Journal of Applied Mathematics and Physics, 2019, 7, 1289-1303
http://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Mathematical Model of the Dynamics of Rumor Propagation

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How to cite this paper: Musa, S. and Fori, M. (2019) Mathematical Model of the Dynamics of Rumor Propagation. *Journal of Applied Mathematics and Physics*, **7**, 1289-1303.
https://doi.org/10.4236/jamp.2019.76088

Received: April 28, 2019
Accepted: June 24, 2019
Published: June 27, 2019

Abstract

In this study, we proposed a deterministic mathematical model that attempts to explain the propagation of a rumor using epidemiological models approach. The population is divided into four classes which consist of ignorant individuals, $I(t)$, spreaders targeting community through media, $M(t)$, spreaders targeting community through verbal communication, $G(t)$ and stiflers, $R(t)$. We explored existence of the equilibria and analyzed its stability. It was established that rumour-free equilibrium E_o is locally asymptotically stable if $R_o < 1$; meaning rumor can seize spreading in a population, and unstable if $R_o > 1$ leads to new rumor spreading in the population. Numerical simulations of the dynamic model are carried out on the system to confirm the analytical results. We see that the dynamics of rumor spreading show similar behavior to that found in the dynamics of infectious diseases except that the spread depends on the classes of spreader.

Keywords

Rumor, Propagation, Rumor-Free Equilibrium, Rumor-Endemic Equilibrium

1. Introduction

The propagation of rumors is a typical form of social interaction that exerts a powerful influence on human affairs. Rumors represent unproven expositions about or interpretations of news, events, or problems that are of public interest. Because rumors are unconfirmed information, it is hard to determine whether they are true or false [1]. Traditionally, rumors are propagated by word of mouth [2] [3] [4]. Nowadays, with the emergence of the internet, rumors spread by instant messengers, emails or publishing blogging and social media that provide a faster speed of transmission [5] [6]. Once a rumor spreads, the truth is at risk of being distorted in the public sphere. Many case studies indicate that most rumors are false news, which exert negative impacts on the public and on social security [7]. Rumour is a generally circulated story, report or statement without facts to confirm its truth; it is like a virus, once it is transmitted to an individual, an outbreak of events happens in a short period of time. It is thus that spreading a rumor is like a contagious disease that does not need more than a few infected individuals to infect the entire population. If a rumor starts in a population, sooner or later everyone will know it [8].

In recent years, online social media is growing rapidly. Social media provides a convenient communication scheme for people. Meanwhile, the scheme enables unreliable sources to spread large amounts of unverified information among people [9] [10] [11] [12] [13]. Rumors are thus possible to spread more quickly and widely through online social media compared to traditional offline social communities. The wide spread of misinformation may bring disorder to people especially when they are facing religion or political crises. This indicates that it is crucial for social media to identify misinformation in time so as to limit the spread of rumors [14].

At the beginning, mathematical models for the rumors were considered merely speculative and imprecise, but for the fact that rumor spreading is now seen like the transmission of disease [15], it leads to a deeper understanding of the future spread of rumor. Rumor transmitted diseases are particularly well suited for modelling because the route of contact is clearly defined: it is through human activities [16] [17]. In addition, there are so many questions that come to play when it comes to addressing the spread of rumor. Modeling is the only practical approach. The spreading of rumor is in many ways similar to the spreading of epidemic infection by the spreader or the infections to notify or infect the susceptible [18] [19] [20] [21] [22]. After notified or infected, that the spreader can become the stifler is similar as the infections can recover after sometimes [23].

2. The Rumor Models

Rumor can be viewed as an infection of the mind [24] [25] [26] [27]. Therefore, the rumor propagation problem can be studied with many modeling techniques used in the study of epidemics. Some of these approaches include deterministic models, stochastic models and complex network [28].

Different possible behaviors in the dynamics of rumor spreading were studied by [25]. In their study, they proposed a deterministic propagation rumor model and detected relevant conditions, derived from local stability analysis of the rumor free equilibrium and rumor-endemic equilibrium.

Rumor propagation through different types of mathematical models was investigated by [29]. In their study, they consider rumor propagation with truth-spreading and determine the threshold which governs the dynamics of the system.

The dynamics of rumor model in complex heterogonous networks was analyzed by [6]. They first introduced a useful stochastic method that allows them to obtain meaningful time profiles for the quantities characterizing the propagation process.

3. Model Formulation

Following the classical assumption on the variables described in Table 1, we formulate a deterministic mathematical model to describe the propagation dynamics of rumors. In the model, the total population is divided into four epidemiological classes of ignorant $I(t)$, spreaders targeting community through media $M(t)$, spreaders targeting community through verbal communication $G(t)$ and stiflers $R(t)$. In the formulation of the model, we wish to clearly state some essential assumptions that:

1) Recruitment into the ignorant compartment is in constant rate,

2) There is no movement between the classes of spreaders,

3) Ignorant individual become spreader after interaction with spreader through media or spreader through verbal communication,

Table 1. Variables and parameters of the model and their descriptions.

Variable and parameter	Description
$I(t)$	Ignorant
$M(t)$	Spreader through media
$G(t)$	Spreader through verbal communication
	Communication
$R(t)$	Stifler
θ	Probability of ignorant to become stifler
β_1	Effective contact rate between the ignorant and the spreaders through media
β_2	Effective contact rate between the ignorant and the spreaders through verbal communication
α_1	Rate at which spreaders through media become stiflers
α_2	Rate at which spreaders through verbal communication become stiflers
σ	Proportion of I that interact with M and G and choose not to spread
π	Recruitment rate into the ignorant population
μ	Death rate

4) The probability of an ignorant to become a spreader through media or verbally are equal,

5) The spreading process are mutually exclusive event,

6) Spreaders through media and verbal communications become stifler at the rate α_1 and α_2 respectivelly,

7) Proportion of ignorant will interact with the spreaders at the rate σ and become stifler.

From the model diagram (**Figure 1**), we have ignorant class $I(t)$ to be individuals who are not aware of the rumor. The model is structured such that ignorant individual can either be infected with rumor by individual in $M(t)$ or $G(t)$. Note that, proportion σ of ignorant individual can easily progress to $R(t)$ without spreading the rumor. The spreader class $M(t)$ are individual who are aware of the rumor and are actively spreading it through media. The spreader class $G(t)$ are individuals who have been informed about the rumor and they are actively spreading it orally to the community. The stifler $R(t)$, are individuals who are individuals who are aware of the rumor, spent time spreading it but are no longer spreading the rumor due to some reasons and they also discourage the spread of the rumor.

Thus, the differential equations for the deterministic model are as follow:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \pi - \mu I - \frac{\theta}{2}\beta_1 IM - \frac{\theta}{2}\beta_2 IG - (M+G)\sigma I \quad (1.1)$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{\theta}{2}\beta_1 IM - \mu M - \alpha_1 M \quad (1.2)$$

$$\frac{\mathrm{d}G}{\mathrm{d}t} = \frac{\theta}{2}\beta_2 IG - \mu G - \alpha_2 G \quad (1.3)$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = (M+G)\sigma I + \alpha_1 M + \alpha_2 G - \mu R \quad (1.4)$$

$$N = I + M + G + R \quad (1.5)$$

4. Basic Properties

Since the model (1.1) to (1.4) monitors human populations, all the variables and the associated parameters are non-negative at all time. It is important to show that the model variables of the model remain non negative for all non-negative initial conditions.

Lemma 1: The region $D = \left\{ (I, M, G, R) \in R_+^4 : I + M + G + R \le \frac{\pi}{\mu} \right\}$ is positively invariant and attract all solutions in R_+^4.

Proof:

Adding all the equations from (1.1) to (1.4), gives the rate of change of the total human population

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \pi - \mu N$$

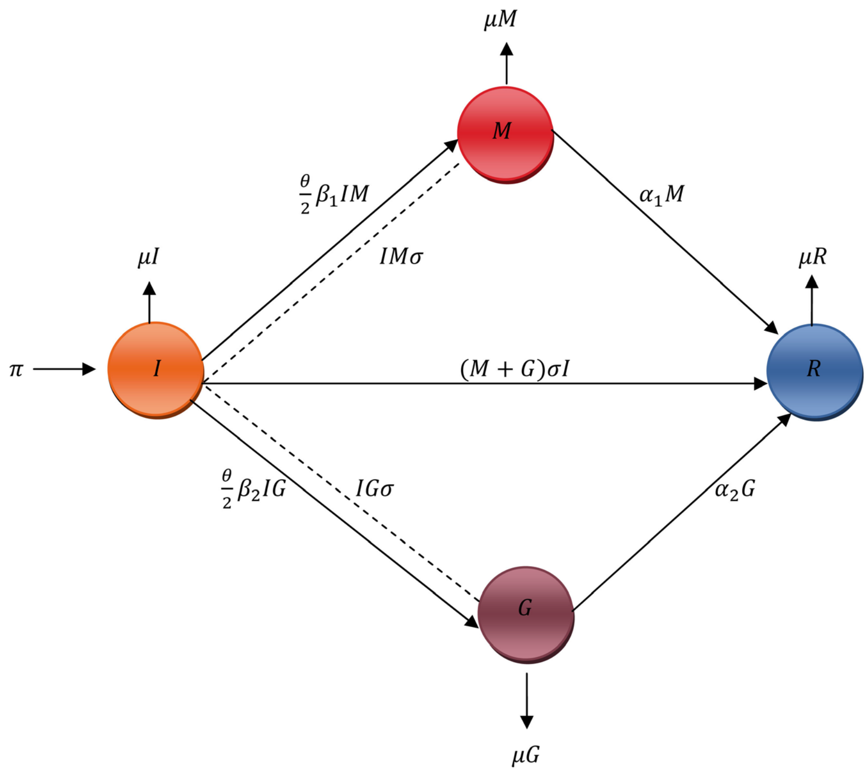

Figure 1. Model flow diagram.

Since $\frac{dN}{dt} = \pi - \mu N$ whenever $N(t) > \frac{\pi}{\mu}$, then $\frac{dN}{dt} < 0$, implying $\frac{dN}{dt}$ is bounded by $\pi - \mu N$.

Thus, a standard comparison theorem by [30] can be used to show that

$$N(t) \le N(0)e^{-\mu t} + \frac{\pi}{\mu}\left(1 - e^{-\mu t}\right)$$

In particular, $N(t) \le \frac{\pi}{\mu}$ if $N(0) \le \frac{\pi}{\mu}$. Thus, R is positively invariant (*i.e.* all solution in D remain in D for all time). Furthermore, if $N(t) > \frac{\pi}{\mu}$ then either the solution enters R in finite time or $N(t)$ approaches $\frac{\pi}{\mu}$ and the spreader variables M and G approaches zero. Hence, D is attracting (*i.e.* all solution in R_+^4 eventually approach, enter or stay in D). Therefore, the model is epidemiologically and mathematically well posed since all the variables remain non-negative for all $t \ge 0$. Hence it is sufficient to study the dynamics of the system (1.1) to (1.4) in D.

5. Analysis of the Model

In this section, we discuss the existence and uniqueness of Rumor Free Equilibrium (RFE) of the model and its analysis. The model Equations (1.1) to (1.4) has an RFE given by

$$E_o = \left(I^*, M^*, G^*, R^*\right) = \left(\frac{\pi}{\mu}, 0, 0, 0\right).$$

The local stability of RFE given will be investigated using the next generation matrix method. We calculate the next generation matrix for the system of the question (1.1) to (1.4) by enumerating the number of ways that: 1) new spreaders arise 2) number of ways that individuals can move but only one way to create a spreader. So, let

F = rate of appearance of new spreaders into the compartment and,

V = rate of transfer into (out) of compartment

$$F = \begin{pmatrix} \frac{\theta}{2}\beta_1 I^* & 0 \\ 0 & \frac{\theta}{2}\beta_2 I^* \end{pmatrix}$$

$$V = \begin{pmatrix} \mu+\alpha_1 & 0 \\ 0 & \mu+\alpha_2 \end{pmatrix}$$

Hence the NGM with large domain is two dimensional and is given by FV^{-1}

$$FV^{-1} = \begin{pmatrix} \frac{\theta\beta_1\pi}{2\mu(\mu+\alpha_1)} & 0 \\ 0 & \frac{\theta\beta_2\pi}{2\mu(\mu+\alpha_2)} \end{pmatrix} \tag{1.6}$$

The dominant eigenvalue of (1.6) is equal to R_o, therefore we evaluate the characteristic equation of (1.6) by using $\left|\left(FV^{-1}\right) - \lambda I\right| = 0$, which gives

$$R_o = R_{o_m} + R_{o_v}$$

$$R_o = \frac{\pi\theta\left[\beta_1(\mu+\alpha_2) + \beta_2(\mu+\alpha_1)\right]}{2\mu(\mu+\alpha_1)(\mu+\alpha_2)}$$

where

$$R_{o_m} = \frac{\theta\beta_1\pi}{2\mu(\mu+\alpha_1)}$$

$$R_{o_v} = \frac{\theta\beta_2\pi}{2\mu(\mu+\alpha_2)}$$

The jacobian of (1.1) to (1.4) at the equilibrium point $E_o = \left(\frac{\pi}{\mu}, 0, 0, 0\right)$ is

$$= \begin{pmatrix} -\mu & -\frac{(\theta\beta_1+\sigma\mu)\pi}{2\mu} & -\frac{(\theta\beta_2+\sigma\mu)\pi}{2\mu} & 0 \\ 0 & \frac{\pi\theta\beta_1}{2\mu} - \mu - \alpha_1 & 0 & 0 \\ 0 & 0 & \frac{\pi\theta\beta_2}{2\mu} - \mu - \alpha_2 & 0 \\ 0 & \alpha_1 + \frac{\sigma\pi}{\mu} & \alpha_2 + \frac{\sigma\pi}{\mu} & -\mu \end{pmatrix} \tag{1.7}$$

Now we try to calculate the eignvalues of (1.7) by finding the characteristic equation using the formula $\left|J_E - \lambda I\right| = 0$

$$\begin{vmatrix} (-\mu-\lambda) & -\frac{(\theta\beta_1+\sigma\mu)\pi}{2\mu} & -\frac{(\theta\beta_2+\sigma\mu)\pi}{2\mu} & 0 \\ 0 & \left(\frac{\pi\theta\beta_1}{2\mu}-\mu-\alpha_1-\lambda\right) & 0 & 0 \\ 0 & 0 & \left(\frac{\pi\theta\beta_2}{2\mu}-\mu-\alpha_2-\lambda\right) & 0 \\ 0 & \alpha_1+\frac{\sigma\pi}{\mu} & \alpha_2+\frac{\sigma\pi}{\mu} & (-\mu-\lambda) \end{vmatrix} = 0 \quad (1.8)$$

Solving (1.8), we have

$$\lambda_1 = -\mu$$

$$\lambda_2 = \left(R_{o_M} - 1\right)\left(\mu + \alpha_1\right)$$

$$\lambda_3 = \left(R_{o_V} - 1\right)\left(\mu + \alpha_2\right)$$

$$\lambda_4 = -\mu$$

Theorem 1: The rumor-free equilibrium of the model equation (1.1) and (1.4) given by E_o, is locally asymptotically stable if $R_o < 1$ and unstable if $R_o > 1$. Thus this theorem 1 implies that for any given rumor in a population, it can be eliminated when $R_o < 1$.

Proof

Having λ_1 and λ_4 to be negative, we also see that λ_2 and λ_3 are both negative too when $R_{o_m} < 1$ and $R_{o_v} < 1$. Since all the eigenvalues of (1.8) have negative real parts when $R_{o_m} < 1$ and $R_{o_v} < 1$, we conclude that rumor-free equilibrium is locally asymptotically stable.

Stability Analysis of the Rumor Endemic Equilibrium

When rumor persists in a population (*i.e.* at least $M \neq 0$ or $G \neq 0$), the model question (1.1) to (1.4) has two equilibrium points denoted by

$$E_1 = \left(I^{\wedge}, M^{\wedge}, G^{\wedge}, R^{\wedge}\right) \text{ and } E_2 = \left(I^{\wedge\wedge}, M^{\wedge\wedge}, G^{\wedge\wedge}, R^{\wedge\wedge}\right)$$

called rumor verbal-endemic equilibrium and rumor media-endemic equilibrium point. For the existence and uniqueness of E_1 and E_2 their coordinate has to satisfy the following $I^{\wedge} > 0$, $M^{\wedge} > 0$ $G^{\wedge} > 0$, $R^{\wedge} > 0$ and $I^{\wedge\wedge} > 0$, $M^{\wedge\wedge} > 0$, $G^{\wedge\wedge} > 0$, $R^{\wedge\wedge} > 0$ respectively.

Equating (1.1) to (1.4) to zero, we obtained

$$\pi - \mu I - \frac{\theta}{2}\beta_1 IM - \frac{\theta}{2}\beta_2 IG - \left(M + G\right) = 0 \quad (1.9)$$

$$\frac{\theta}{2}\beta_1 IM - \mu M - \alpha_1 M = 0 \quad (1.10)$$

$$\frac{\theta}{2}\beta_2 IG - \mu G - \alpha_2 G = 0 \quad (1.11)$$

$$\left(M+G\right)\sigma I+\alpha_1 M+\alpha_2 G-\mu R=0 \tag{1.12}$$

From (1.10), we have

$$I^{\wedge}=\frac{2(\mu+\alpha_1)}{\theta\beta_1} \tag{1.13}$$

$$M^{\wedge}=0 \tag{1.14}$$

Substituting (1.13) and (1.14) in (1.9), we have

$$G^{\wedge}=\frac{\pi\theta\beta_1-2\mu\left(\mu+\alpha_1\right)}{\beta_2\theta\left(\mu+\alpha_1\right)+2\sigma\left(\mu+\alpha_1\right)} \tag{1.15}$$

Using (1.13), (1.14) and (1.15) in (1.12), we obtained

$$R^{\wedge}=\left(\frac{2\sigma\left(\mu+\alpha_1\right)+\alpha_1\theta\beta_1}{\theta\beta_1\mu}\right)\left(\frac{\pi\theta\beta_1-2\mu\left(\mu+\alpha_1\right)}{\beta_2\theta\left(\mu+\alpha_1\right)+2\sigma\mu\left(\mu+\alpha_1\right)}\right) \tag{1.6}$$

Thus $E_1=\left(I^{\wedge},M^{\wedge},G^{\wedge},R^{\wedge}\right)$

Local Stability of rumor verbal-endemic equilibrium

We used the Jacobian Stability approach to prove the stability of the rumor verbal-endemic equilibrium.

The Jacobian of (1.9) to (1.12) at the equilibrium point $E_1=\left(I^{\wedge},M^{\wedge},G^{\wedge},R^{\wedge}\right)$ is

$$\begin{pmatrix} -\mu-\frac{\theta}{2}\beta_1 M^{\wedge}-\frac{\theta}{2}\beta_2 G^{\wedge}-\left(M^{\wedge}+G^{\wedge}\right)\sigma & -\frac{\theta}{2}\beta_1 I^{\wedge}-\sigma I^{\wedge} & -\frac{\theta}{2}\beta_2 I^{\wedge}-\sigma I^{\wedge} & 0 \\ \frac{\theta}{2}\beta_1 M^{\wedge} & \frac{\theta}{2}\beta_1 I^{\wedge}-\mu-\alpha_1 & 0 & 0 \\ \frac{\theta}{2}\beta_2 G^{\wedge} & 0 & \frac{\theta}{2}\beta_2 I^{\wedge}-\mu-\alpha_2 & 0 \\ \left(M^{\wedge}+G^{\wedge}\right)\sigma & \alpha_1+\sigma I^{\wedge} & \alpha_2+\sigma I^{\wedge} & -\mu \end{pmatrix} \tag{1.17}$$

If we evaluate (1.17) at E_1 and find the eigenvalue using characteristic equation $\left|J_{(E_1)}-\lambda I\right|=0$, we will obtain

$$\lambda_1=-\mu-\left(\frac{\theta\beta_2+2\sigma}{2}\right)\left(\frac{\pi\theta\beta_1-2\mu\left(\mu+\alpha_1\right)}{\theta\beta_2\left(\mu+\alpha_1\right)+2\sigma\sigma\left(\mu+\alpha_1\right)}\right)$$

$$\lambda_2=0$$

$$\lambda_3=\frac{\beta_2\left(\mu+\alpha_1\right)}{\beta_1}-\mu-\alpha_2$$

$$\lambda_4=-\mu$$

From the above questions, it is observed clearly that only $\lambda_1<0$ and $\lambda_4<0$, therefore we conclude that rumor verbal-endemic equilibrium is unstable since some of the eigenvalue are greater than zero. Furthermore, once a rumor spreads, the truth is at risk of being distorted in the public sphere, therefore it must spread first before it decline naturally or by counter rumor.

Rumor media-endemic equilibrium

To obtain the rumor media endemic equilibrium we use the same approach as in rumor verbal-endemic equilibrium.

From (1.11), we have

$$I^{\wedge\wedge} = \frac{2(\mu+\alpha_1)}{\theta\beta_2} \tag{1.18}$$

$$G^{\wedge\wedge} = 0 \tag{1.19}$$

Substituting (1.18) and (1.19) in (1.9), we have

$$M^{\wedge\wedge} = \frac{\pi\theta\beta_2 - 2\mu(\mu+\alpha_2)}{\beta_1\theta(\mu+\alpha_2) + 2\sigma(\mu+\alpha_2)} \tag{1.20}$$

Using (1.18), (1.19) and (1.20) in (1.12), we obtained

$$R^{\wedge\wedge} = \left(\frac{2\sigma(\mu+\alpha_1)+\alpha_1\theta\beta_1}{\theta\beta_1\mu}\right)\left(\frac{\pi\theta\beta_1 - 2\mu(\mu+\alpha_1)}{\beta_2\theta(\mu+\alpha_1)+2\sigma\mu(\mu+\alpha_1)}\right) \tag{1.21}$$

Thus $E_2 = \left(I^{\wedge\wedge}, M^{\wedge\wedge}, G^{\wedge\wedge}, R^{\wedge\wedge}\right)$

Local Stability of Rumor Media-endemic Equilibrium

We also use the Jacobin stability approach to prove the stability of the rumor media endemic. The Jacobian of (1.9) to (1.12) at equilibrium point $E_2 = \left(I^{\wedge\wedge}, M^{\wedge\wedge}, G^{\wedge\wedge}, R^{\wedge\wedge}\right)$ is

$$\begin{pmatrix} -\mu-\frac{\theta}{2}\beta_1 M^{\wedge\wedge}-\frac{\theta}{2}\beta_2 G^{\wedge\wedge}-\left(M^{\wedge\wedge}+G^{\wedge\wedge}\right)\sigma & -\frac{\theta}{2}\beta_1 I^{\wedge\wedge}-\sigma I^{\wedge\wedge} & -\frac{\theta}{2}\beta_2 I^{\wedge\wedge}-\sigma I^{\wedge\wedge} & 0 \\ \frac{\theta}{2}\beta_1 M^{\wedge\wedge} & \frac{\theta}{2}\beta_1 I^{\wedge\wedge}-\mu-\alpha_1 & 0 & 0 \\ \frac{\theta}{2}\beta_2 G^{\wedge\wedge} & 0 & \frac{\theta}{2}\beta_2 I^{\wedge\wedge}-\mu-\alpha_2 & 0 \\ \left(M^{\wedge\wedge}+G^{\wedge\wedge}\right)\sigma & \alpha_1+\sigma I^{\wedge\wedge} & \alpha_2+\sigma I^{\wedge\wedge} & -\mu \end{pmatrix} \tag{1.22}$$

If we evaluate (1.22) at E_2 and find the eigenvalue using characteristic equestion $\left|J_{(E_1)} - \lambda I\right| = 0$, we will obtain

$$\lambda_1 = -\mu - \left(\frac{\theta\beta_1 + 2\sigma}{2}\right)\left(\frac{\pi\theta\beta_2 - 2(\mu+\alpha_2)}{\theta\beta_1(\mu+\alpha_2)+2\sigma(\mu+\alpha_2)}\right)$$

$$\lambda_2 = \frac{\beta_1(\mu+\alpha_2)}{\beta_2} - \mu - \alpha_1$$

$$\lambda_3 = 0$$

$$\lambda_4 = -\mu$$

Since some of the eigenvalues are not less than zero, we conclude that rumor through media-endemic equilibrium is unstable. Consequently, when rumor occurs in an ignorant population it must spread first before it declines either naturally or by counter rumor.

6. Numerical Solution and Discussion of Results

This section deals with the numerical studies of the developed model and discussion of the simulation result using estimated parameters and adopted some from other models.

Model Simulation

We performed some numerical experiments using ode45 function from MATLAB to study the behavior of the systems on the ignorant, spreaders through media (spreader 1), spreader through verbal communication (spreader 2) and stiflers population. The parameter values used in the simulation of this model are presented in Table 2.

The results of the simulation of the model (1.1) to (1.4) with parameter value in Table 2 are presented in the figures below: (Figures 2-7).

Experiment 1: Investigating the impact of contact rates between the ignorant and spreaders.

Experiment 2: Impact of rate at which spreaders become stiflers.

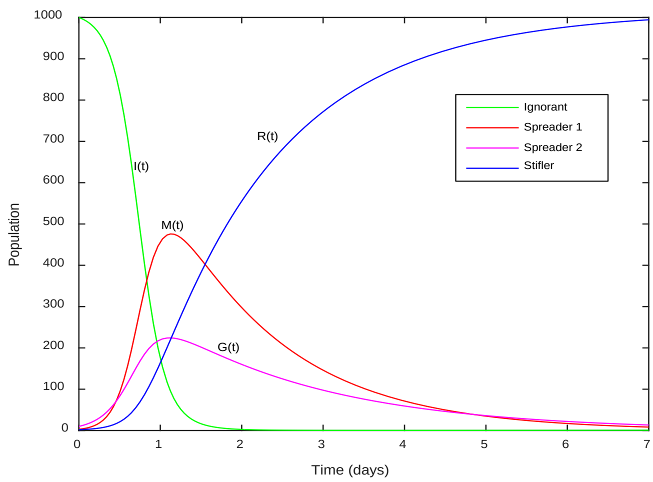

Figure 2. Time evolution of ignorant, spreader 1, Spreader 2 and Stifler population when $\beta_1 = 0.8$ and $\beta_2 = 0.5$.

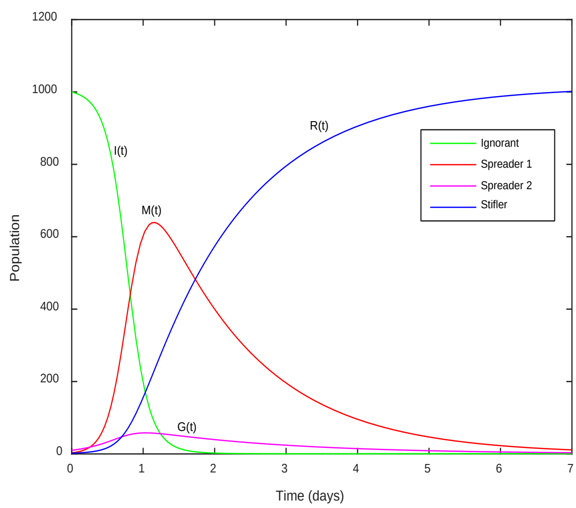

Figure 3. Time evolution of ignorant, spreader 1, Spreader 2 and Stifler population when $\beta_1 = 0.8$ and $\beta_2 = 0.3$.

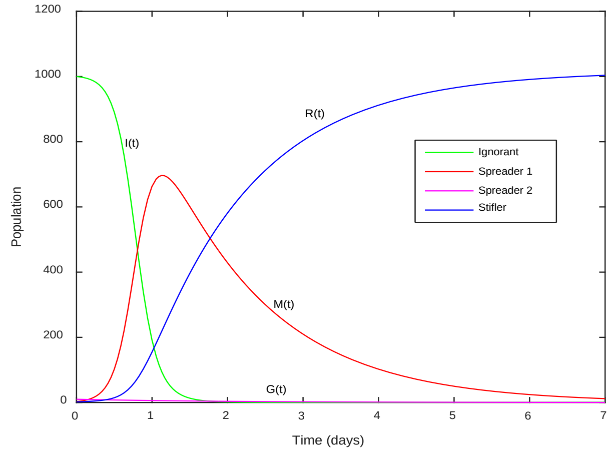

Figure 4. Time evolution of ignorant, spreader 1, Spreader 2 and Stifler population when $\beta_1 = 0.8$, $\beta_2 = 0.3$.

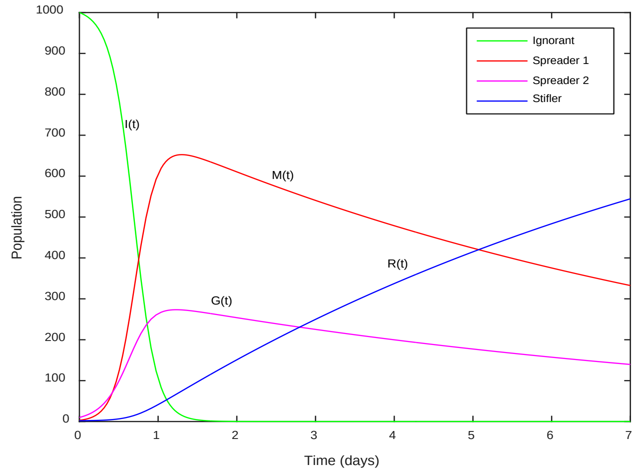

Figure 5. Time evolution of ignorant, spreader 1, Spreader 2 and Stifler population $\alpha_1 = 0.122$ and $\alpha_2 = 0.120$.

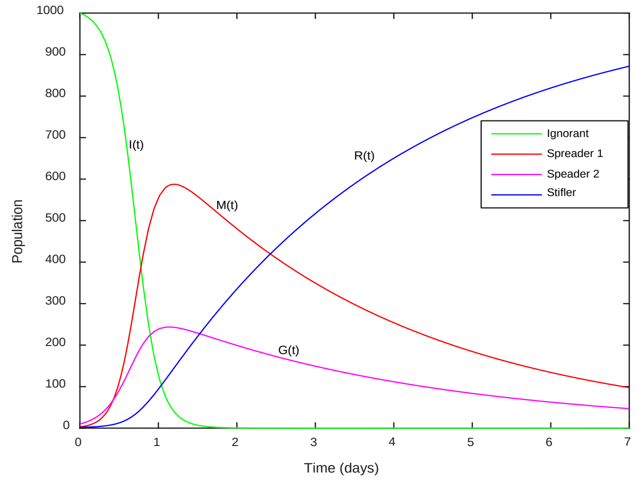

Figure 6. Time evolution of ignorant, spreader 1, Spreader 2 and Stifler population when $\alpha_1 = 0.32$, $\alpha_2 = 0.29$.

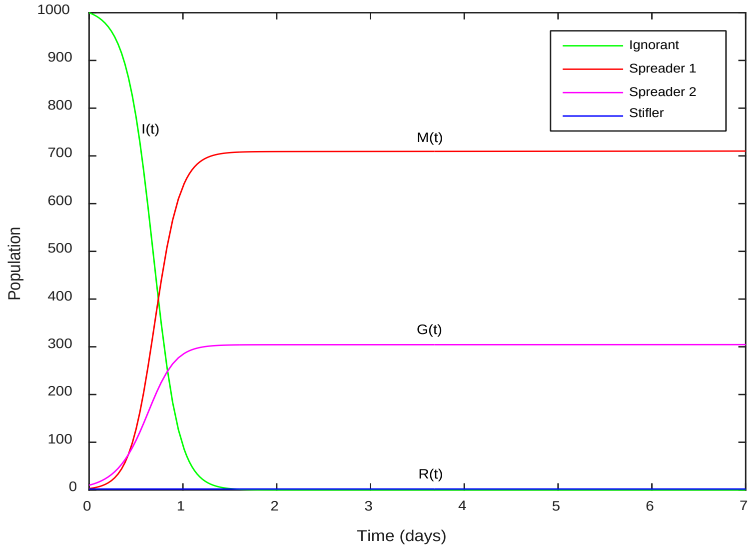

Figure 7. Time evolution of ignorant, spreader 1, Spreader 2 and Stifler population when $\alpha_1 = 0$, $\alpha_2 = 0$.

Table 2. Estimated and adopted parameter values and variables used in the simulation of the model.

Parameter	Value	Source
π	0.3	[6]
μ	0.000005	Assumed
θ	0.02	[31]
β_1	0.8	[31]
β_2	0.5	[31]
σ	0.00001	[6]
α_1	0.72	[22]
α_2	0.5	[31]
I	1000	Assumed
M	3	Assumed
G	10	Assumed
R	2	Assumed

7. Discussion of Results

Here we discuss the results obtained in the analytical studies and the numerical experiments.

Analytical Results

The modified model consists of a 4-dimensional system of ordinary differential equations. The Rumour Free Equilibrium was established for the system (1.1) to (1.4). We also obtained the reproduction number of the two spreader comportments using next generation matrix method and we established the stability of the rumor Free Equilibrium and Endemic Equilibrium of the modified model using linearization methods. We observed that all the eigenvalues have negative real parts, implying that the rumor free equilibrium is locally asymptotically stable and unstable otherwise. For the endemic equilibrium point, it is observed that some of the eigenvalues of polynomial (1.17) and (1.22) are positive and we conclude that rumor-endemic equilibrium is unstable (**Figure 5-7**). That is if there exists a rumor in population, it has to spread through first before it decline naturally or suppress by another rumor.

It is observed that the impact of R_{o_M} is the result of the interaction of each parameters and it is easy to decrease R_{o_M} by regulating and controlling a few parameters so also R_{o_V}. The parameter with the most positive impact on R_{o_M} is the rate of contact between the spreaders targeting the community through media and the ignorant. Similarly the most positive impact on R_{o_V} is the rate of contact between the spreaders targeting the community through verbal communication and the ignorant. While the parameter that has moderate influence is the recruitment rate. In this study, we proposed two classes of spreaders and we observed that the propagation of rumor through media and verbal communication are mutually exclusive. Therefore, the appropriate increase in the propagation of rumor through media is conducive to the elimination of the propagation of rumor through verbal communication and vice versa (**Figures 2 to 7**). On the other hand, the propagation of rumor through media and verbal communication coexist for long time under certain condition (**Figure 4**). At this point, the improvement of the propagation of the rumor through media is also conducive to the propagation of rumor through verbal communication in a certain range (**Figures 2 to 7**).

8. Conclusions

Deterministic model on rumor propagation was analyzed to get insight into its dynamical features and to obtain associated spreaders thresholds. Some of the theoretical findings of the study are as follows:

1) rumor spreading has a life span which depends on the type of class of the spreader [see (**Figures 2-4**)].

2) the analysis carried out shows that the increase in stiflers population increased the rate at which spreaders convinced.

Contribution

The model can be used to control rumor propagation by varying the parameters in the model, not only that it can also be modified to manage some other social negative behaviors like smoking, drug abuse, corruption, prostitution, gang, etc.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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