Retraction Notice

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Expression of Concern:
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Correction:
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Non-Markovian Effects on the Dynamics of Entanglement in Real Environment

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Abstract

By considering the influence of the environmental disorder in the non-local decoherence system, we investigate the non-Markovian dynamics of two independent qubits. With the simulation of the environment noise as external entities, we find that the preservation of entanglement depends on the environmental disorder. Reasonable explanation is given with the comparison of the situation in Ref. [1].

Keywords

Non-Markovian, Entanglement, Non-Local Decoherence

1. Introduction

All realistic quantum systems are unavoidable to interact with its environments that occurs the so-called dissipation and decoherence, the dissipation and decoherence quantum dynamics of quantum cavities have been studied for decades because it's the bases of quantum communication and information [2] [3] [4] [5] [6]. Taking into account the influence of the environment on the quantum dynamics evolution makes the systems more complicate and interesting [7] [8]. We can mainly classify the quantum dynamical processes into Markovian processes with no memory effect and non-Markovian processes with memory effect, according to the environmental characters.

Most of the studies are interested with the information between the system and reservoir, to investigate their characters. For memoryless Markovian open systems, the system information losses to the environment monotonically, and the last time is relatively short. So it has many limitations for further investigation. However, in the non-Markovian case, employing the memory effect, the lost information will return to the system at a later time during the interaction.

This phenomenon arouses researchers much interest, and many works related non-Markovian dynamics has been investigated [9] [10] [11] [12] recently. Between the works, Breuer *et al.* raised a theorem called the degree of non-Markovianity which can be used to study the non-Markovian dynamics quantitatively in Ref. [13]. They calculate the changing rate of trace distance as a criterion to distinguish the states. The increment is interpreted as the return back information flow from the reservoir to the system.

However, the realistic environment is not easy to control, and it will cause decoherence. In this paper, we import a system M to simulate the environment and use the amplitude damping channel to research its characteristic. The article is organized as follows: In Section 2, we introduce our mode and build the initial states. Then the time-dependent evolution is calculated in Section 3. In Section 4 we use concurrence to study entanglement evolution and the analysis is given, too. At last, we conclude the paper simply.

2. Model

First, we introduce our model: Consider a system of two identical non-interacting two-level atoms, each of them coupled with its own reservoir which is initially in the vacuum state. Due to the independence of each atom, we only need to discuss the problem of a single atom interacting with its corresponding reservoir. The Hamiltonian of the interaction between a single atom and N-mode reservoir under the rotating-wave approximation can be written as:

$$H = \frac{1}{2}\omega_0\sigma_+\sigma_- + \sum_k \omega_k b_k^+ b_k + (\sigma_- B^+ + \sigma_+ B) \tag{1}$$

With $B = \sum g_k b_k$, where ω_0 is the transition frequency of the two-level atom, and $\sigma_\pm$ are the Pauli raising and lowering operators for the atom respectively. b_k^+ and b_k are the creation and annihilation operators with frequency ω_k for the reservoir mode *k*, and g_k is the corresponding coupling constant.

We prepare the initial state as the not quite perfect Bell state:

$$\left|\Phi\right\rangle_{AB} = \cos\theta\left|e\right\rangle_A\left|e\right\rangle_B + \sin\theta\left|g\right\rangle_A\left|g\right\rangle_B \tag{2}$$

where $\left|e\right\rangle_A$ and $\left|g\right\rangle_A$ are orthogonal states of qubit A, and the same as B, and mean as the excited and ground states. As we know, in reality, every system cannot avoid the affect of its environment, and a state will deviate from the ideal target state. Now we construct a new model to describe how this might occur. It is coupled to an environment that will cause post-preparation decoherence, and also still entangled with marginal entities denotes by [14]. We write the initial state as:

$$\left|\Phi(0)\right\rangle = [\cos\theta\left|e\right\rangle_A\left|e\right\rangle_B\left|m_1\right\rangle + \sin\theta\left|g\right\rangle_A\left|g\right\rangle_B\left|m_2\right\rangle] \otimes \left|\phi_0\right\rangle_a\left|\phi_0\right\rangle_b \tag{3}$$

where $\left|m_1\right\rangle$ and $\left|m_2\right\rangle$ are normalized states of the marginal system M, and $\left|\phi_0\right\rangle_a$ and $\left|\phi_0\right\rangle_b$ are the normalized states of the environmental reservoirs *a* and *b*, respectively. Imperfect control of the preparation leads to a possibly

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mixed rather than pure initial state with the relation $\eta = \langle m_2 | m_1 \rangle$, here η represents the environmental disorder with a span $0 \le \eta \le 1$. $\eta = 0$ means the environment is totally disordered, and $\eta = 1$ means the ideal environment. It is easy to write out the initial two-qubit reduced density matrix:

$$\rho = \begin{pmatrix} \rho_{11}(0) & 0 & 0 & \rho_{14}(0) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{41}(0) & 0 & 0 & \rho_{44}(0) \end{pmatrix} = \begin{pmatrix} \cos 2\theta^2 & 0 & 0 & \cos\theta\sin\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cos\theta\sin\theta & 0 & 0 & \sin 2\theta^2 \end{pmatrix} \tag{4}$$

3. States Evolution

Now we consider the time-dependent dynamics of the two-qubit entanglement to see how it is affected by the initial state specification. Each qubit takes the same amplitude damping process [1] [15] [16]:

$$|g\rangle_A |\phi_0\rangle_a \to |g\rangle_A |\phi_0\rangle_a \tag{5}$$

$$|e\rangle_A |\phi_0\rangle_a \to \sqrt{P_a(t)}\,|e\rangle_A |\phi_0\rangle_a + \sqrt{1-P_a(t)}\,|g\rangle_A |\phi_0\rangle_a \tag{6}$$

where $|\phi_1\rangle_a$ is another environmental reservoir state with ${}_a\langle\phi_1|\phi_0\rangle_b = 0$, and $P_a(t)$ is the probability that qubit A remains in its excited state, here $P_a(t)$ obeys the differential equation:

$$\dot{P}_a(t) = -\int_0^t dt_1 f(t-t_1) P_a(t) \tag{7}$$

and $f(t-t_1)$ is the correlation function which is related to the spectral density $J(\omega)$ of the reservoir:

$$f(t-t_1) = \int d\omega J(\omega) \exp[i(\omega_0 - \omega)(t-t_1)] \tag{8}$$

The exact form of $P_a(t)$ depends on the particular choice for the spectral density of the reservoir. We take $J(\omega)$ as the Lorentzian form:

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_0 - \omega)^2 + \lambda^2} \tag{9}$$

The parameter λ defines the spectral width of the reservoir and its connected to the reservoir correlation time $\tau_R = \lambda^{-1}$. On the other hand, the parameter γ_0 can be shown to be related to the decay of the excited state of the qubit in the Markovian limit of a flat spectrum. The relaxation time scale τ_s, over which time state of the system changes is then related to γ_0 by $\tau_s = \gamma_0^{-1}$. Using the spectral density of Equation (5), we must distinguish a weak and a strong coupling regime. For a strong coupling regime $\gamma_0 > \frac{\lambda}{2}$ or $\tau_R < 2\tau_S$, the reservoir correlation time is greater than the relaxation time, the non-Markovian effects become evident. For this reason, we are mainly interested in this regime. The weak regime $\gamma_0 > \frac{\lambda}{2}$ ($\tau_R < 2\tau_S$) is the Markovian regime we just make some simple comparison with the non-Markovian regime.

We solve the $P_a(t)$ as:

$$P_a(t) = e^{-\frac{\lambda t}{2}}[\cos(\frac{dt}{2}) + \frac{\lambda}{d}\sin(\frac{dt}{2})] \tag{10}$$

where $d = \sqrt{2\gamma_0\lambda - \lambda^2}$. $P_a(t)$ presents oscillations describing the fact that the decay of the atom excited state is induced by the coherent processes between the system and the reservoir.

Qubit B and reservoir b are characterized similarly as Equation (5), Equation (6) with decay probability $P_b(t)$.

From Equations (3), (5), (6) we can calculate the time dependent state $|\Phi(t)\rangle$ easily. For the exact expressions of $|\Phi(t)\rangle$ is too complex, we do not show it here. Here because of the fact that the marginal entity has ceased interaction with the qubit system after $t = 0$, the time dependent marginal states preserve the overlap relation $\langle m_2(t)|m_1(t)\rangle = \langle m_2(0)|m_1(0)\rangle$. Then the two-qubit time dependent reduced density matrix $\rho_{AB}(t)$ is obtained by tracing off the reservoir and marginal states, yielding

$$\begin{pmatrix} \rho_{11}(0)P_a(t)P_b(t) & 0 & 0 & \rho_{14}(0)\sqrt{P_a(t)P_b(t)} \\ 0 & \rho_{11}P_a(t)(1-P_b(t)) & 0 & 0 \\ 0 & 0 & \rho_{11}(1-P_a(t))P_b(t) & 0 \\ \rho_{41}(0)\sqrt{P_a(t)P_b(t)} & 0 & 0 & \rho_{11}(1-P_a(t))(1-P_b(t)) + \rho_{44} \end{pmatrix} \tag{11}$$

4. Entanglement Evolution and Analysis

In order to follow the entanglement dynamics of the bipartite system, we use Wotters concurrence [17]. This is obtained from the density matrix ρ^T for qubit A and B as $C_{\rho(t)} = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$, where the quantities λ_i are the eigenvalue of the matrix ζ:

$$\zeta = \rho^T(\sigma_y^A \otimes \sigma_y^B)\rho^{*T}(\sigma_y^A \otimes \sigma_y^B) \tag{12}$$

arranged in decreasing order. Here ρ^{*T} denotes the complex conjugation of ρ^T in the standard basis, and σ_y is the well-known Pauli matrix expressed in the same basis. The concurrence varies from $C = 0$ for a disentangled state to $C = 1$ for a maximally entangled state. The two-party entanglement obeys the X-state formula [18]

$$C(t) = \max\left\{0, 2\sqrt{P_a(t)P_b(t)}|\rho_{14}| - 2\sqrt{P_a(t)P_b(t)(1-P_a(t))(1-P_a(t))}\rho_{11}\right\} \tag{13}$$

Now we analyze the evolution regulation of entanglement. We draw **Figure 1** to make a comparison between the Markovian and non-Markovian regime. It is obvious that in Markovian regime the concurrence $C(t)$ decays exponentially but in non-Markovian regime the evolution curve appears revival effect which is already explained in Ref. [2]. In order to investigate the parameter η, we draw **Figure 2**. The two figures' parameters have no difference but the value of η. Making an easy comparison with them we find that η is greatly affected the concurrence.

When $\eta = 1$, it represents the environment is an ideal external system, this result returns to Ref. [1]. We both have the identical conclusion. Then we order $\eta = 0.6$ for the right figure, the maximum value of $C(t)$ turns down and the non-Markovian effect is not as obvious as before. It is not hard to explain this phenomenon, the critical factor is the environmental disorder.

Then environment is more disorder, the more information could loss, and the revival effect is not obvious naturally. To proof our conclusion, we draw **Figure 3** to give a more clear explanation go a step. From **Figure 3** we can see that the bigger η is, the larger is $C(t)$, which follows the same regulation.

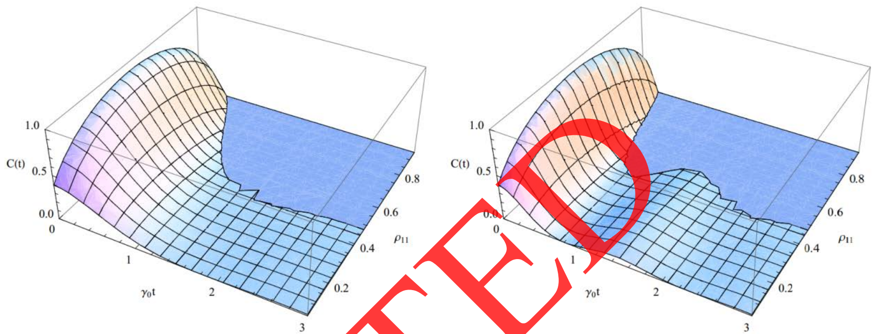

Figure 1. Time-dependent surfaces showing entanglement evolution parameterized in two regimes: Left for $\lambda = 5\gamma_0$ which is in the Markovian regime; Right for $\lambda = 0.1\gamma_0$ which is in the non-Markovian regime. Here we order $\eta = 0.9$.

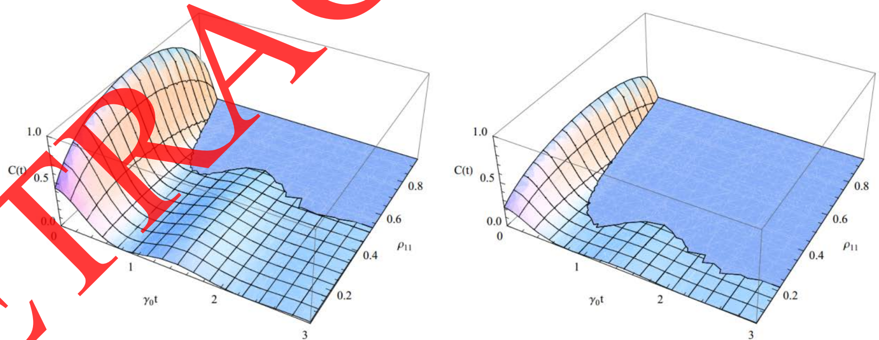

Figure 2. Time-dependent surfaces showing entanglement evolution in non-Markovian regime with $\lambda = 0.1\gamma_0$; Left for $\eta = 1$; Right for $\eta = 0.6$.

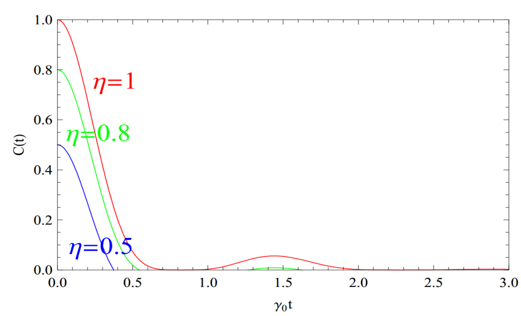

Figure 3. Concurrence against with the dimensionless quantity $\gamma_0 t$ in non-Markovian regime at $\rho_{11} = 0.5$. The red line is for $\eta = 1$, the green line is for $\eta = 0.8$ and the blue line is for $\eta = 0.5$.

5. Conclusions

In conclusion, we have investigated the behavior of non-local decoherence in this paper.

We choose the not quite perfect Bell state as the initial condition and employ a system M to consider the affect of the environment. After the amplitude damping process, we trace off the external entities to get the reduced density matrix which has the X-style. Considering the concurrence of entanglement in the Markovian and non-Markovian regime, we find that the entanglement is greatly affected by the environmental disorder. The results are identical with Ref. [1]. The work provides a method to transfer entangled communication effectively.

Supported

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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