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Robust Finite-Time H_∞ Filtering for Discrete-Time Markov Jump Stochastic Systems

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Abstract

This study is concerned with the problem of finite-time H_∞ filter design for uncertain discrete-time Markov Jump stochastic systems. Our attention is focused on the design of mode-dependent H_∞ filter to ensure the finite-time stability of the filtering error system and preserve a prescribed H_∞ performance level for all admissible uncertainties. Sufficient conditions of filtering design for the system under consideration are developed and the corresponding filter parameters can be achieved in terms of linear matrix inequalities (LMI). Finally, a numerical example is provided to illustrate the validity of the proposed method.

Keywords

Markov Jump Stochastic Systems, Finite-Time Stability, Filter Design, Linear Matrix Inequality

1. Introduction

Since Markov Jump systems is important class of stochastic dynamic systems, it has drawn a lot of attention. Many contributions for Markov Jump systems have been reported in the literature. Robust stability and stabilization control, H_∞ control, H_∞ filtering design, passive control and so on have been widely studied [1]-[9]. Robust stabilization problem and H_∞ control for Markov Jump Linear Singular Systems with Wiener Process was studied in [1]. The problems of stability and robust stabilization for stochastic fuzzy systems were addressed in [2], which designed a robust stochastic fuzzy controller with H_∞ performance for a class of Markov Jump nonlinear systems. Some results on delay-dependent H_∞ filtering for discrete-time singular Markov Jump systems were reported in [3]. The authors investigated delay-dependent robust stability and corresponding control problems for Markov Jump linear systems in [4]. In [5] [6] [7] [8] [9], some methods of H_∞ filtering design for Markov Jump systems or switched systems were proposed.

As well known, Lyapunov asymptotic stability theory focuses on the steady-state behavior of plants over an infinite-time interval. But in many practical systems, it is only required that the system states remain within the given bounds. This motivated the introduction of finite-time stability or short-time stability, which has received considerable attention [10]-[19]. The authors investigated the sufficient conditions of finite-time stability for a class of stochastic nonlinear systems in [10]. The problem of robust finite-time stabilization for impulsive dynamical linear systems was investigated in [11]. In [12] fuzzy control method was adopted to solve finite-time stabilization of a class of stochastic system. A robust finite-time filter was established for singular discrete-time stochastic system in [13]. Some related works for finite-time problems were discussed in [14]-[19]. To the best of the author' knowledge, the problem of robust finite-time filtering for discrete-time Markov Jump stochastic systems has not been fully investigated. This motivates us to investigate the present study. One application of these new results could be used to detect generation of residuals for fault diagnosis problems.

In this paper, we introduce the definition of finite-time stochastic stable (FTSS) into a class of discrete-time Markov Jump stochastic systems with parametric uncertainties. The main purpose of this research is to construct a detection filter such that the resulting filter error augmented system is FTSS. A central problem that we consider is the design of a detection filter that generates a residual signal to estimate the fault signal and detect failure. Sufficient conditions for FTSS of the filter error system is established by applying the Lyapunov-Krasovskii functional candidate combined with LMIs. The desired FTSS filter can be received by solving a set of LMIs. A numerical example is given to demonstrate the applicability and validity of the proposed theoretical method.

The structure of the paper is organized as follows. Some preliminaries and the problem formulation are introduced in Section 2. In Section 3, a sufficient condition for FTSS of the corresponding filtering error system is established and the method to design a finite-time filter is presented. Section 4 presents a numerical example to demonstrate the effectivity of the mentioned methodology. Some conclusions are drawn in Section 5.

We use R^n to denote the n-dimensional Euclidean space. The notation $X > Y$ (respectively, $X \geq Y$, where X and Y are real symmetric matrices, means that the matrix $X - Y$ is positive definite (respectively, positive semi-definite). I and 0 denote the identity and zero matrices with appropriate dimensions. $\lambda_{\max}(R)$ and $\lambda_{\min}(R)$ denotes the maximum and the minimum of the eigenvalues of a real symmetric matrix R. The superscript T denotes the transpose for vectors or matrices. The symbol * in a matrix denotes a term that is defined by symmetry of the matrix.

2. Model Descriptions and Preliminaries

We shall consider the following uncertain discrete-time Markov Jump stochastic system:

$$x_{k+1} = \left[A(\eta_k) + \Delta A(\eta_k)\right]x_k + \left[B(\eta_k) + \Delta B(\eta_k)\right]v_k \tag{1a}$$

$$y_k = \left[C(\eta_k) + \Delta C(\eta_k)\right]x_k + \left[D(\eta_k) + \Delta D(\eta_k)\right]v_k \tag{1b}$$

$$z_k = L(\eta_k)x_k \tag{1c}$$

$$x(0) = x_0 \in R^n \tag{1d}$$

where $x_k \in R^n$, $y_k \in R^m$ are the state vector and the measurement or output vector, $z_k \in R^q$ is the controlled output, and v_k is a one-dimensional zero-mean process which satisfies $\Xi[v_k] = 0, \Xi[v_i v_j] = 0, i \neq j, \Xi[v_k^2] = \alpha$, which is assumed to be independent of the system mode $\{\eta_k\}$. Ξ is the expected value. Here $\alpha > 0$ is a known scalar.

The random form process $\{\eta_k\}$ is a discrete-time Markov process taking values in a finite set $S \triangleq \{1, 2, \cdots, s\}$. The set S comprises the operation modes of the system. The transition probabilities for the process $\{\eta_k\}$ are defined as

$$p_{ij} = \text{Prob}(\eta_{k+1} = j \mid \eta_k = i) \tag{2}$$

where $p_{ij} > 0$ is the transition probability rate from mode i to mode j, for $\forall i, j \in S, \sum_{j \in S} p_{ij} = 1$.

For each possible value of $\eta_k = i, i \in S$ in the succeeding discussion, we denote the matrices with the ith mode by

$$A_i \triangleq A(\eta_k),\ B_i \triangleq B(\eta_k),\ C_i \triangleq C(\eta_k),\ D_i \triangleq D(\eta_k),\ L_i \triangleq L(\eta_k),$$

$$\Delta A(\eta_k) \triangleq \Delta A_i,\ \Delta B(\eta_k) \triangleq \Delta B_i,\ \Delta C(\eta_k) \triangleq \Delta C_i,\ \Delta D(\eta_k) \triangleq \Delta D_i$$

where A_i, B_i, C_i, D_i, L_i for any $i \in S$ are known constant matrices of appropriate dimensions $\Delta A_i, \Delta B_i, \Delta C_i, \Delta D_i$ are matrices that represent the time-varying parameter uncertainties and are assumed to be of the form:

$$\begin{bmatrix} \Delta A_i \\ \Delta C_i \end{bmatrix} = \begin{bmatrix} H_{1i} \\ H_{2i} \end{bmatrix} F_k \begin{bmatrix} G_1 \\ G_2 \end{bmatrix},\quad \begin{bmatrix} \Delta B_i \\ \Delta B_i \end{bmatrix} = \begin{bmatrix} H_{3i} \\ H_{4i} \end{bmatrix} F_k \begin{bmatrix} G_3 \\ G_4 \end{bmatrix}. \tag{3}$$

The matrices $H_{1i}, H_{2i}, H_{3i}, H_{4i}, G_1, G_2, G_3, G_4$ are known and provide the structure of the uncertainty. F_k is arbitrary except for the bound on F_k which satisfies $F_k^{\mathrm{T}} F_k < I$.

Where $A_{i,k}, A_{1i,k}, B_{i,k}, B_{1i,k}, D_{1i,k}, D_{2i,k}, C_{i,k}, D_{i,k}$ for any $i \in S$ and $k \in N$ are known constant matrices of appropriate dimensions.

We now summarize several needed results from the literature.

Definition 1 ([20]) The discrete-time Markovian Jump stochastic system (1) is said to be finite-time stochastic stable (**FTSS**) with respect to (c_1, c_2, P, N), where $P > 0, 0 < c_1 < c_2$ and N is a positive integer, if $\Xi\{x_0^{\mathrm{T}} P x_0\} < c_1$ implies $\Xi\{x_k^{\mathrm{T}} P x_k\} < c_2$ for all $k = 1, 2, \cdots, N$.

The next two Lemmas will play a key role in what follows.

Lemma 1 ([21]) Let M, N and F be matrices of appropriate dimension, and

$F^{\mathrm{T}}F \le I$. Then for any scalar $\varepsilon > 0$, $MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} \le \varepsilon MM^{\mathrm{T}} + \varepsilon^{-1}N^{\mathrm{T}}N$.

Lemma 2 (Schur complement [22] [23])

Given a symmetric matrix $\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$, the following three conditions are equivalent to each other:

1) $\phi < 0$;

2) $\phi_{11} < 0$ and $\phi_{22} - \phi_{12}^{\mathrm{T}}\phi_{11}^{-1}\phi_{12} < 0$;

3) $\phi_{22} < 0$ and $\phi_{11} - \phi_{12}\phi_{22}^{-1}\phi_{12}^{\mathrm{T}} < 0$.

We now consider the following filter:

$$\hat{x}_{k+1} = A_{fi}\hat{x}_k + B_{fi}y_k \tag{4a}$$

$$\hat{z}_k = L_{fi}\hat{x}_k \tag{4b}$$

where $\hat{x}_k \in R^n$ is the filter state, and matrices A_{fi}, B_{fi}, L_{fi} are filter parameters with compatible dimensions to be determined. It is assumed that A_{fi} is non-singular. Define $\xi_k^{\mathrm{T}}(t) = \begin{bmatrix} x_k & \hat{x}_k \end{bmatrix}^{\mathrm{T}}$, $e_k = z_k - \hat{z}_k$. Then the filtering error system is

$$\xi_{k+1} = \left(\bar{A}_i + \Delta\bar{A}_i\right)\xi_k + \left(\bar{B}_i + \Delta\bar{B}_i\right)v_k \tag{5a}$$

$$e_k = \bar{L}\xi_k \tag{5b}$$

where, $\bar{A}_i = \begin{bmatrix} A_i & 0 \\ B_{fi}C_i & A_{fi} \end{bmatrix}$,

$$\Delta\bar{A}_i = \begin{bmatrix} \Delta A_i & 0 \\ B_{fi}\Delta C_i & 0 \end{bmatrix} = \begin{bmatrix} H_{1i} & 0 \\ 0 & B_{fi}H_{2i} \end{bmatrix} F_k \begin{bmatrix} G_1 & 0 \\ G_2 & 0 \end{bmatrix} = \bar{H}_i F_k \bar{G},$$

$$\Delta\bar{B}_i = \begin{bmatrix} \Delta B_i \\ B_{fi}\Delta D_i \end{bmatrix} = \begin{bmatrix} H_{3i} \\ B_{fi}H_{4i} \end{bmatrix} F_k \begin{bmatrix} G_3 \\ G_4 \end{bmatrix} = \tilde{H}_i F_k \tilde{G},\ \bar{B}_i = \begin{bmatrix} B \\ B_f D \end{bmatrix},\ \bar{L} = \begin{bmatrix} L & -L_f \end{bmatrix} \tag{6}$$

Then the problem to be presented in this paper can be summarized as follows.

Given a scalar $\gamma > 0$, design a filter (4) for the system (1), such that

1) the filtering error system (5) is FTSS,

2) the filtering error e_k satisfies

$$\Xi\left[e_k^{\mathrm{T}}e_k\right] \le \gamma^2\Xi\left[v_k^2\right], \tag{7}$$

where the prescribed value γ is the attenuation level.

3. Robust H∞ Filter Design

In this section we address the problems of admissibly finite-time stochastic stability analysis and the filter design of the discrete-time Markov Jump stochastic system. A sufficient condition of the filter existence and the design technique is proposed in the following theorems.

Theorem 1: The error system in (5) is robust FTSS with respect to (c_1, c_2, P, N) and (7) is satisfied if there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\mu > 1$, $\gamma > 0$ and symmetric positive-definite matrix P, Q_i, $i \in S$ so that if $R_i = P^{\frac{1}{2}}Q_iP^{\frac{1}{2}}$ the following condition holds:

$$\Theta = \begin{bmatrix} \Phi & \Psi \end{bmatrix} < 0 \tag{8}$$

where Θ is

$$\Phi=\begin{bmatrix} \Theta_{11} & * & * & * & * & * & * & * & * \\ 0 & \Theta_{22} & * & * & * & * & * & * & * \\ \bar{G} & 0 & -\varepsilon_1^{-1}I & * & * & * & * & * & * \\ \bar{H}_i^{\mathrm{T}}R_i\bar{A}_i & 0 & 0 & -\varepsilon_1 I & * & * & * & * & * \\ \bar{G} & 0 & 0 & 0 & \Psi=-\lambda_{\max}^{-1}\left(\bar{H}_i^{\mathrm{T}}R_i\bar{H}_i\right)I & * & * & * & * \\ 0 & \tilde{G} & 0 & 0 & 0 & -\varepsilon_2^{-1}I & * & * & * \\ 0 & \tilde{H}_i^{\mathrm{T}}R_i\bar{B} & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * & * \\ 0 & \tilde{G} & 0 & 0 & 0 & 0 & 0 & -\lambda_{\max}^{-1}\left(\tilde{H}_i^{\mathrm{T}}R_i\tilde{H}_i\right)I & * \\ L_i & -C_{fi} & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} \tag{9}$$

where $\Theta_{11}=\bar{A}_i^{\mathrm{T}}R_i\bar{A}_i-\mu R_i$, $\Theta_{22}=\bar{B}_i^{\mathrm{T}}R_i\bar{B}_i-\mu I-\gamma^2 I$ and
$\dfrac{\mu^N\lambda_{\max}(Q_i)c_1+\sum_{k=1}^{N}\mu^k\alpha}{\lambda_{\min}(Q_i)}\le c_2$.

Proof: Let us consider the following Lyapunov function candidate for system (5):

$$V_i=V(\xi_k,i)=\xi_k^{\mathrm{T}}R_i\xi_k. \tag{10}$$

Then, we compute that

$$\begin{aligned}
&\Xi\left[V(\xi_{k+1},i)-\mu V(\xi_k,i)-\mu v_k^{\mathrm{T}}v_k\right]=\Xi\left\{\xi_{k+1}^{\mathrm{T}}R_i\xi_{k+1}-\mu\xi_k^{\mathrm{T}}R_i\xi_k-\mu v_k^{\mathrm{T}}v_k\right\}\\
&=\begin{bmatrix}\xi_k\\ v_k\end{bmatrix}^{\mathrm{T}}\left\{\begin{bmatrix}\left(\bar{A}_i+\Delta\bar{A}_i\right)^{\mathrm{T}}R_i\left(\bar{A}_i+\Delta\bar{A}_i\right)-\mu R_i & 0\\ 0 & \left(\bar{B}_i+\Delta\bar{B}_i\right)^{\mathrm{T}}R_i\left(\bar{B}_i+\Delta\bar{B}_i\right)-\mu I\end{bmatrix}\right\}\begin{bmatrix}\xi_k\\ v_k\end{bmatrix}\\
&=\begin{bmatrix}\xi_k\\ v_k\end{bmatrix}^{\mathrm{T}}\left\{\begin{bmatrix}\bar{A}_i^{\mathrm{T}}R_i\bar{A}_i-\mu R_i & 0\\ 0 & \bar{B}_i^{\mathrm{T}}R_i\bar{B}_i-\mu I\end{bmatrix}+\begin{bmatrix}\Delta\bar{A}_i^{\mathrm{T}}\\ 0\end{bmatrix}R_i\begin{bmatrix}\bar{A}_i & 0\end{bmatrix}\right.\\
&\quad+\begin{bmatrix}\bar{A}_i^{\mathrm{T}} & 0\end{bmatrix}^{\mathrm{T}}R_i^{\mathrm{T}}\begin{bmatrix}\Delta\bar{A}_i^{\mathrm{T}}\\ 0\end{bmatrix}^{\mathrm{T}}+\begin{bmatrix}\Delta\bar{A}_i^{\mathrm{T}}\\ 0\end{bmatrix}R_i\begin{bmatrix}\Delta\bar{A}_i & 0\end{bmatrix}+\begin{bmatrix}0\\ \Delta\bar{B}_i^{\mathrm{T}}\end{bmatrix}R_i\begin{bmatrix}0 & \bar{B}_i\end{bmatrix}\\
&\quad\left.+\begin{bmatrix}0 & \bar{B}_i^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}R_i^{\mathrm{T}}\begin{bmatrix}0\\ \Delta\bar{B}_i^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}+\begin{bmatrix}0\\ \Delta\bar{B}_i^{\mathrm{T}}\end{bmatrix}R_i\begin{bmatrix}0 & \Delta\bar{B}_i\end{bmatrix}\right\}\begin{bmatrix}\xi_k\\ v_k\end{bmatrix}=\begin{bmatrix}\xi_k\\ v_k\end{bmatrix}^{\mathrm{T}}\Theta\begin{bmatrix}\xi_k\\ v_k\end{bmatrix}
\end{aligned} \tag{11}$$

Note that

$$\begin{bmatrix} \Delta\bar{A}_i^{\mathrm{T}} \\ 0 \end{bmatrix} R_i \begin{bmatrix} \Delta\bar{A}_i & 0 \end{bmatrix} \le \lambda_{\max}\left(\bar{H}_i^{\mathrm{T}} R_i \bar{H}_i\right) \begin{bmatrix} \bar{G}^{\mathrm{T}} \\ 0 \end{bmatrix} \begin{bmatrix} \bar{G} & 0 \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} 0 \\ \Delta\bar{B}_i^{\mathrm{T}} \end{bmatrix} R_i \begin{bmatrix} 0 & \Delta\bar{B}_i \end{bmatrix} \le \lambda_{\max}\left(\tilde{H}_i^{\mathrm{T}} R_i \tilde{H}_i\right) \begin{bmatrix} 0 \\ \tilde{G}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 & \tilde{G} \end{bmatrix}. \tag{13}$$

Then by two applications of Lemma 2, we have

$$\begin{aligned} &\begin{bmatrix} \Delta\bar{A}_i^{\mathrm{T}} \\ 0 \end{bmatrix} R_i \begin{bmatrix} \bar{A}_i & 0 \end{bmatrix} + \begin{bmatrix} \bar{A}_i^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}} R_i^{\mathrm{T}} \begin{bmatrix} \Delta\bar{A}_i^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}} \\ &\le \varepsilon_1 \begin{bmatrix} \bar{G}^{\mathrm{T}} \\ 0 \end{bmatrix} \begin{bmatrix} \bar{G} & 0 \end{bmatrix} + \varepsilon_1^{-1} \begin{bmatrix} \bar{A}_i^{\mathrm{T}} R_i^{\mathrm{T}} \bar{H}_i \\ 0 \end{bmatrix} \begin{bmatrix} \bar{H}_i^{\mathrm{T}} R_i \bar{A}_i & 0 \end{bmatrix} \end{aligned} \tag{14}$$

and

$$\begin{aligned} &\begin{bmatrix} 0 \\ \Delta\bar{B}_i^{\mathrm{T}} \end{bmatrix} R_i \begin{bmatrix} 0 & \bar{B}_i \end{bmatrix} + \begin{bmatrix} 0 & \bar{B}_i^{T} \end{bmatrix}^{\mathrm{T}} R_i^{\mathrm{T}} \begin{bmatrix} 0 \\ \Delta\bar{B}_i^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ &\le \varepsilon_2 \begin{bmatrix} 0 \\ \tilde{G}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 & \tilde{G} \end{bmatrix} + \varepsilon_2^{-1} \begin{bmatrix} 0 \\ \bar{B}_i^{\mathrm{T}} R_i^{\mathrm{T}} \tilde{H}_i^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 & \tilde{H}_i R_i \bar{B}_i \end{bmatrix} \end{aligned}. \tag{15}$$

Applying the Schur Complement, the condition (8) contains the following inequality:

$$\begin{aligned} &\begin{bmatrix} \bar{A}_i^{\mathrm{T}} R_i \bar{A}_i - \mu R_i & 0 \\ 0 & \bar{B}_i^{\mathrm{T}} R_i \bar{B}_i - \mu I \end{bmatrix} + \varepsilon_1 \begin{bmatrix} \bar{G}^{\mathrm{T}} \\ 0 \end{bmatrix} \begin{bmatrix} \bar{G} & 0 \end{bmatrix} + \varepsilon_1^{-1} \begin{bmatrix} \bar{A}_i^{\mathrm{T}} R_i^{\mathrm{T}} \bar{H}_i \\ 0 \end{bmatrix} \begin{bmatrix} \bar{H}_i^{\mathrm{T}} R_i \bar{A}_i & 0 \end{bmatrix} \\ &+\lambda_{\max}\left(\bar{H}_i^{\mathrm{T}} R_i \bar{H}_i\right) \begin{bmatrix} \bar{G}^{\mathrm{T}} \\ 0 \end{bmatrix} \begin{bmatrix} \bar{G} & 0 \end{bmatrix} + \varepsilon_2 \begin{bmatrix} 0 \\ \tilde{G}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 & \tilde{G} \end{bmatrix} + \varepsilon_2^{-1} \begin{bmatrix} 0 \\ \bar{B}_i^{\mathrm{T}} R_i^{\mathrm{T}} \tilde{H}_i^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 & \tilde{H}_i R_i \bar{B}_i \end{bmatrix} \\ &+\lambda_{\max}\left(\tilde{H}_i^{\mathrm{T}} R_i \tilde{H}_i\right) \begin{bmatrix} 0 \\ \tilde{G}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} 0 & \tilde{G} \end{bmatrix} < 0 \end{aligned}$$

Then

$$\Theta < 0. \tag{16}$$

With the conditions (10) and (11), it then also follows that

$$\Xi\left[V\left(\xi_k, i\right)\right] = \Xi\left[\xi_k^{\mathrm{T}} R_i \xi_k\right] \ge \lambda_{\min}\left(Q_i\right) \Xi\left[\xi_k^{\mathrm{T}} P \xi_k\right]. \tag{17}$$

Proceeding in an iterative fashion, we obtain the following inequality:

$$\begin{aligned} \Xi\left[V\left(\xi_k, i\right)\right] &\le \Xi\left[\mu V\left(\xi_{k-1}, i\right) + \mu v_{k-1}^{\mathrm{T}} v_{k-1}\right] \\ &\le \mu^k \lambda_{\max}\left(Q_i\right) \Xi\left[\xi_0^{\mathrm{T}} P \xi_0\right] + \sum_{k=1}^{N} \mu^k \alpha \\ &\le \mu^N \lambda_{\max}\left(Q_i\right) c_1 + \sum_{k=1}^{N} \mu^k \alpha \end{aligned}$$

Thus we have that

$$\Xi\left[\xi_k^{\mathrm{T}} P \xi_k\right] \le \frac{\mu^N \lambda_{\max}\left(Q_i\right) c_1 + \sum_{k=1}^{N} \mu^k \alpha}{\lambda_{\min}\left(Q_i\right)} \le c_2. \tag{18}$$

Obviously, (8) indicates that

$$\Xi\left[V\left(\xi_{k+1}, i\right) - \mu V\left(\xi_k, i\right) - \mu v_k^{\mathrm{T}} v_k + e_k^{\mathrm{T}} e_k - \gamma^2 v_k^{\mathrm{T}} v_k\right] \le 0. \tag{19}$$

Then we can conclude that (7) holds.

Theorem 2 The filtering error system (5) is FTSS with respect to (c_1, c_2, P, N) and the error signal satisfies (7), if there exist positive definite matrix Q_i and matrices $\Omega_{1i}, \Omega_{2i}, \Omega_{3i}$, $R = \begin{bmatrix} R_{1i} & 0 \\ 0 & R_{2i} \end{bmatrix}$, $R_i = P^{\frac{1}{2}} Q_i P^{\frac{1}{2}}$, $i \in S$ satisfying:

$$\bar{\Theta} = \begin{bmatrix} \bar{\Phi} & \Psi \end{bmatrix} < 0 \tag{20}$$

where Ψ is from (8) and $\bar{\Phi}$ is the same as Φ in (8) except that

$$\bar{\Phi}_{11} = \begin{bmatrix} A_i^{\mathrm{T}} R_{1i} A_i + \Omega_{2i}^{\mathrm{T}} R_{2i} \Omega_{2i} & \Omega_{2i}^{\mathrm{T}} \Omega_{1i} \\ \Omega_{1i}^{\mathrm{T}} \Omega_{2i} & \Omega_{1i}^{\mathrm{T}} R_{2i}^{-\mathrm{T}} \Omega_{1i} \end{bmatrix},$$

$$\bar{\Phi}_{22} = \begin{bmatrix} B_i^{\mathrm{T}} R_{1i} & D_i^{\mathrm{T}} C_i^{-\mathrm{T}} \Omega_{2i}^{\mathrm{T}} R_{2i} \Omega_{2i} C_i^{-1} D_i \end{bmatrix} - \mu I - \gamma^2 I$$

$$\bar{\Phi}_{41} = \begin{bmatrix} H_{1i}^{\mathrm{T}} R_{1i} A_i + H_{2i}^{\mathrm{T}} R_{2i} \Omega_{2i} & H_{2i}^{\mathrm{T}} \Omega_{1i} \end{bmatrix} \text{ and } \bar{\Phi}_{72} = H_{3i} R_{1i} B_i + H_{4i} R_{2i} \Omega_{2i} C_i^{-1} \tag{21}$$

Moreover, the suitable filter parameters A_{fi}, B_{fi}, L_{fi} in system (4) can be given by

$$A_{fi} = R_{2i}^{-1} \Omega_{1i}, B_{fi} = \Omega_{2i} C_i^{-1}, C_{fi} = \Omega_{3i}. \tag{22}$$

Proof: By Theorem 1, the terms in (9) can be rewritten as follows:

$$\bar{A}_i^{\mathrm{T}} R_i \bar{A}_i = \begin{bmatrix} A_i^{\mathrm{T}} R_{1i} A_i + C_i^{\mathrm{T}} B_{fi}^{\mathrm{T}} R_{2i} B_{fi} C_i - \mu R_{1i} & C_i^{\mathrm{T}} B_{fi}^{\mathrm{T}} R_{2i} A_{fi} \\ A_{fi}^{\mathrm{T}} R_{2i} B_{fi} C_i & A_{fi}^{\mathrm{T}} R_{2i} A_{fi} - \mu R_{2i} \end{bmatrix},$$

$$\bar{B}_i^{\mathrm{T}} R_i \bar{B}_i = \begin{bmatrix} B_i^{\mathrm{T}} R_{1i} & D_i^{\mathrm{T}} B_{fi}^{\mathrm{T}} R_{2i} B_{fi} D_i \end{bmatrix}$$

and $\bar{H}_i^{\mathrm{T}} R \bar{A}_i = \begin{bmatrix} H_{1i}^{\mathrm{T}} R_{1i} A_i + H_{2i}^{\mathrm{T}} R_{2i} B_{fi} C_i & H_{2i}^{\mathrm{T}} R_{2i} A_{fi} \end{bmatrix}$,

while

$$\tilde{H}_i^{\mathrm{T}} R_i \bar{B}_i = H_{3i} R_{1i} B_i + H_{4i} R_{2i} B_{fi} D_i.$$

Let $\Omega_{1i} = R_{21} A_{fi}, \Omega_{2i} = R_{21} B_{fi}, \Omega_{3i} = L_{fi}$, then the condition (8) is equivalent to (20).

4. Numerical Example

We now give a numerical example to illustrate the proposed approach. In this example, we choose the following coefficients for the discrete-time Markov Jump stochastic system in the form of (1):

$$A_1 = \begin{bmatrix} -1 & 0.8 \\ 0.9 & -2 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & 0.6 \\ 0.8 & -1.8 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.3 \\ -0.3 \end{bmatrix},$$

$C_1 = \begin{bmatrix} 0.4 & -0.2 \end{bmatrix}$, $C_2 = \begin{bmatrix} -0.1 & 0.3 \end{bmatrix}$, $D_1 = 0.2$, $D_2 = 0.1$, $x(0) = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}^{\mathrm{T}}$, $\Xi\left[\omega^2(k)\right] = 0.1$ and use the Matlab LMI Toolbox.

$$H_{11} = \begin{bmatrix} 0.02 \\ -0.03 \end{bmatrix},\ H_{21} = -0.02,\ H_{12} = \begin{bmatrix} 0.04 \\ -0.01 \end{bmatrix},\ H_{22} = 0.05,$$

$$H_{31} = \begin{bmatrix} -0.04 \\ 0.02 \end{bmatrix},\ H_{41} = 0.01,\ H_{32} = \begin{bmatrix} 0.03 \\ -0.04 \end{bmatrix},\ H_{42} = 0.03$$

$$G_1 = [-0.03 \quad -0.02],\ G_2 = [-0.02 \quad -0.01],\ G_3 = [-0.02 \quad -0.02],$$
$$G_4 = [-0.02 \quad -0.05],\ L_1 = [0.03 \quad -1.2],\ L_2 = [0.02 \quad -1.5].$$

Suppose $\mu = 1.04$, $\gamma = 3.2$, $P = diag\{1.2, 1.2\}$, $\alpha = 0.01$, $c_1 = 0.7$, $c_2 = 2.4$, $N = 20$, $\varepsilon_1 = 0.3$, $\varepsilon_2 = 0.2$ and apply Theorem 1, we find that LMIs (5) is feasible. Thus the system is finite-time stochastic stable with respect to $(0.7, 2.4, P, 20)$ for all N. Moreover, applying Theorem 2, we can obtain the corresponding filter parameters as follows:

$$A_{f1} = \begin{bmatrix} 0.7435 & 0.3269 \\ 0.3269 & 0.4336 \end{bmatrix},\ B_{f1} = \begin{bmatrix} 2.4634 \\ -0.6429 \end{bmatrix},\ L_{fi} = [0.1036 \quad -0.0264],$$
$$A_{f2} = \begin{bmatrix} 0.6844 & 0.4758 \\ 0.4758 & 0.2927 \end{bmatrix},\ B_{f2} = \begin{bmatrix} 2.8547 \\ -0.7341 \end{bmatrix},\ L_{f2} = [0.0074 \quad -0.0159].$$

The necessary LMI's are solved in MATLAB using the LMI capabilities of the Robust Control Toolbox.

5. Conclusion

In this paper, we have investigated the H_∞ filtering problems for discrete-time Markov Jump stochastic systems. Stochastic Lyapunov function method is adopted to establish sufficient conditions for the FTSS of the filter error system. The design of H_∞ filter is constructed in a given finite-time interval in the form of LMIs with some fixed parameters. An example is given to demonstrate the validity of the proposed method.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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