatively large heat sink, and then assumes a heat spreading area inside the board. While high-performance CFD simulators provide detailed numerical information, such a simplified calculation is also effective in a practical situation because the estimation can be made quickly without using high-performance computers. In addition, the rough design can be made easily by changing design parameters such as dimensions, physical properties and operating conditions, etc.

According to Vermeersch and Mey [1], the early paper about this spreading angle goes back to the publication by Balents *et al.* in 1969. Balents *et al.* [2] used the constant value of 45˚ as the heat spreading angle and conducted an approximate heat transfer analysis concerning a heat dissipating element mounted directly on a ceramic substrate which was bonded to a metal baseplate. After a while, the 45˚ heat spreading model of Balents *et al.* was used by Cook *et al.* [3] in 1976, and thereafter many attempts have been made to modify the value or extend the theory of the heat spreading angle. Nevertheless, the above-mentioned 45˚ heat spreading angle is still familiar among thermal designers.

The applicability of the 45˚ heat spreading angle was described by several researchers. Masana [4] showed the comparison between the 45˚ heat spreading approach and the Fourier solution for a substrate (thickness: w) on which a square (dimensions: $l \times l$) heat dissipating element was mounted. It was described that the common assumption of 45˚ for the spreading angle got close to the Fourier solution for large w/l. Guenin [5] also compared the 45˚ heat spreading analysis to the finite element analysis, and then concluded that the 45˚ heat spreading approximation provided the reasonable accuracy for boards in which the size of the heat source area was greater than the board thickness. Malhammer [6] described briefly that if a 20% error could be accepted, the 45 degree rule was always relevant for small heat sources. Lasance [7] reported that the 45˚ heat spreading angle was acceptable in a single layer board for $h/k < 1$ and $d > 2$ mm, where h was the heat transfer coefficient at the cooled section, k and d were the thermal conductivity and the thickness of the board, respectively. Furthermore, concerning the limitation, Lasance [7] and Ha and Graham [8] mentioned that the 45˚ heat spreading approach could not be used for multi-layer boards.

The above descriptions are guides in conducting the heat transfer calculation using the 45˚ heat spreading angle. However, they seem to be incoherent; accordingly, the comprehensive investigation is needed for the rational thermal design using the 45˚ heat spreading angle. In the present study, therefore, the extensive numerical investigation is conducted on the applicability of the 45˚ heat spreading angle. For simplicity, the two-dimensional mathematical model is considered and the calculation is conducted in a cylindrical coordinate system. Based on the numerical results, the validity of the 45˚ heat spreading angle and its relation with design parameters are discussed.

2. Analytical Method

2.1. Mathematical Modeling

Figure 1 shows the analytical system. The heat transfer characteristics inside the board are analyzed in a cylindrical ($r - z$) coordinate system. This board is essentially desk-shaped, and has the thickness of z_b. The center (radius: r_h) of the top is heated by a heat source while the bottom (radius: r_c) is entirely cooled by a heat sink. The 45˚ heat spreading angle is shown in **Figure 2**.

The governing equation is given by

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0, \tag{1}$$

and the following boundary conditions are applied for the heated section ($0 \le r \le r_h$, $z = 0$) and the cooled section ($0 \le r \le r_c$, $z = z_b$), respectively:

$$-\lambda\left(\partial T/\partial z\right) = q \text{ at heated section,} \tag{2}$$

$$-\lambda\left(\partial T/\partial z\right) = \alpha\left(T - T_f\right) \text{ at cooled section,} \tag{3}$$

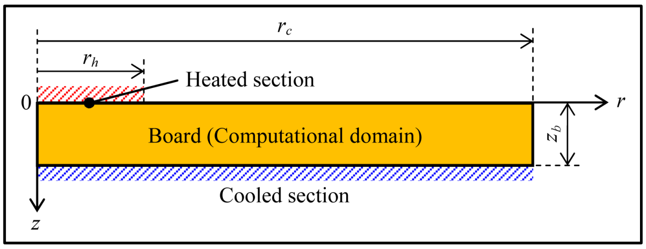

Figure 1. Analytical system.

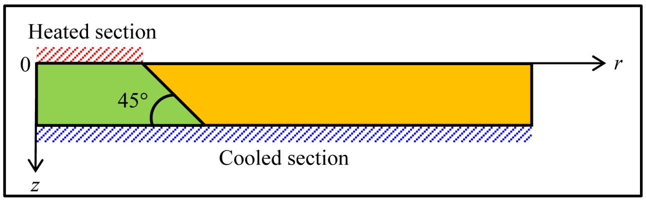

Figure 2. 45˚ heat spreading angle.

where λ is the thermal conductivity of the board, q the heat flux from the heat source, α the heat transfer coefficient at the cooled section and T_f the cooling fluid temperature. The following adiabatic boundary condition is applied at the surfaces except for the heated and the cooled section:

$$\partial T/\partial n = 0, \tag{4}$$

where n is the coordinate normal to the boundary surface.

The governing equation is discretized by the control volume method [9] and solved numerically with the boundary conditions. The analytical solution of Equations (1)-(4) having infinite series and Bessel functions was also shown by Lee *et al.* [10]. However, because the analytical solution is complicated and time-consuming, the numerical procedure is adopted here to obtain the results quickly. The numerical calculation is conducted by changing the parameters as shown in **Table 1** at $z_b = 4.0$ mm, $q = 1.0$ W/cm^2 and $T_f = 20$˚C.

2.2. Dimensionless Parameters

The dimensionless temperature, θ, and the dimensionless coordinates, R, Z, N, are defined as follows:

$$\theta = \frac{T - T_f}{q\left(\dfrac{z_b}{\lambda} + \dfrac{1}{\alpha}\right)}, \quad R = \frac{r}{z_b}, \quad Z = \frac{z}{z_b}, \quad N = \frac{n}{z_b}. \tag{5}$$

Accordingly, the dimensionless governing equation and the dimensionless boundary conditions for the heated section ($0 \le R \le R_h$, $Z = 0$), the cooled section ($0 \le R \le R_c$, $Z = 1$) and the other adiabatic section are expressed as follows:

$$\frac{\partial^2\theta}{\partial R^2} + \frac{1}{R}\frac{\partial\theta}{\partial R} + \frac{\partial^2\theta}{\partial Z^2} = 0, \tag{6}$$

$$-\partial\theta/\partial Z = Bi/(1+Bi) \ \text{ at heated section}, \tag{7}$$

$$-\partial\theta/\partial Z = Bi\,\theta \ \text{ at cooled section}, \tag{8}$$

$$\partial\theta/\partial N = 0 \ \text{ at adiabatic section}, \tag{9}$$

where R_h, R_c and Bi are the following dimensionless radii and the Biot number:

$$R_h = \frac{r_h}{z_b}, \quad R_c = \frac{r_c}{z_b}, \quad Bi = \frac{\alpha z_b}{\lambda}. \tag{10}$$

It should be noted that the heat transfer behavior inside the present model is characterized by R_h, R_c, and Bi.

Table 1. Numerical conditions.

r_h [mm]	4, 8, 16
r_c [mm]	20, 30, 40, 50, 60
λ [W/(m·K)]	0.4 - 400
α [W/(m^2·K)]	100 - 10,000

3. Results and Discussion

3.1. Temperature Distribution

The representative numerical results are shown in **Figures 3** and **4**. The effect of λ is shown at α = 100 W/(m^2·K) and α = 5000 W/(m^2·K). **Figure 3** is the temperature contour inside the board, and **Figure 4** the temperature distribution at the cooled section (the top surface of the board). It should be noted that in **Figure 4**, the scales of the vertical axis for α = 100 W/(m^2·K) and α = 5000 W/(m^2·K) are different. In addition, if the conductive thermal resistance inside the board is neglected, the temperature of the board becomes uniform and then the uniform temperature, $\bar{T}$, is obtained by the following equation:

$$\bar{T} = \frac{q}{\alpha}\left(\frac{r_h}{r_c}\right)^2 + T_f . \tag{11}$$

The value of $\bar{T}$ is also shown in **Figure 4**. When α = 100 W/(m^2·K) and λ = 0.4 W/(m·K), it is observed that the contour lines inside the board are crowded near the heated section. Accordingly, the temperature at the cooled section changes conspicuously near the heated section and then approaches the cooling temperature at T_f = 20˚C. In this case, because the conductive thermal resistance inside the board is relatively large, the heat flow is not spread out but concentrated near the heated section. When α = 100 W/(m^2·K) and λ = 400 W/(m·K), on the other hand, it is observed that except for the region near the heated section, the contour lines are almost perpendicular to the top/bottom surfaces and then the temperature at the cooled section is very close to the uniform temperature at $\bar{T} = 24.0^\circ\text{C}$. In this case, the heat is spread out over the board. It is also confirmed that the temperature distribution becomes flatter and approaches $\bar{T}$ with the increase in λ.

When α = 5000 W/(m^2·K), it is observed that the temperature inside the board is entirely lower than that in the case of α = 100 W/(m^2·K). Furthermore, the temperature at the cooled section is very close to the uniform temperature at $\bar{T} = 20.1^\circ\text{C}$ even in the case of λ = 0.4 W/(m·K). This is attributed to the decrease in the convective thermal resistance at the cooled section, which enhances the heat transfer in z direction.

The temperature distribution at the cooled section is moreover shown in **Figure 5** in the dimensionless form. The numerical results are shown by changing all dimensionless parameters of R_h, R_c and Bi. It should be noted that the scales of the vertical axis for R_h = 1, R_h = 2, R_h = 4 are different. In addition, the value of Bi is also indicated in **Figure 3** for reference. Based on Bi, the heat transfer behavior inside the board is characterized as follows. When Bi is very small, namely Bi = 0.001, because the conductive thermal resistance inside the board is very small compared to the convective thermal resistance at the cooled section, the temperature is almost uniform irrespective of R_h and R_c. When Bi = 0.1, the temperature distribution is conspicuous due to the increase in the conductive thermal resistance inside the board. Furthermore, when Bi = 10, because of the decrease in the convective thermal resistance at the cooled section, the temperature is entirely lower than that in the case of Bi = 0.1.

Because the heated and the cooled area are prescribed by R_h and R_c respectively, it is expected that the temperature entirely increases with the increase in R_h or the decrease in R_c. However, due to the concentration of heat flow near the heated section, it is found that the temperature distribution is hardly affected by R_c, namely, the board width when Bi is large (Bi = 10).

3.2. 45˚ Heat Spreading Characteristics

In order to evaluate the 45˚ heat spreading characteristics, the index, η, is defined here by the ratio of the heat transfer rate inside the 45˚ heat spreading area to the total heat transfer rate through the board. η is calculated by,

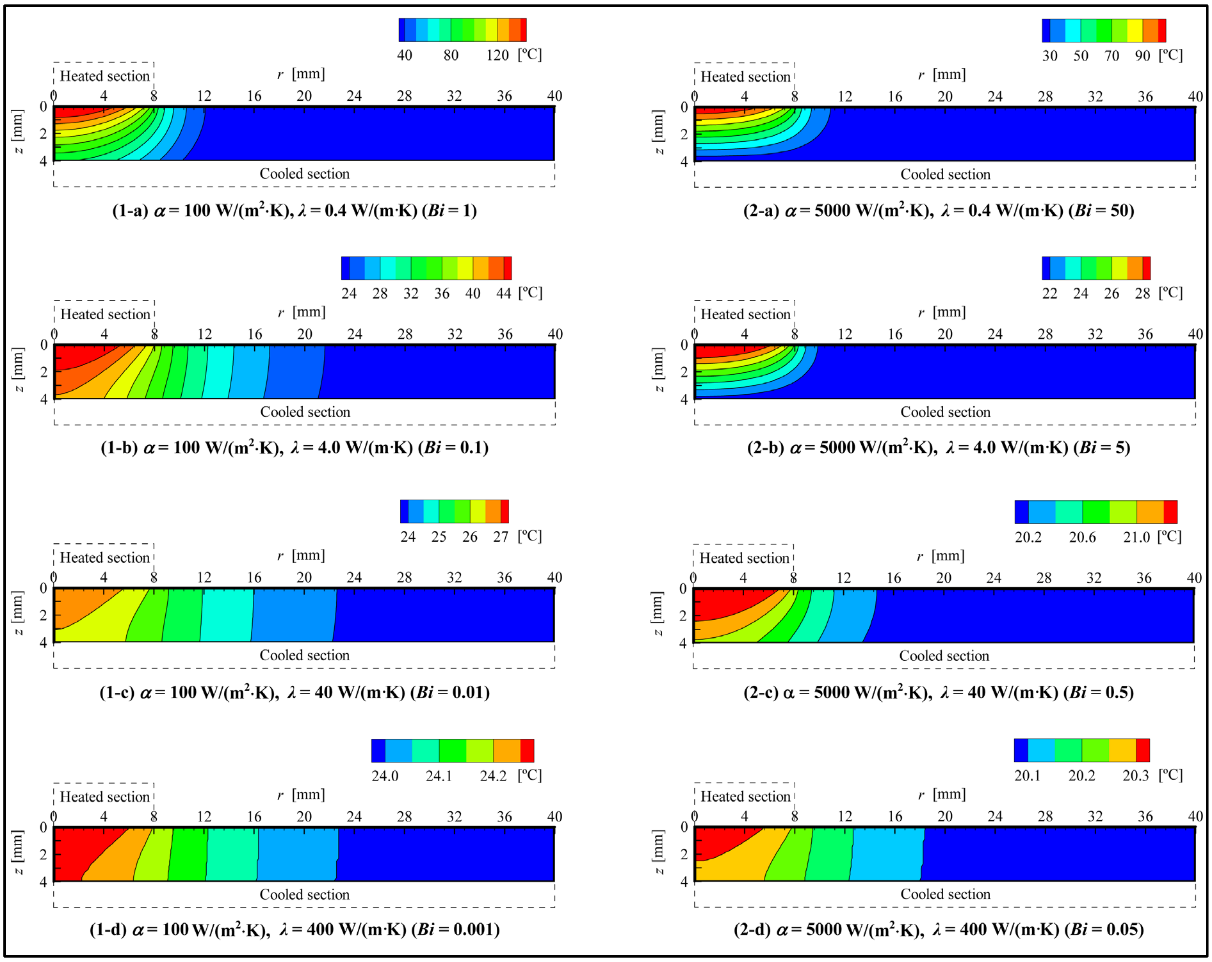

Figure 3. Temperature distribution inside board (r_h = 8 mm, r_c = 40 mm).

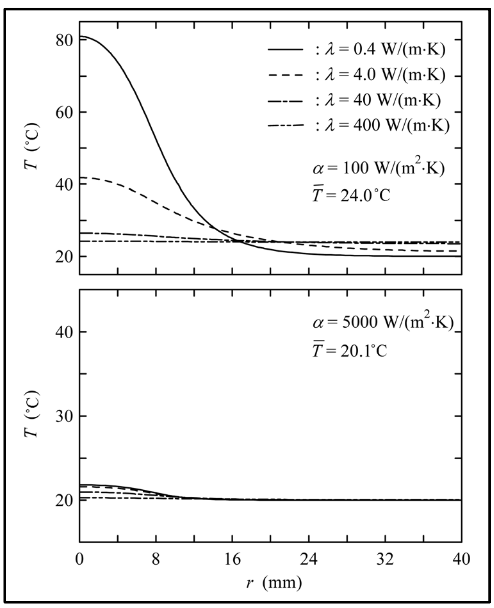

Figure 4. Temperature distribution at cooled section (r_h = 8 mm, r_c = 40 mm).

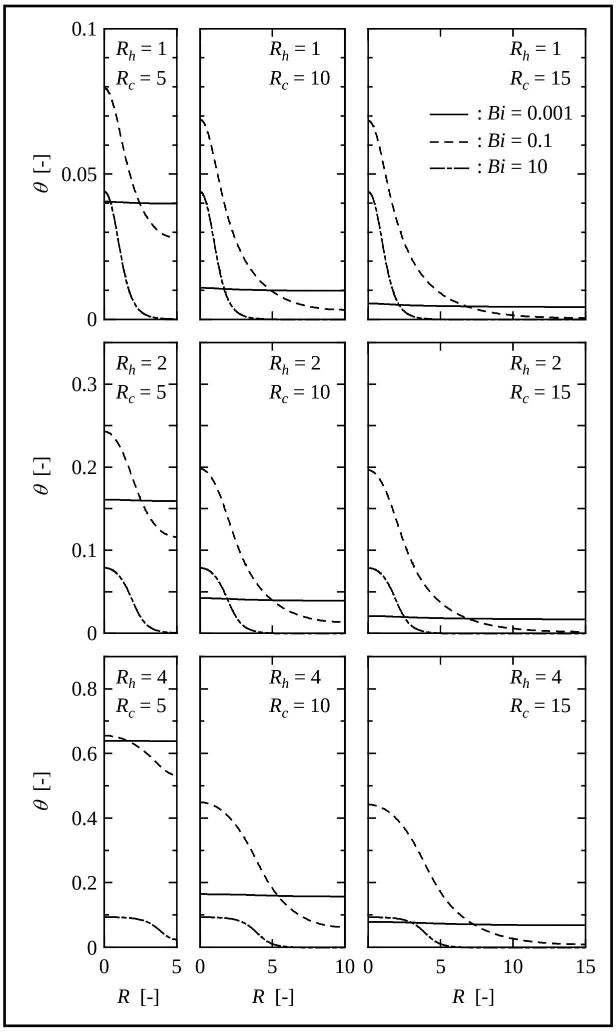

Figure 5. Dimensionless temperature distribution at cooled section.

$$\eta = \frac{\int_0^{r_h+z_b} \alpha \left(T\big|_{z=z_b} - T_f \right) 2\pi r \mathrm{d}r}{q\pi r_h^2}. \tag{12}$$

Figure 6 shows the relation between η and λ changing α and r_c as parameters. As reference, the values of η = 0.80 and η = 0.90 are indicated by lines in this figure. As expected, it is observed that η increases with the decrease in λ because of the concentration of heat flow near the heated section; η increases with α because of the enhancement of heat transfer in z direction. It is also confirmed that the high value of η such as η = 0.80 and η = 0.90 is obtained for small λ and large α, which corresponds to relatively large Bi.

The numerical calculation is moreover conducted by using the following isothermal boundary conditions:

$$T = T_f. \tag{13}$$

This boundary condition is applied at the cooled section and then the numerical result of η is also shown in **Figure 6**. It is found that irrespective of the numerical conditions, the high value of η ($\eta \approx 0.91$) is obtained under Equation (13), implying that the 45˚ heat spreading angle is applicable when the isothermal boundary condition is used at the cooled section.

By changing all dimensionless parameters of R_h, R_c and Bi, η is rearranged as shown in **Figure 7** in the dimensionless form. It should be noted that η = 1 is obtained irrespective of Bi when R_h = 4 and R_c = 5, because

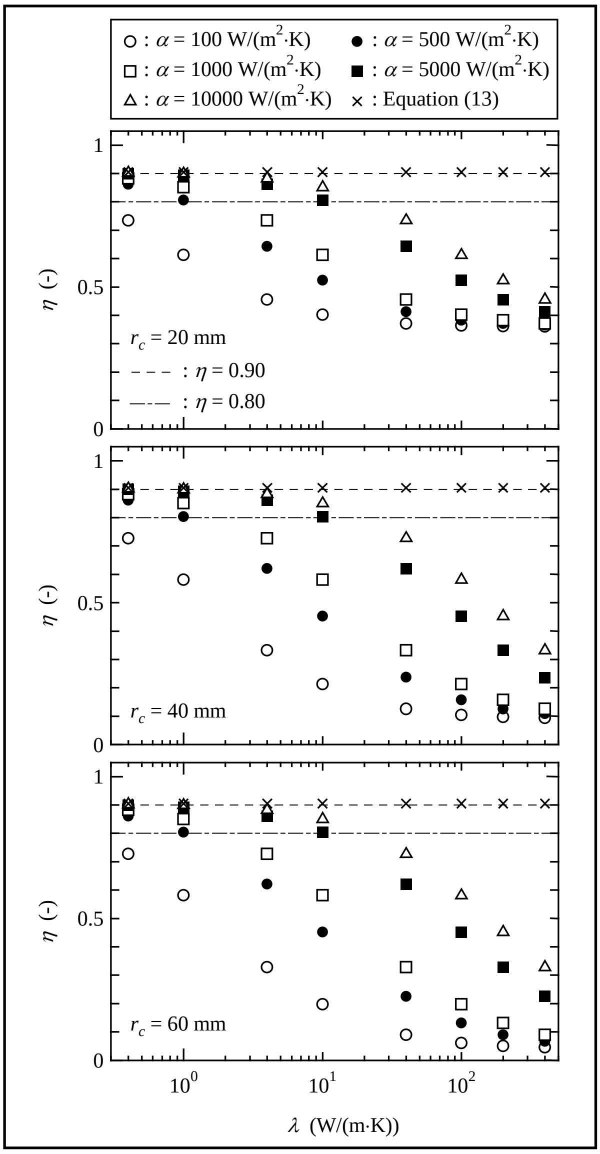

Figure 6. 45˚ heat spreading characteristics (r_h = 8 mm).

the cooled section is completely included within the 45˚ heat spreading area. This trivial case is not shown in this figure. The values of η = 0.80 and η = 0.90 are also indicated by lines in this figure. As shown, η is well characterized by R_h, R_c and Bi. It is observed that irrespective of R_h and R_c, η increases with Bi and then relatively large increase in η is obtained between Bi = 0.01 and Bi = 10. Furthermore, it is also found that the value of η is not affected by R_c when Bi is large. This is due to the concentration of heat flow near the heated section.

From **Figure 7**, the values of Bi when η = 0.80 and η = 0.90 are obtained, and then these values denoted by $Bi|_{\eta=0.80}$ and $Bi|_{\eta=0.90}$ respectively are shown in **Figure 8**. Since η is not affected by R_c for large Bi as mentioned above, $Bi|_{\eta=0.80}$ and $Bi|_{\eta=0.90}$ depend on only R_h. Moreover, the relations between $Bi|_{\eta=0.80}$, $Bi|_{\eta=0.90}$ and R_h are expressed as follows:

$$Bi\big|_{\eta=0.80} = \left(0.0450R_h^2 + 0.333R_h - 0.327\right)^{-1} \quad \left(1 \le R_h \le 5\right), \tag{14}$$

$$Bi\big|_{\eta=0.90} = \left(0.0104R_h^2 + 0.176R_h - 0.372\right)^{-1} \quad \left(2 \le R_h \le 5\right). \tag{15}$$

Equations (14) and (15) are also shown in **Figure 8**. It is confirmed that the 45˚ heat spreading angle is appli-

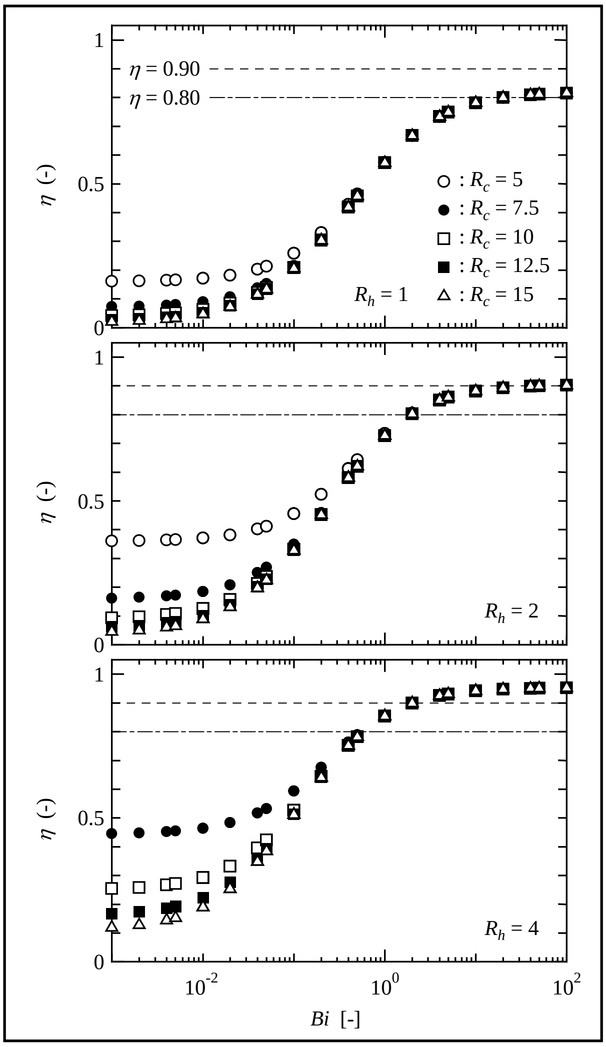

Figure 7. 45˚ heat spreading characteristics in dimensionless form.

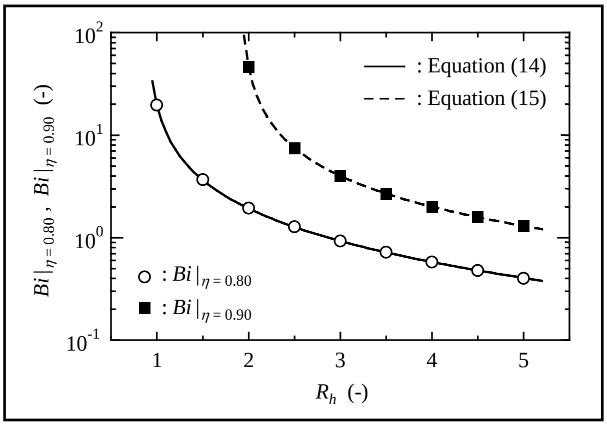

Figure 8. Biot number index to use 45˚ heat spreading angle.

cable when *Bi* is relatively large as expected, and then the specific numerical values of *Bi* are obtained by Equations (14) and (15) when 20% and 10% errors are accepted, respectively.

3.3. Thermal Resistance

When the 45˚ heat spreading angle is used, the thermal resistance of the board, R_{45}, is simply obtained as

$$R_{45} = \frac{z_b}{\pi \lambda r_h \left(r_h + z_b \right)}. \tag{16}$$

The derivation of Equation (16) is shown in **Appendix**. From the numerical results, on the other hand, the thermal resistance of the board, R_b, is also obtained by subtracting the convective thermal resistance at the cooled section from the total thermal resistance. R_b is calculated by,

$$R_b = \frac{T_{h,ave} - T_f}{q \pi r_h^2} - \frac{1}{\alpha \pi r_c^2}, \tag{17}$$

where $T_{h,ave}$ is the average temperature at the heated section.

By using the ratio of R_{45} to R_b, the comparison between these thermal resistances is shown in **Figure 9** changing all dimensionless parameters of R_h, R_c and Bi. Irrespective of the numerical conditions, it is found that the value of R_{45}/R_b is less than 1, implying that R_{45} is smaller than R_b. This is owing to the fact that the effect of the conductive thermal resistance inside the board in r direction is neglected in Equation (16). It is also observed that the change in R_{45}/R_b with Bi and R_c is similar to that in η; while in the range of $R_{45}/R_b > 0.8$, the effect of R_h on R_{45}/R_b is smaller than the case of η. Therefore, $Bi = 10$ is the index to use Equation (16) when 20% error is accepted.

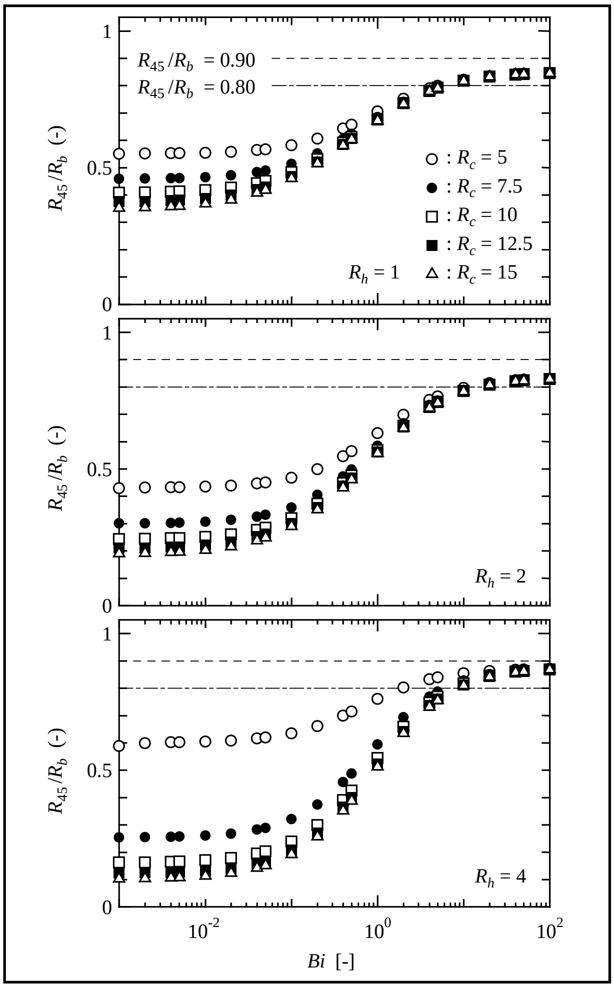

Figure 9. Thermal resistance of board: comparison between 45˚ heat spreading approach and numerical results.

4. Conclusions

The numerical analysis is conducted on the applicability of the 45˚ heat spreading angle inside the board. In the present calculation range, the main findings can be summarized as follows:

1) The heat transfer behavior inside the board is well characterized by the three dimensionless parameters: the two dimensionless widths of the heated and the cooled section, and the Biot number.

2) However, when the Biot number is large, the heat transfer behavior is hardly affected by the dimensionless width of the cooled section due to the concentration of heat flow near the heated section.

3) The validity of the 45˚ heat spreading angle is confirmed when the isothermal boundary condition is used at the cooled section.

4) The 45˚ heat spreading angle is applicable when the Biot number is large. The equations are proposed to calculate the Biot number index to use the 45˚ heat spreading angle.

5) The thermal resistance of the board obtained by the 45˚ heat spreading angle can be used with 20% error when the Biot number is 10.

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Appendix

The thermal resistance, R, is defined by

$$R = -\int \frac{dT}{Q(z)}, \tag{18}$$

where $Q(z)$ is the heat transfer rate inside the board at the position of z. When the 45˚ heat spreading angle is used, $Q(z)$ is expressed as

$$Q(z) = \pi (r_h + z)^2 \left(-\lambda \frac{\mathrm{d}T}{\mathrm{d}z} \right). \tag{19}$$

Substituting Equation (19) into Equation (18) and then integrating from $z = 0$ to $z = z_b$, R_{45} is obtained as shown in Equation (16).

Nomenclature

Bi : Biot number (-);
N : dimensionless coordinate normal to surface (-);
n : coordinate normal to surface (m);
Q : heat transfer rate (W);
q : heat flux (W/cm^2, W/m^2);
R : thermal resistance (K/W); dimensionless radial coordinate (-); dimensionless radius (-);
r : radial coordinate (mm, m); radius (mm, m);
T : temperature (˚C);
T_f : cooling temperature (˚C);
$\overline{T}$: uniform temperature (˚C);
Z : dimensionless axial coordinate (-);
z : axial coordinate (mm, m); thickness (mm, m).

Greek Symbols

α : heat transfer coefficient ($W/(m^2 \cdot K)$);
η : index defined by Equation (12) (-);
λ : thermal conductivity ($W/(m \cdot K)$);
θ : dimensionless temperature (-).

Subscripts

45 : 45˚ heat spreading angle;
ave: average;
b : board;
c : cooled section;
h : heated section.