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Use of the Wavelet Transform for Digital Terrain Model Edge Detection (Special Issue—Wavelet Analysis)

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Abstract

The purpose of this work is to analyze the feasibility of using the wavelet transform in the edge detection of digital terrain models (DTM) obtained by Laser Scanner. The Haar wavelet transform and the edge detection method called Wavelet Transform Modulus Maxima (WTMM), both implemented in Matlab language, were used. In order to validate and verify the efficiency of WTMM, the edge detection of the same DTM was performed by the Roberts, Sobel-Feldman and Canny methods, chosen due to the wide use in the scientific community in the area of Image Processing and Remote Sensing. The comparison of the results showed superior performance of WTMM in terms of processing time.

Keywords

Digital Terrain Model, Edge Detection, Wavelets Transform, Canny, Roberts, Sobel, Sobel-Feldman

1. Introduction

Edge detection of images and digital terrain models (DTM) obtained by Remote Sensing is an important research problem for mapping, especially in large scales (1:10,000 and larger), due to the need for greater detailing of the features and the large amount of digital data of the inputs used for these scales.

The wavelet transform (WT) had its origin in 1909 with the work of Alfred Haar [1] [2] [3] [4], who described the first orthonormal function system and used it for multiscale analysis in his doctoral thesis. However, the formalization of the theory was consolidated only from the 1980s, with the works of Morlet, Mallat, Meyer and Daubechies, among others [1] [5] [6] [7].

The wavelet transform has been widely applied in the areas of knowledge that require analysis and visualization at different scales, which makes its proper use in image processing, digital terrain modeling, cartographic generalization and geoprocessing.

The use of WT for edge detection appears in this context as a tool with great potential due to the characteristics of ease of implementation, simplicity of algorithms and speed of processing. In this paper, we used the edge detection method called Wavelet Transform Modulus Maxima (WTMM), described by [8].

Among the traditional edge detection methods, Canny, Roberts and Sobel-Feldman are widely used in the Remote Sensing and Geoprocessing areas and for this reason they were used as reference for the validation, in this work, of the results obtained by edge detection by WTMM.

2. Fundamentals

In this section are made a brief study of the theoretical foundations and the main works carried out in the context of the edge detection with the operators of Canny, Roberts, Sobel-Feldman and WTMM. Several studies, such as those of [9] [10] [11] [12] [13], compared the edge detection operators of Roberts, Sobel-Feldman and Canny, among others. The general conclusion of these studies is that the Canny operator presented better results than those of Roberts and Sobel-Feldman, but it is a more complex algorithm with a higher computational cost.

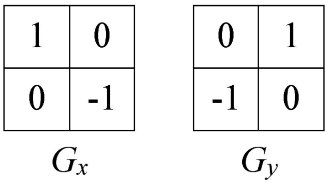
2.1. Roberts Edge Detector

The Roberts edge detector was created by Lawrence Gilman Roberts in his Ph.D. thesis at MIT in 1963 [14]. It is a classic edge detector that uses the directional derivative of the first order. Its implementation consists of a pair of 2 × 2 dimension convolution masks (matrices) as shown in **Figure 1** for the detection of gradients.

These matrices provide maximum responses to the edges with a 45˚ direction in relation to the pixel grid and can be applied separately to the input grid to produce separate measurements of the gradient component in each orientation (called G_x and G_y). The results can then be agreed to find the absolute magnitude of the gradient at each point and the orientation of that gradient. The magnitude of the gradient is given by:

$$|G| = \sqrt{G_x^2 + G_y^2} \tag{1}$$

And the direction of the gradient is given by:



1	0
0	-1

G_x

0	1
-1	0

G_y

Figure 1. Roberts operator template.

$$\theta = \arctan\left(\frac{G_x}{G_y}\right) - \frac{3\pi}{4} \tag{2}$$

Roberts edge detector has the advantage of simplicity of the algorithm and disadvantage of noise susceptibility.

2.2. Sobel-Feldman Edge Detector

The Sobel edge detector operator, more properly called the Sobel-Feldman operator [15] uses the first derivative, similarly to the Roberts operator, for the detection of gradients and their orientation. The Sobel-Feldman operator consists of a pair of convolution masks (3 × 3) as shown in **Figure 2**, which produces responses to the 0˚ and 90˚ direction edges in relation to the pixel grid. Like the Roberts operator, the matrices can be applied separately to the input grid to produce separate measurements of the gradient components in each orientation (G_x and G_y) and then agreed. By using a larger matrix, the Sobel-Feldman operator has the advantage of reducing errors due to the noise's effects [16].

The magnitude of the gradient is given by:

$$|G| = \sqrt{G_x^2 + G_y^2} \tag{3}$$

And the direction of the gradient is given by:

$$\theta = \arctan\frac{G_x}{G_y} \tag{4}$$

2. 3. Canny Edge Detector

The Canny edge detector [17] was created by John Canny in 1986 and is also known as Canny's Great Edge Detector. It performs better than Roberts and Sobel-Feldman, but requires more computational effort. In his work, Canny considered three criteria [18]:

1) Low probability of failure: the ideal detector must find all edges and not find false edges.

2) Location Criteria: The location of the detected edges must match the actual edges.

3) Simple Response Criteria: The detector should provide a single response to a single edge.

The Canny detector implementation consists of 5 steps:

1) Application of a filter for smoothing the image, to reduce noise.

1	2	1
0	0	0
-1	-2	-1

G_x

-1	0	1
-2	0	2
-1	0	1

G_y

Figure 2. Sobel-Feldman operator template.

2) Obtaining the gradient with the use of convolution matrices similar to those of Roberts or Sobel-Feldman. In this step, four convolution matrices are generally applied to obtain the edges in 0˚, 45˚, 90˚ and 135˚ directions.

3) Computation of the direction of the gradient.

4) Non-maximum suppression, which consists of thinning edges by suppressing pixels that are not local maxima.

5) Threshold with hysteresis, which consists in the use of thresholds for suppression of unconnected edges.

The Canny operator has more accurate edge detection, but its implementation is more complex and its computational cost is higher.

2.4. WTMM: Wavelet Transform Modulus Maxima for Edge Detector

According to Mallat and Hwang [8], if a wavelet function $\psi^1(x)$ has a null moment, it can be taken as the first derivative of a smoothing function $\theta(x)$, where $\theta(x)$ is a function such that $\theta(x)=O\left(1/\left(1+x^2\right)\right)$, that is, $\theta(x)\to 0$ when $x\to\pm\infty$. In **Figure 3** are shown examples of functions of the type $\theta(x)$ and $\psi^1(x)$.

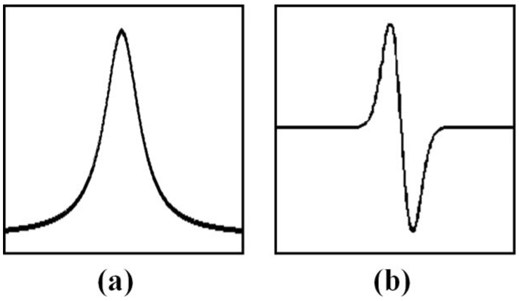
Let $W^1 f(x)=f*\psi^1(x)$ the wavelet transform of a $f(x)$ function performed by the application of wavelet functions of type $\psi^1(x)$. In the work of [8] it was demonstrated that $W^1 f(x)$ is proportional to the first derivative of the $f(x)$ function smoothed by the convolution with the smoothing function $\theta(x)$. This means that maximum $W^1 f(x)$ localities can be used as detectors of abrupt points of variation (edges) and discontinuities of the function $f(x)$. In two-dimensions, [8] showed that WTMM can be used for edge detection in images.

For two-dimensional functions $\left(z=f(x,y)\right)$, the magnitude of the gradient is given by:

$$|G|=\sqrt{G_x^2+G_y^2} \quad (3)$$

where G_x is the derivative in the horizontal direction $\left(\partial\Psi_1(x,y)/\partial x\right)$ and G_y is the derivative in the vertical direction $\left(\partial\Psi_1(x,y)/\partial y\right)$ of the grid;

And the direction of the gradient is given by:



Source: Adapted from [8].

Figure 3. Wavelet Transform Modulus Maxima. (a) Function *wavelet* $\theta(x)$; (b) $\psi^1(x)$: first derivative of $\theta(x)$.

$$\theta = \arctan \frac{G_x}{G_y} \tag{4}$$

3. Methods

The data used in the experiments are related to the DTM of an area of the Polytechnic Center of the Federal University of Paraná (**Figure 4**). The DTM was obtained by laser profiling (Lidar: Laser Dectection and Ranging) by the Institute of Technology for Development (LACTEC) with the LIDAR OPTECH ALTM 2050 System.

The grid obtained consists of 400 × 400 points of altimetry, with planimetric resolution of 1.00 meters and altitudes varying from 888.90 to 944.69 meters. The data are found in the SAD-69 (South American Datum 1969) reference geodetic system and the Universal Transverse Mercator coordinate system (UTM), in the UTM zone No. 22 (Central Meridian −51˚).

The Roberts, Sobel-Feldman, Canny and WTMM edge detectors were applied to the original DTM grid. The detectors of Roberts, Sobel-Feldman and Canny used were those in the Matlab library. The WTMM was implemented in Matlab language, from the Haar wavelet transform, also implemented in the same language. The data was processed on a PC/Desktop computer, with an Athlon XP processor of 2600 MHz and 1 (one) Gigabyte of RAM.

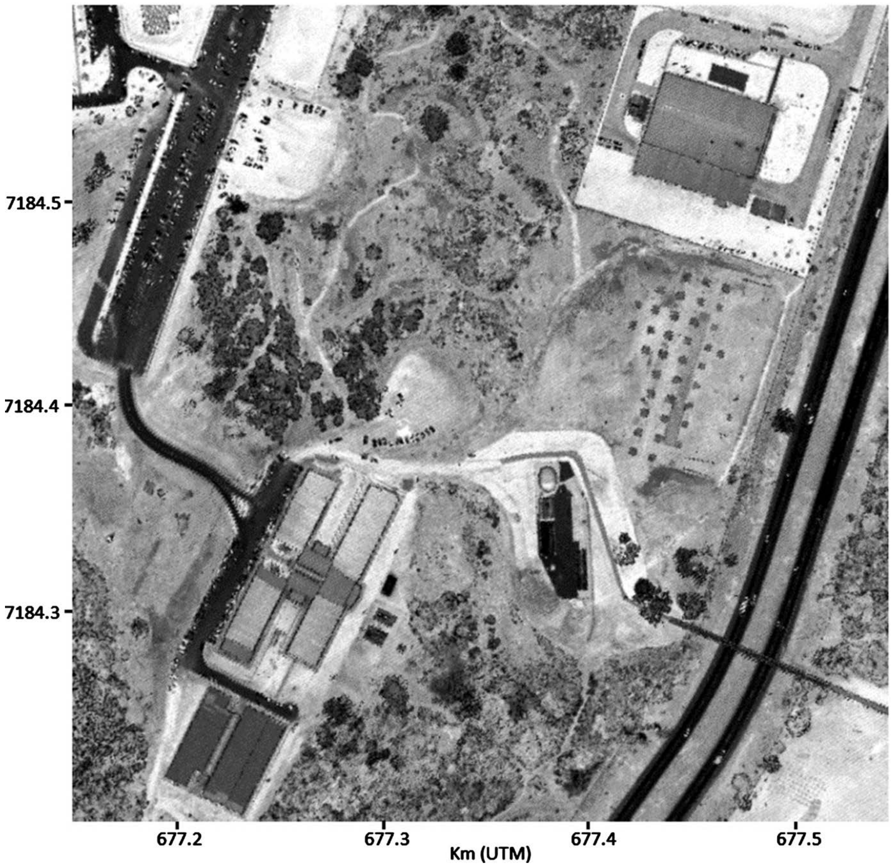


Figure 4. DTM of the study area.

4. Results

Figures 5-8 are shown the results obtained in the edge detections and in **Table 1** are shown the respective processing times. From the analysis of the figures and the table we can conclude that the Canny detector presented a better result, mainly in the detection of circular edges (northwest sector of the study region), but at a much higher computational cost. On the other hand, WTMM presented a similar result of Sobel-Feldman and better than Roberts, but with a much lower computational cost, although a user-made computer program is usually slower than libraries. The processing time of WTMM was about three times less than Roberts and Sobel-Feldman and 18 times less than Canny.

It was also observed that the WTMM has thicker edges than the other detectors.

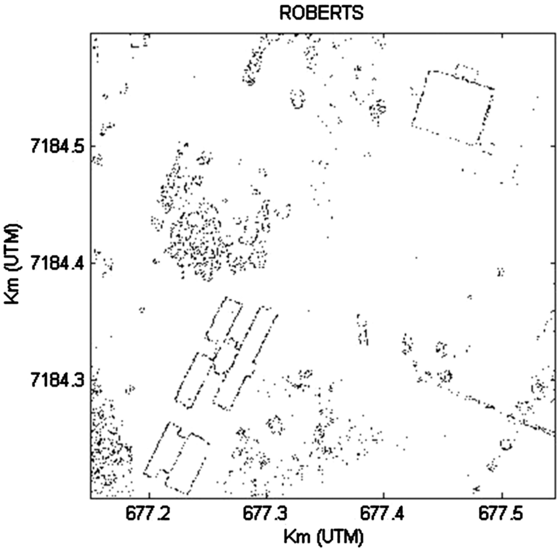


Figure 5. Edges obtained by the Roberts detector.

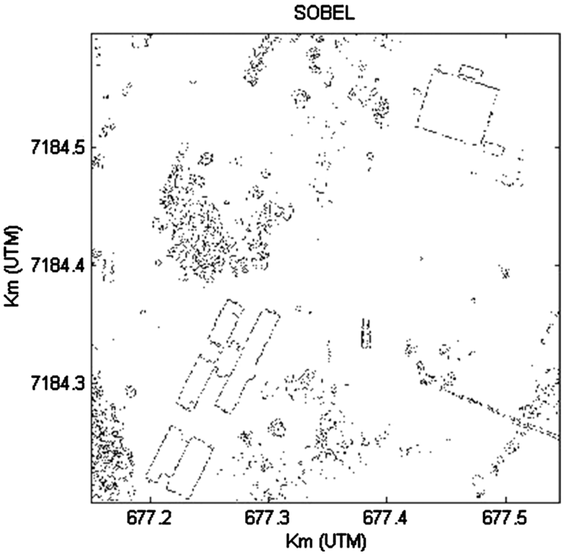


Figure 6. Edges obtained by the Sobel-Feldman detector.

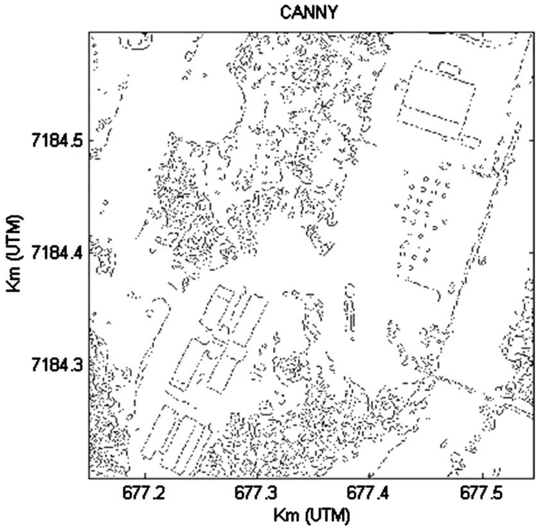


Figure 7. Edges obtained by the Canny detector.

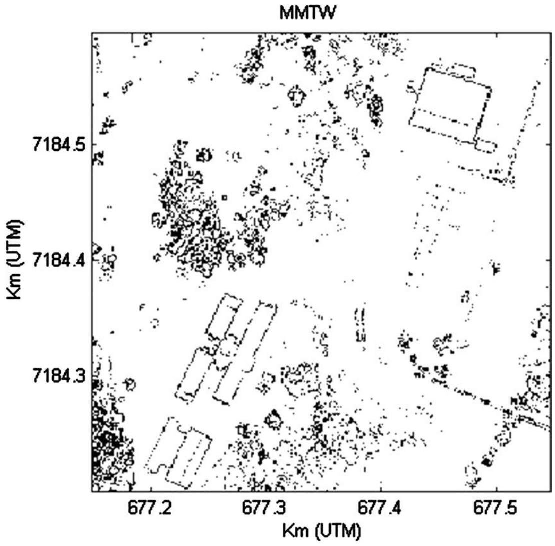


Figure 8. Edges obtained by the WTMM detector.

Table 1. Processing time.

Method	Processing time (miliseconds)
Roberts	6.5
Sobel-Feldman	7.0
Canny	36.0
WTMM	2.0

5. Conclusion

The experiments carried out in this research had the objective of verifying the feasibility of using the wavelet transform in the edge detection of the digital terrain model obtained by Laser Scanner. The results show that edge detection by using WTMM can be seen as an alternative to edge detection due to its characteristics of simplicity of the algorithms, ease of implementation and low computational cost.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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