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Abundant Lump Solutions and Interaction Phenomena to the Kadomtsev-Petviashvili-Benjamin-Bona-Mahony Equation

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Abstract

In this paper, we obtained a kind of lump solutions of the Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) equation with the assistance of Mathematica. Some contour plots with different determinant values are sequentially made to show that the corresponding lump solutions tend to zero when $x^2 + y^2 \to \infty$. Particularly, lump solutions with specific values of the include parameters are plotted, as illustrative examples. Finally, a combination of stripe soliton and lump soliton is discussed to the KP-BBM equation, in which such a solution presents two different interesting phenomena: lump-kink and lump-soliton. Simultaneously, breather rational soliton solutions are displayed.

Keywords

Lump Solution, KP-BBM Equation, Hirota Bilinear Form, Interaction Phenomenon, Breather Soliton

1. Introduction

Soliton, rogue waves, lump solutions, breather waves and interaction solutions of nonlinear evolution equations (NLEEs) have attracted more and more attention [1] [2] [3] [4], and lump solutions are a kind of rational function and localized in all directions of space. Lump-soliton solutions have the meromorphic structures which can guarantee their stability [5]. Furthermore, lump solutions can be regarded as the localized wave configurations which decay rationally to the asymptotic values, and lump solitons move with the uniform velocities [6]. Lump solution may not be able to maintain its amplitude and shape through the interaction of the soliton solution. This interaction is inelastic [7]. Lump solutions have been found for kinds of integrable equation [8] [9] [10]. Lump solutions emerge the non-linear patterns, such as optic media, plasma, shallow water wave, and Bose-Einstein condensate [11].

Recently Ma W. X. found out that the approach to solving the lump solutions of NLEEs use the Hirota bilinear form [12]. The study of lump solution has been lack of development because of the complexity in the process that the lump solution of NLEEs can be solved. He successfully proved form of the solution and its existence [13]. By using this method, some researchers perfectly constructed the lump solutions and the interaction solutions [14]-[38] and breather waves [11], [39] of NLEEs.

In the following, we consider the Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) equation

$$u_{xt}+u_{xx}+\alpha\left(u^2\right)_{xx}+\beta u_{xxxt}+\gamma u_{yy}=0. \tag{1}$$

The KP-BBM equation is formulated using the KP equation, derived from the standard BBM equation [40]. Some of the previous studies have been done [41] [42] [43] [44].

In the present paper, we would like to focus on KP-BBM equation. It has a Hirota bilinear form, and so, we will do a search for the positive quadratic function solutions to the corresponding bilinear KP-BBM equation. Firstly, we will obtain five different classes of positive quadratic function solutions. Secondly, the interaction solution constructed through symbolic computations beginning with a linear combination ansatz, in which such solution presents two different interesting phenomena: lump-kink and lump-soliton. Thirdly, breather rational soliton solutions are derived. Finally, some conclusions will be drawn at the end of this article.

2. Lump Solution

Under the bilinear transformation

$$u=2\left(\ln f\right)_{xx}. \tag{2}$$

Equation (1) is turned into the following Hirota bilinear form:

$$\begin{aligned}&\text{KP-BBM}(f):\left(D_xD_t+D_x^2+\beta D_x^3D_t+\gamma D_y^2+\alpha D_x^2-3\beta D_xD_t\right)(f\cdot f)\\&=2\left[ff_{xt}-f_tf_x+f_{xx}f-f_x^2+\beta\left(ff_{xxxt}-3f_xf_{xxt}+3f_{xx}f_{xt}-f_tf_{xxx}\right)\right.\\&\quad\left.+\gamma\left(f_{yy}f-f_y^2\right)+\alpha\left(f_{xx}f-f_x^2\right)-3\beta\left(f_{xt}f-f_tf_x\right)\right]=0,\end{aligned} \tag{3}$$

where the operator D is defined:

$$\begin{aligned}D_t^lD_x^kD_y^nf\cdot g=&\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^l\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^k\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^nf(x,y,t)\\&\cdot g\left(x',y',t'\right)|\,x=x',y=y',t=t'.\end{aligned}$$

To search for the quadratic function solution of the bilinear KP-BBM equation, we suppose

$$f = g^2 + h^2 + a_9, g = a_1 x + a_2 y + a_3 t + a_4, h = a_5 x + a_6 y + a_7 t + a_8, \tag{4}$$

where $a_i (i = 1, \cdots, 9)$ are real parameters to be determined later. To figure out a set of algebraic equations in $a_i (i = 1, \cdots, 9)$, we substitute (4) into (3) and equate all the coefficients of different polynomials of x, y, t to zero. We obtain a set of algebraic equations in $a_i (i = 1, \cdots, 9)$. Solving the set of algebraic equations, we can find the following sets of solutions.

Case 1:

$$a_1 = \frac{a_4 a_5}{a_8}, a_2 = -\frac{a_6 a_8}{a_4}, a_3 = \frac{a_4 a_6 \alpha \sqrt{\gamma}}{\sqrt{3\beta a_4^2 (\alpha + 3\beta)}}, a_4 = a_4, a_5 = \frac{\sqrt{3} a_6 a_8 \sqrt{\beta} \sqrt{\gamma}}{\sqrt{a_4^2 (\alpha + 3\beta)}},$$

$$a_6 = a_6, a_7 = \frac{a_6 a_8 \alpha \sqrt{\gamma}}{\sqrt{3 a_4^2 \beta (\alpha + 3\beta)}}, a_8 = a_8, a_9 = -\frac{9 a_6^2 (a_4^2 + a_8^2) \alpha \beta^2 \gamma}{a_4^2 (\alpha + 3\beta)^2},$$

where a_4, a_6, a_7, a_8 are arbitrary constants, and all these constants are satisfied with the condition as follows:

$$a_4 a_8 \beta (\alpha + 3\beta) \neq 0, \alpha\gamma < 0. \tag{5}$$

This set leads to a kind of positive quadratic function solutions to the bilinear KP-BBM equation in (3)

$$\begin{aligned} f = & \left[\frac{a_4 a_5}{a_8} x - \frac{a_6 a_8}{a_4} y + \frac{a_4 a_6 \alpha \sqrt{\gamma}}{\sqrt{3\beta a_4^2 (\alpha + 3\beta)}} t + a_4 \right]^2 \\ & + \left[\frac{\sqrt{3} a_6 a_8 \sqrt{\beta} \sqrt{\gamma}}{\sqrt{a_4^2 (\alpha + 3\beta)}} x + a_6 y + \frac{a_6 a_8 \alpha \sqrt{\gamma}}{\sqrt{3 a_4^2 \beta (\alpha + 3\beta)}} t + a_8 \right]^2 \\ & - \frac{9 a_6^2 (a_4^2 + a_8^2) \alpha \beta^2 \gamma}{a_4^2 (\alpha + 3\beta)^2} \end{aligned} \tag{6}$$

and the resulting is a kind of positive quadratic function solutions. In turn, we gain the lump solution to the KP-BBM Equation in (1) by using the transformation (2)

$$u_1(x, y, t) = \frac{4(a_1^2 + a_5^2) f - 8(a_1 g + a_5 h)^2}{f^2}, \tag{7}$$

where the function f is defined by (6), and the functions g and h are given as follows:

$$\begin{aligned} g &= \frac{a_4 a_5}{a_8} x - \frac{a_6 a_8}{a_4} y + \frac{a_4 a_6 \alpha \sqrt{\gamma}}{\sqrt{3\beta a_4^2 (\alpha + 3\beta)}} t + a_4, \\ h &= \frac{\sqrt{3} a_6 a_8 \sqrt{\beta} \sqrt{\gamma}}{\sqrt{a_4^2 (\alpha + 3\beta)}} x + a_6 y + \frac{a_6 a_8 \alpha \sqrt{\gamma}}{\sqrt{3 a_4^2 \beta (\alpha + 3\beta)}} t + a_8. \end{aligned} \tag{8}$$

Figure 1 show the profile of u_1 and its density plots.

Note that the lump solution in (7) are analytic if the parameter satisfy $a_4 \beta (\alpha + 3\beta) \neq 0$ and $\alpha\gamma < 0$. We find that at any given time *t*, the above lump

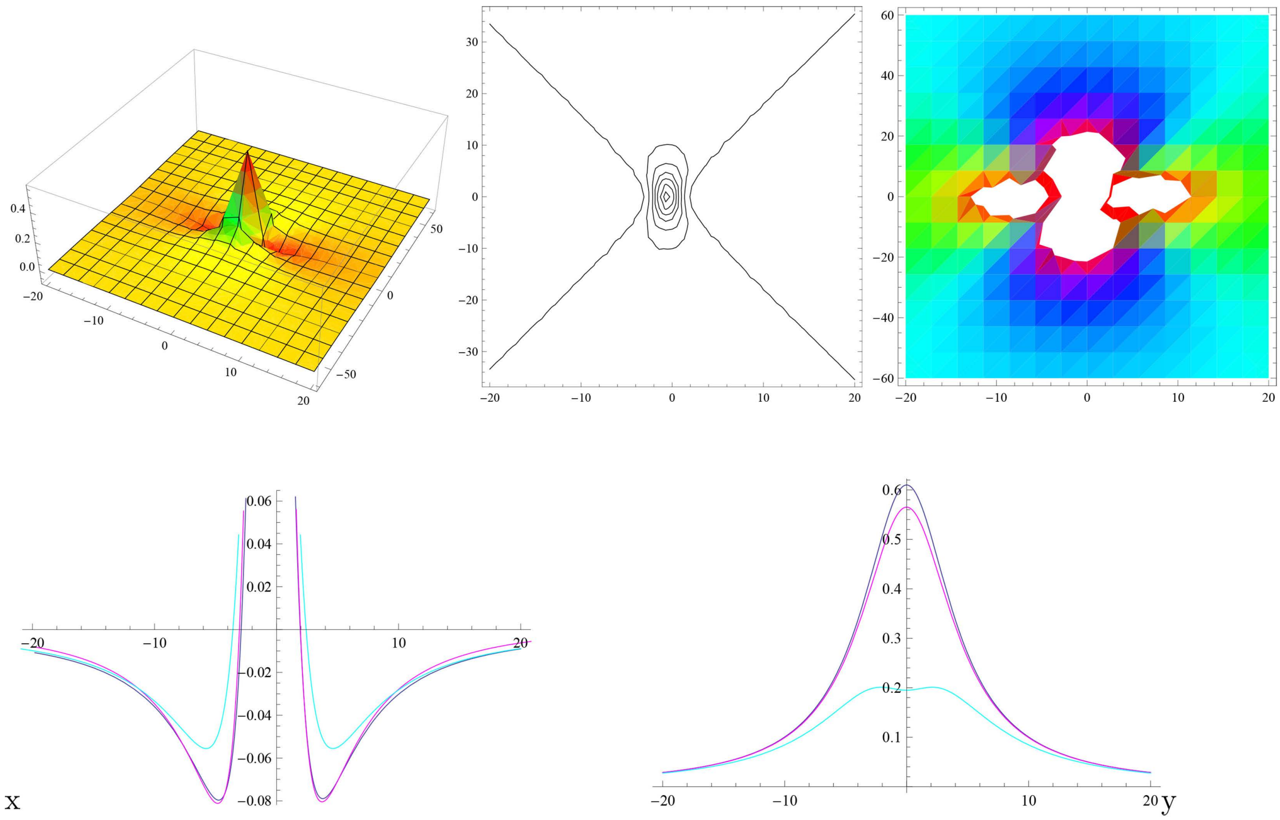

Figure 1. The profiles of $u_1(x,y,t)$ with $t=0$, density plot and *x*,*y*-curves with $a_4=1, a_6=1, a_8=1, \alpha=-2, \beta=1, \gamma=1$.

solutions $u \to 0$ if the corresponding sum of squares $g^2+h^2 \to \infty$.

Case 2:

$$a_1=\frac{3a_3\beta}{\alpha}, a_2=-\frac{a_6a_5\alpha}{3a_3\beta}, a_3=-\frac{a_6\alpha\sqrt{\gamma}}{\sqrt{3}\sqrt{\beta}\sqrt{\alpha+3\beta}}, a_4=a_4, a_5=a_5, a_6=a_6,$$

$$a_7=\frac{a_1^2\alpha+a_5^2\alpha-3a_1a_3\beta}{3a_5\beta}, a_8=a_8, a_9=-\frac{3\alpha\beta\left(a_5^2(\alpha+3\beta)+3a_6^2\beta\gamma\right)}{(\alpha+3\beta)^2},$$

where a_4, a_5, a_6, a_8 are arbitrary constants, and all these constants are satisfied with the condition as follows:

$$\gamma\beta \neq 0, \alpha+3\beta \neq 0, 3\alpha\beta\left(a_5^2(\alpha+3\beta)+3a_6^2\beta\gamma\right)<0.$$

This set leads to a kind of positive quadratic function solutions to the bilinear KP-BBM equation in (3)

$$f=\left[a_8+a_6y+a_5\left(x+\frac{t\alpha}{3\beta}\right)\right]^2-\frac{3\alpha\beta\left(a_5^2(\alpha+3\beta)+3a_6^2\beta\gamma\right)}{(\alpha+3\beta)^2} +\frac{\left[\sqrt{3}a_5y(\alpha+3\beta)+3a_4\sqrt{\beta\gamma(\alpha+3\beta)}-\sqrt{3}a_6(t\alpha+3x\beta)\gamma\right]^2}{9\beta\gamma(\alpha+3\beta)}. \tag{9}$$

Analogously, we obtain the following lump solution to Equation (1)

$$u_2(x,y,t) = \left[-\frac{8\left(3a_5a_8\beta(\alpha+3\beta)+a_5^2(\alpha+3\beta)(t\alpha+3x\beta)-3\sqrt{3}a_4a_6\beta^{\frac{3}{2}}\sqrt{(\alpha+3\beta)\gamma}+3a_6^2\beta(t\alpha+3x\beta)\gamma\right)^2}{9f^2\beta^2(\alpha+3\beta)^2}\right] + \left[4\left(a_5^2+\frac{3a_6^2\beta\gamma}{\alpha+3\beta}\right)\right]\frac{1}{f}, \quad (10)$$

where the function f is defined by (9).

Case 3:

$$a_1=a_1, a_2=\frac{a_4a_6}{a_8}, a_3=\frac{a_1\alpha}{3\beta}, a_4=a_4, a_5=a_5, a_6=\frac{a_1\sqrt{\alpha+3\beta}}{\sqrt{3}\sqrt{\gamma}\sqrt{\beta}},$$
$$a_7=\frac{a_5\alpha}{3\beta}, a_8=-\frac{a_1a_4}{a_5}, a_9=-\frac{3\left(a_1^2+a_5^2\right)\alpha\beta}{\alpha+3\beta},$$

where a_1, a_4, a_5 are arbitrary constants, and all these constants are satisfied with $a_5a_8\beta\gamma\neq 0$, $a_2^2a_5-2a_1a_2a_6-a_6^2a_5\neq 0$ and $\frac{\alpha\beta}{\alpha+3\beta}<0$.

This leads to

$$f=\left[a_1\left(x+\frac{t\alpha}{3\beta}\right)-\frac{a_5y\sqrt{\alpha+3\beta}}{\sqrt{3\beta\gamma}}+a_4\right]^2 + \left(a_5x+\frac{a_1y\sqrt{\alpha+3\beta}}{\sqrt{3}\sqrt{\gamma}\sqrt{\beta}}+\frac{a_5t\alpha}{3\beta}-\frac{a_1a_4}{a_5}\right)^2-\frac{3\left(a_1^2+a_5^2\right)\alpha\beta}{\alpha+3\beta}. \quad (11)$$

Analogously, we obtain the following lump solution to Equation (1)

$$u_3(x,y,t)=-\frac{36a_5^2\beta^2(\alpha+3\beta)\gamma\left[6\sqrt{3}a_4a_5y\beta^{\frac{3}{2}}\sqrt{\gamma}(\alpha+3\beta)^{\frac{3}{2}}-9a_4^2\beta^2(\alpha+3\beta)\gamma+a_5^2(-3p+q)\right]}{-6\sqrt{3}a_4a_5y\beta^{\frac{3}{2}}(\alpha+3\beta)^{\frac{3}{2}}\sqrt{\gamma}+9a_4^2\beta^2(\alpha+3\beta)\gamma+a_5^2(3p+q)}, \quad (12)$$

where

$$p=y^2\beta(\alpha+3\beta)^2,$$
$$q=t^2\alpha^2(\alpha+3\beta)+6tx\alpha\beta(\alpha+3\beta)+9\beta^2\left[-3\alpha\beta+x^2(\alpha+3\beta)\right]\gamma. \quad (13)$$

Case 4:

$$a_1=-\frac{3a_6a_7\beta}{a_2\alpha}, a_2=a_2, a_3=\frac{a_1^2\alpha+a_5^2\alpha-3a_5a_7\beta}{3a_1\beta}, a_4=\frac{a_2a_8}{a_6}, a_5=\frac{3a_7\beta}{\alpha},$$
$$a_6=a_6, a_7=\frac{a_2\alpha\sqrt{\gamma}}{\sqrt{3\beta}\sqrt{\alpha+3\beta}}, a_8=a_8, a_9=-\frac{9\left(a_2^2+a_6^2\right)\alpha\beta^2\gamma}{(\alpha+3\beta)^2}, \quad (14)$$

where a_2, a_6, a_8 are arbitrary constants, and all these constants are satisfied with $\gamma\alpha<0$ and $a_1a_2a_6(\alpha+3\beta)\alpha\beta\neq 0$.

This leads to

$$f=\left(a_8^2+\frac{a_2^2a_8^2}{a_6^2}\right)+\frac{2\left(a_2^2+a_6^2\right)a_8 y}{a_6}+\frac{\left(a_2^2+a_6^2\right)(3p+q)}{3\beta(\alpha+3\beta)^2}. \tag{15}$$

Analogously, we obtain the following lump solution to Equation (1)

$$u_4(x,y,t)=-\frac{36a_6^2\beta^2(\alpha+3\beta)\gamma\left[-3a_8^2\beta(\alpha+3\beta)^2-6a_6a_8y\beta(\alpha+3\beta)^2+a_6^2(-3p+q)\right]}{\left[3a_8^2\beta(\alpha+3\beta)^2+6a_6a_8y\beta(\alpha+3\beta)^2+a_6^2(3p+q)\right]^2}, \tag{16}$$

where the functions p,q is defined by (13).

Figure 2 show the profile of u_4 and its density plots.

Case 5:

$$a_1=\frac{3a_5a_7a_8\beta}{2a_4a_5\alpha-3a_4a_7\beta}, a_2=-\frac{3a_7^2a_8\beta(\alpha+3\beta)}{a_4a_6\alpha^2\gamma}, a_3=\frac{a_7a_8}{a_4}, a_4=a_4, a_5=\frac{3a_7\beta}{\alpha},$$

$$a_6=\frac{\sqrt{3}a_7a_8\sqrt{\beta}\sqrt{\alpha+3\beta}}{a_4\alpha\sqrt{\gamma}}, a_7=a_7, a_8=a_8, a_9=-\frac{27a_7^2\left(a_4^2+a_8^2\right)\beta^3}{a_4^2\alpha(\alpha+3\beta)},$$

where a_4,a_7,a_8 are arbitrary constants, and all these constants are satisfied with $(\beta+3\alpha)\gamma\alpha a_4a_6\neq 0$, $2a_4a_5\alpha-3\beta a_4a_7\neq 0$ and $\frac{\beta^3}{\alpha^2+3\alpha\beta}<0$,

$$f=-\frac{27a_7^2\left(a_4^2+a_8^2\right)\beta^3}{a_4^2\alpha(\alpha+3\beta)}+\frac{\left[a_4^2\alpha+a_7a_8(t\alpha+3x\beta)-\frac{\sqrt{3}a_4a_7y\sqrt{\beta}\sqrt{\alpha+3\beta}}{\sqrt{\gamma}}\right]^2}{a_4^2\alpha^2}+\left[a_8+a_7\left(t+\frac{3x\beta}{\alpha}\right)+\frac{\sqrt{3}a_7a_8y\sqrt{\beta}\sqrt{\alpha+3\beta}}{a_4\alpha\sqrt{\gamma}}\right]^2. \tag{17}$$

Analogously, we obtain the following lump solution to Equation (1)

$$u_5(x,y,t)=\frac{36a_7^2\beta\left(-2\left(a_7a_8^2(t\alpha+3x\beta)+a_4^2(2a_8\alpha+a_7t\alpha+3a_7x\beta)\right)^2+a_4^2\left(a_4^2+a_8^2\right)\alpha^2 f\right)}{a_4^2\alpha^4 f^2}, \tag{18}$$

where the function f is defined by (17).

3. Interaction Solutions

3.1. Lump-Kink Solutions

In this section, we will study the interaction between a lump and a stripe of the KP-BBM Equation (1). For search the interaction between rational solution and solitary wave solution, we turn the above function $f(x,y,t)$ into the following new form

$$f=(a_1x+a_2y+a_3t+a_4)^2+(a_5x+a_6y+a_7t+a_8)^2+\exp(a_9x+a_{10}y+a_{11}t+a_{12})+a_{13}, \tag{19}$$

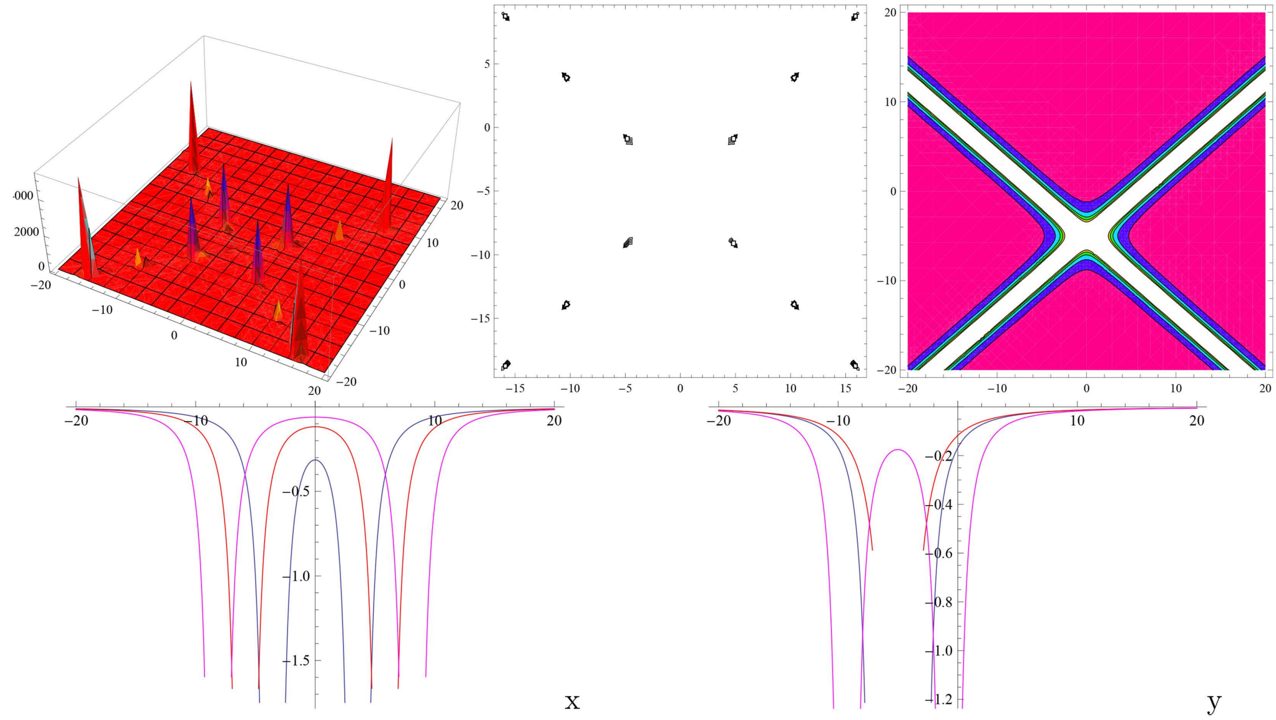

Figure 2. The profiles of $u_4\left(x,y,t\right)$ with $t=0$, density plot and *x*-curves with $a_2=1, a_6=1, a_8=5, \gamma=\beta=1, \gamma=-1$.

where $a_i\left(i=1,\cdots,13\right)$ are real constants to be determined and $a_{13}>0$.

For Equation (1), substituting (19) into the corresponding bilinear form (3), direct Mathematica symbolic computations generate the following set of solutions for the parameters:

$$\begin{aligned}&a_1=-\frac{a_5a_7}{a_3}, a_9=0, \gamma=0, \beta=\frac{2}{3}, \alpha=-2,\\&a_i=a_i\ \left(i=2,3,4,5,6,7,8,10,11,12,13\right),\end{aligned}\tag{20}$$

where $a_3\neq 0$. Then the exact interaction solution of u is expressed as follows:

$$\begin{aligned}u_1&=2\left(\ln f\right)_{xx}\\&=2\left[-\frac{\left(a_9\exp\left(a_{12}+a_{11}t+a_9x+a_{10}y\right)+2a_1\left(a_4+a_3t+a_1x+a_2y\right)+2a_5\left(a_8+a_7t+a_5x+a_6y\right)\right)^2}{\left(a_{13}+\exp(a_{12}+a_{11}t+a_9x+a_{10}y)+\left(a_4+a_3t+a_1x+a_2y\right)^2+\left(a_8+a_7t+a_5x+a_6y\right)^2\right)^2}\right.\\&\left.+\frac{2a_1^2+2a_5^2+a_9^2\exp\left(a_{12}+a_{11}t+a_9x+a_{10}y\right)}{a_{13}+\exp\left(a_{12}+a_{11}t+a_9x+a_{10}y\right)+\left(a_4+a_3t+a_1x+a_2y\right)^2+\left(a_8+a_7t+a_5x+a_6y\right)^2}\right].\end{aligned}\tag{21}$$

To obtain the collision phenomenon, $a_3^2+a_7^2+a_{11}^2\neq 0$ is necessary. So the asymptotic behavior of u can be obtained, when

$$\lim_{t\to\infty}u\left(x,t,y\right)=0.$$

The asymptotic behavior shows that the lump is finally submerged drowned or swallowed up by the stripe along with the change of time. **Figure 3** exhibit the interaction between the lump soliton and kink soliton. The interaction between two solitary waves is inelastic.

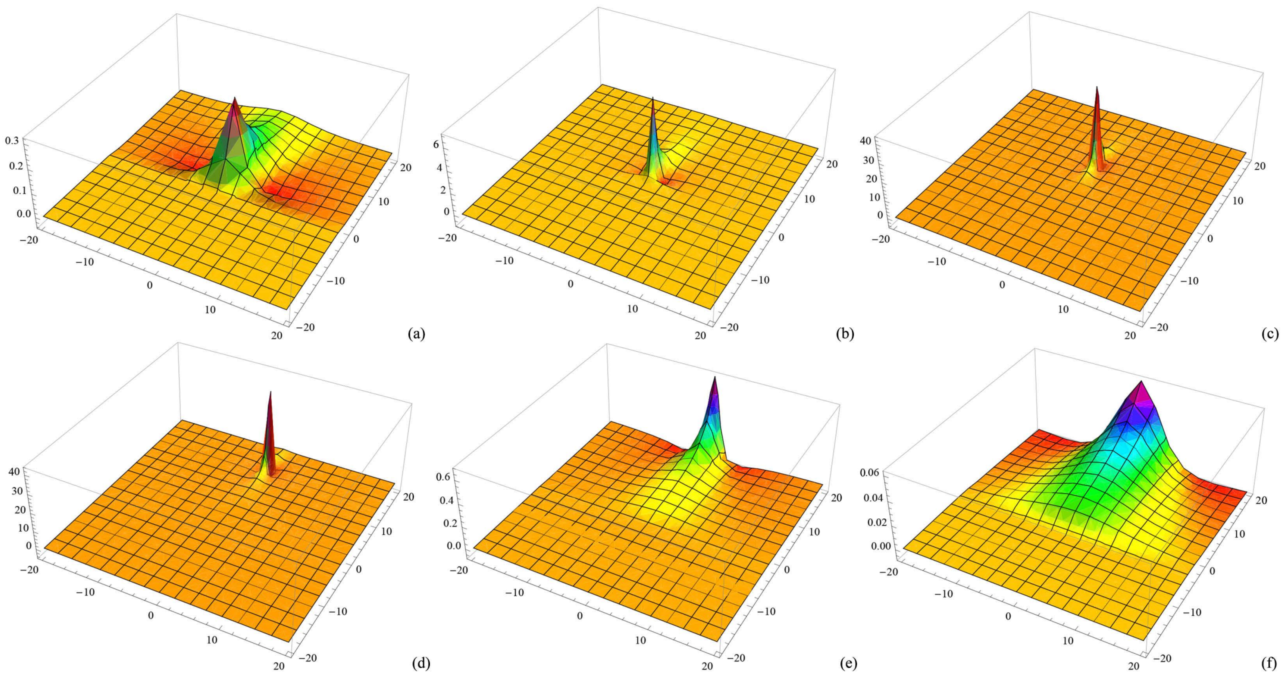

Figure 3. The interaction solution Equation (21) for $u_1(x,y,t)$ with $a_2=-1$, $a_3=2$, $a_4=0$, $a_5=-\frac{5}{2}$, $a_6=-1$, $a_7=2$, $a_8=0$, $a_{10}=-3$, $a_{11}=-\frac{1}{3}$, $a_{12}=5$, $a_{13}=1$ when $t=3$ (a)(d), $t=0$ (b)(e), $t=-3$ (c)(f).

3.2. Lump-Soliton Solutions

In the following, we compute interaction solutions between lumps and solitons to the KP-BBM Equation (1). It is combined functions of positive quadratic functions and hyperbolic cosine, and then we explored nonlinear phenomenon. We suppose

$$\begin{aligned} f=&\left(a_1x+a_2y+a_3t+a_4\right)^2+\left(a_5x+a_6y+a_7t+a_8\right)^2 \\ &+\cosh\left(a_9x+a_{10}y+a_{11}t+a_{12}\right)+a_{13}, \end{aligned} \tag{22}$$

where $a_i\left(i=1,\cdots,13\right)$ are real constants to be determined and $a_{13}>0$. Substituting (22) into the Hirota bilinear form (3), direct symbol calculation results in a kind of solutions:

$$\begin{aligned} &a_1=\frac{4a_2a_6\gamma}{a_7\left(-2+3\beta\right)}, a_2=-\frac{a_3a_6}{a_7}, a_3=\frac{a_7}{\sqrt{3}}, \\ &a_4=0, a_5=0, a_6=a_6, a_7=a_7, a_8=a_8, \\ &a_9=\frac{\sqrt{-2+3\beta}}{\sqrt{2\beta}}, a_{10}=0, a_{11}=0, \alpha=-2, \\ &a_{12}=a_{12}, a_{13}=\frac{64a_6^4\beta\gamma^2}{3a_7^2\left(-2+3\beta\right)^3}. \end{aligned} \tag{23}$$

Then we can obtain the soliton-lump soliton to Equation (1) with the transformation $u_2=2\left(\ln f\right)_{xx}$, where

$$f=\left(a_8+a_7t+a_6y\right)^2+\frac{64a_6^4\beta\gamma^2}{3a_7^2\left(-2+3\beta\right)^3}$$
$$+\frac{\left[a_6a_7y\left(2-3\beta\right)+a_7^2t\left(-2+3\beta\right)-4a_6^2x\gamma\right]^2}{3a_7^2\left(2-3\beta\right)^2}+\cosh\left[a_{12}+\frac{x\sqrt{-2+3\beta}}{\sqrt{2\beta}}\right]. \quad (24)$$

According to the Ref. [45], we can calculate the general formula of the original coordinates of lumps.

$$\left(\frac{a_2a_9t-a_4a_7t-a_5a_7}{a_1a_7-a_2a_6},-\frac{a_1a_9t-a_4a_6t-a_5a_6}{a_1a_7-a_2a_6}\right) \quad (25)$$

where $a_1a_7-a_2a_6\neq 0$. Then, substituting $a_5=0$ into formula (25). From formula (23), we know that the initial velocities in x direction and y direction of lump are $v_x=-\frac{a_4a_7-a_2a_9}{a_1a_7-a_2a_6}$ and $v_y=-\frac{a_1a_9-a_4a_6}{a_1a_7-a_2a_6}$. But in the collision, the lump and the solitary wave will exchange the energy, which will result in that the lump and the solitary wave will not be moving in the original trajectories or moving at the same speeds [46].

To illustrate the interaction phenomena between a lump and a stripe, we select the following parameters:

$$a_6=-3, a_7=5, a_8=\frac{1}{2}, a_{12}=\frac{3}{5}, \beta=2, \gamma=1. \quad (26)$$

Figure 4 and **Figure 5** show the profile of $u_2\left(x,y,t\right)$ and its contour plot with the parameters (26).

4. Breather-Wave Solutions

To construct the breather-wave solutions [47] [48], we assume that

$$\begin{cases} f=\exp\left(-p_1\mu\right)+\delta_1\cos\left(p\psi\right)+\delta_2\exp\left(p_1\mu\right), \\ \mu=x+a_1y+b_1t, \\ \psi=x+a_2y+b_2t. \end{cases} \quad (27)$$

where μ and ψ are the linear functions of x,y,t, while p,a_i,b_i (i=1,2) and p_1 are real constants to be determined. To get the following results, we substitute the expression (27) into Bilinear Form (3) and eliminate the coefficients of $\exp\left(p_1\mu\right),\exp\left(-p_1\mu\right),\cos\left(p\psi\right)$.

$$eq[1]=-2p^2\delta^2\left(1+b_2+\alpha-3b_2\beta-4b_2p^2\beta+a_2^2\gamma\right),$$
$$eq[2]=8p_1^2\delta_2\left(1+b_1+\alpha-3b_1\beta+4b_1p_1^2\beta+a_1^2\gamma\right),$$
$$eq[3]=2p^2\delta_1^2\left(-1-b_2-\alpha+3b_2\beta+4b_2p^2\beta-a_2^2\gamma\right),$$
$$eq[4]=2\delta_1\left(b_2p^4\beta+p_1^2\left(1+b_1+\alpha-3b_1\beta+b_1p_1^2\beta+a_1^2\gamma\right)\right.$$
$$\left.-p^2\left(1+b_2+\alpha-3b_2\beta+3b_1p_1^2\beta+3b_2p_1^2\beta+a_2^2\gamma\right)\right),$$
$$eq[5]=-2pp_1\delta_1\left[b_1\left(1-3\beta-p^2\beta+3p_1^2\beta\right)\right]$$
$$+b_2\left(1-3\beta-3p^2\beta+p_1^2\beta\right)+2\left(1+\alpha+a_1a_2\gamma\right).$$

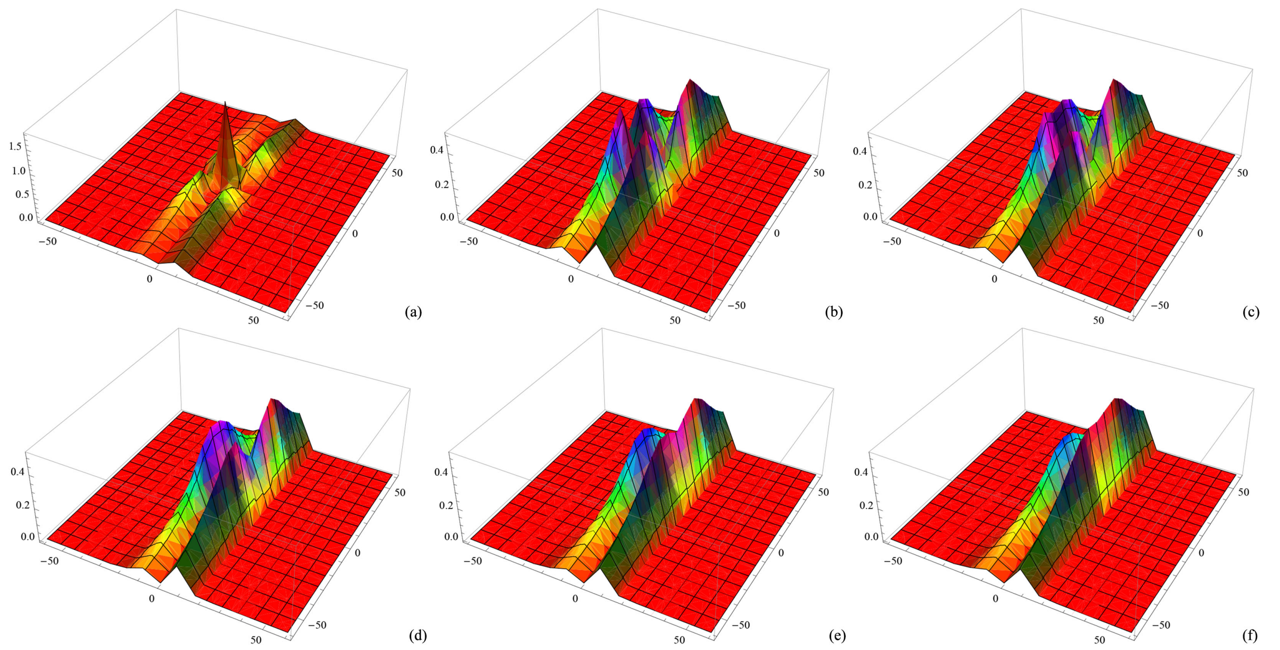

Figure 4. Profiles of $u_2(x,y,t)$ with the parameters (26) at times $t=0$ (a), $t=2$ (b), $t=4$ (c), $t=7$ (d), $t=10$ (e), $t=12$ (f).

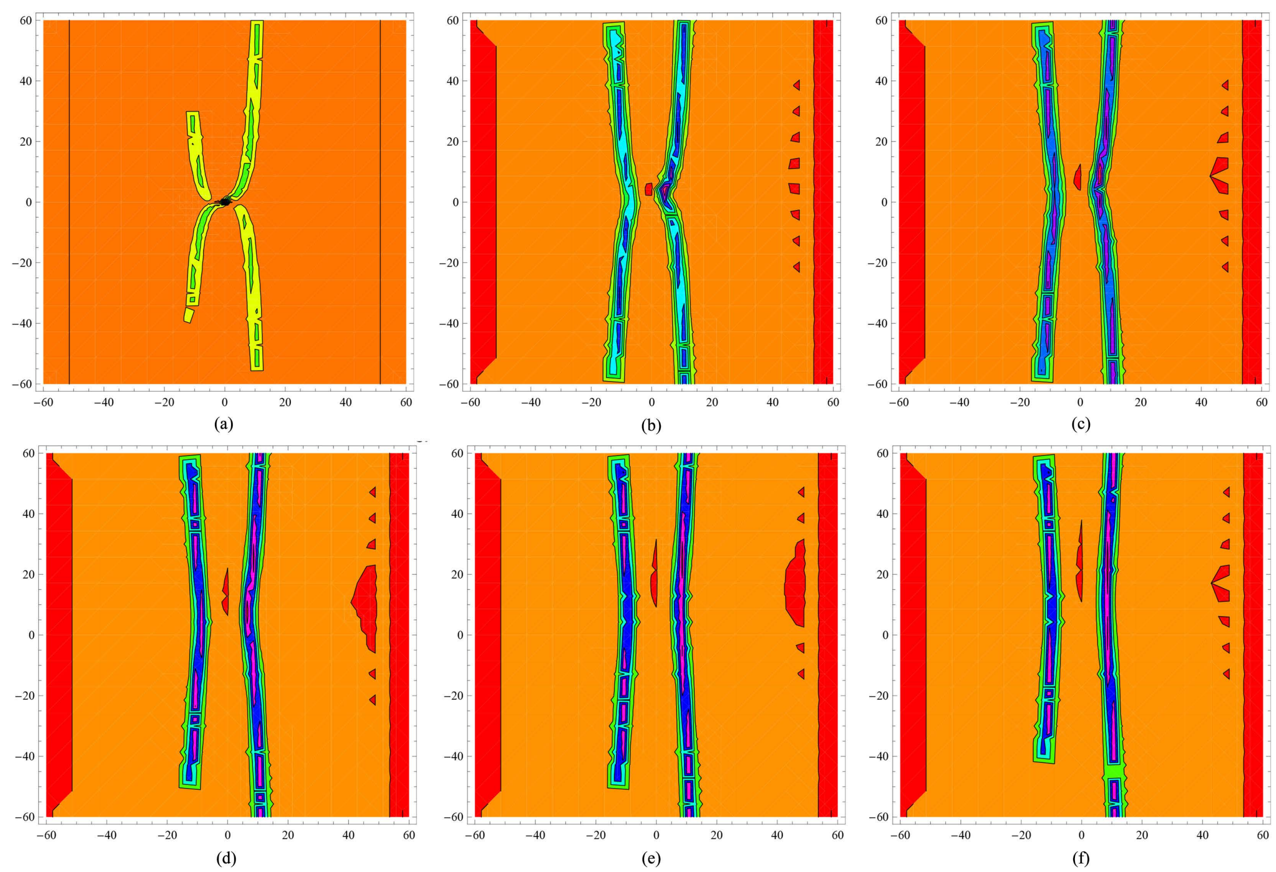

Figure 5. Contour plot of the $u_2(x,y,t)$ with the parameters $a_6=-3$, $a_7=5$, $a_8=\frac{1}{2}$, $a_{12}=\frac{3}{5}$, $\beta=2$, $\gamma=1$ when $t=0$ (a), $t=2$ (b), $t=4$ (c), $t=7$ (d), $t=10$ (e), $t=12$ (f).

Then, we get the following restricted condition:

$$b_1 = -\frac{1+\alpha+a_1^2\gamma}{1-3\beta+4p_1^2\beta}, b_2 = \frac{1+\alpha+a_2^2\gamma}{-1+3\beta+4p^2\beta}, a_1 = a_2,$$
$$p = \frac{p_1\sqrt{-1+3\beta}\sqrt{1+\alpha+a_1^2\gamma}}{\sqrt{1+\alpha-3\alpha\beta+a_2^2\gamma+\beta\left(-3-4a_1^2p_1^2\gamma+a_2^2\left(-3\gamma+4p_1^2\gamma\right)\right)}}. \tag{28}$$

and the expression (27) can be rewritten as

$$f(x,y,t) = 2\sqrt{\delta_2}\cosh\left[p_1\left(x+a_2y-\frac{1+\alpha+a_1^2\gamma}{1-3\beta+4p_1^2\beta}\right)+\frac{1}{2}\ln\delta_2\right]$$
$$+\delta_1\cos\left[p\left(x+a_2y-\frac{t\left(1+\alpha+a_2^2\gamma\right)}{1-3\beta-4p_1^2\beta}\right)\right]. \tag{29}$$

When we take $p = p_1$, lead to

$$u(x,y,t) = \frac{2p_1^2\sinh\left(p_1\mu+\frac{1}{2}\ln\delta_2\right)}{\left[2\sqrt{\delta_2}\cosh\left(p_1\mu+\frac{1}{2}\ln\delta_2\right)+\delta_1\cos\left(p_1\psi\right)\right]^2}. \tag{30}$$

5. Conclusions

In this paper, through Hirota transformation and Mathematica symbolic computation, we have presented five kinds of lump solutions and two classes of interaction phenomena to the KP-BBM Equation (1). Firstly, the analyticity and localization of the resulting solutions are ensured by some determinant conditions, and a sub-class of abundant lump solution includes different choices of the parameters and coefficient. Secondly, some contour plots with different determinant values which are sequentially made to show that the corresponding lump solutions tend to zero when $x^2+y^2\to\infty$. Thirdly, we explored two classes of interaction phenomenon in the literature. It showed in this work illustrate that the exponential-algebraic wave solution is unstable. At last, we obtain the breather-wave.

Solitons always have a connection between amplitude and width, so that it is independent. It is a different shape or profile. As we can see from the picture, lump solutions don't have such connection between amplitude and widths, and the profile shape can be quite random. Therefore, in the shape and formation lump solution process, the three-dimensional solitons solutions are more free than one-dimensional ones.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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