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Addendum to "On an Intrinsically Local Gauge Symmetric *SU*(3) Field Theory for Quantum Chromodynamics"

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Abstract

A much simpler and self-consistent derivation of the non-linear component $G_\mu \times G_\nu$ of the quantum chromodynamic *SU*(3) field tensor is given which does not require the postulate of color confinement to complete the derivation and which mirrors *SU*(2)'s formal development.

Keywords

SU(3) Lagrangian, Local Gauge Invariance, Quantum Chromodynamics, Normed Division Algebras

1. Introduction

In this author's previously published, referenced paper [1][1], a derivation of the non-linear component $G_\mu \times G_\nu$ of the *SU*(3) field tensor for quantum chromodynamics was given which was elaborate and which required the somewhat artificial postulate of color confinement to complete the derivation. A much simpler and mathematically direct derivation which does not rely on color confinement and which mirrors *SU*(2)'s development exists and is given herein. The mathematical methodology used is taken from the subject original paper, which is covered in detail therein [1].

2. The Derivation

The gauge field "cross product" for the non-linear term of the *SU*(3) field tensor has the form [1]

$$(\boldsymbol{B} \times \boldsymbol{C})_i = \sum f_{ijk} B_j C_k \tag{1}$$

[1]See Ref. [1], Sec. 3.i.

where i = 0 – 7 and the f_{iik} are the structure constants of the Gell-Mann commutation relation $\left[\lambda_i, \lambda_j\right] = 2if_{ijk}\lambda_k$. A bijective relation between the Gell-Mann generators $\{\lambda_a\}$ and the octonion basis elements $\{e_a\}$ was given with structure constants existing for the terms [1]

$$f_{ijk} \quad \forall ijk = 123,147,246,257,345,165,376; \tag{2}$$

$$f_{ijk} \quad \forall ijk = 450,670.$$

Using the formalism's unique division-algebraic coupling equation [1]

$$(v_0, \boldsymbol{v})(w_0, \boldsymbol{w}) = (v_0 w_0 - \boldsymbol{v} \cdot \boldsymbol{w}, v_0 \boldsymbol{w} + \boldsymbol{v} w_0 + \boldsymbol{v} \times \boldsymbol{w}), \tag{3}$$

we now consider the coupled operator $\bar{\eta}\eta$ (where $\bar{\eta}$ defines the involution $\bar{\eta} = \gamma_0 - \boldsymbol{\gamma}_{SU(3)}$ of $\eta = \gamma_0 + \boldsymbol{\gamma}_{SU(3)}$) instead of the coupled operator $\eta\eta$ as was considered in the original paper. Setting $\boldsymbol{\gamma} \equiv \boldsymbol{\gamma}_{SU(3)} = \gamma_a e_a; a = 1\text{-}7$, we have for the applicable vector portion $\bar{\eta}\eta_{\wedge SU(3)} \equiv v_0 \boldsymbol{w} + \boldsymbol{v} w_0 + \boldsymbol{v} \times \boldsymbol{w}$ of the coupled operator

$$\bar{\eta}\eta_{\wedge SU(3)} = \gamma_0 \boldsymbol{\gamma} - \boldsymbol{\gamma}\gamma_0 + (\boldsymbol{\gamma} \times \boldsymbol{\gamma}) = 2(\gamma_0 \times \boldsymbol{\gamma}) + (\boldsymbol{\gamma} \times \boldsymbol{\gamma}), \tag{4}$$

in which we have used $a \times b = \frac{1}{2}[a,b]$. As we are using the $\mathbb{O}$-based coupling equation, both terms of Equation (4) are 7-dimensional cross products.

The term $(\boldsymbol{\gamma} \times \boldsymbol{\gamma})$ has components $f'_{iik}\gamma_i\gamma_k$. Since the 7-dim cross product only sums from i = 1 – 7, setting $f'_{iik} = f_{iik}$ only covers the structure constants

$$f_{ijk} \quad \forall ijk = 123,147,246,257,345,165,376.$$

To cover the remaining $f_{iik} \; \forall ijk = 450,670$ we look to the term $(\gamma_0 \times \boldsymbol{\gamma})$, which has components $c'_{i0k}\gamma_0\gamma_k$. Recalling the total asymmetry of f_{iik}, we simply set $c'_{i0k} = -\frac{1}{2} f_{0ik}$ for $ik = \{45,67\}$ and $c'_{i0k} = \frac{1}{2} f_{0ik}$ for $ik = \{54,76\}$, with $c'_{i0k} = 0$ for all other ik and the $\frac{1}{2}$ being required due to the 2 in $2(\gamma_0 \times \boldsymbol{\gamma})$.

The bijective mapping between eight Clifford fields $\tilde{G}_j\tilde{G}_k$ and the eight $SU(3)$ gauge fields G_jG_k follows as in the original paper, with

$$\sum d_{ijk}\gamma_j\gamma_k\tilde{G}_j\tilde{G}_k \Leftrightarrow \sum f_{ijk}G_jG_k, \tag{5}$$

$$d_{ijk} = c'_{ijk} \quad \forall ijk = 450,670;$$

$$d_{ijk} = f'_{ijk} \quad \forall ijk = 123,147,246,257,345,165,376;$$

$$d_{ijk} = 0 \quad \text{otherwise},$$

thus generating the non-linear component $G_\mu \times G_\nu$.

3. Results and Discussion

The derivation herein of the non-linear portion of $SU(3)$'s field tensor is more direct and mathematically straightforward than the original paper's derivation. Further, it mirrors the $SU(2)$ formalism's use of $\bar{\eta}\eta_{\wedge SU(2)}$ in generating the

$W_\mu \times W_\nu$ portion of the $SU(2)$ field tensor and does not require the somewhat artificial postulate of color confinement for the mathematical derivation. Lastly, given this derivation the previously established bijective relation between the octonion basis $\{e_a\}$ and the Gell-Mann generators $\{\lambda_a\}$ [1] is now seen to be unnecessary and superfluous to the octonionic development of $SU(3)$ gauge theory, since the vector section $\bar{\eta}\eta_{\wedge SU(3)} \equiv v_0\boldsymbol{w} + \boldsymbol{v}w_0 + \boldsymbol{v}\times\boldsymbol{w}$ of Equation (3) generates the entirety of $SU(3)$'s Lie algebra structure constants while residing solely within the $\{e_a\}$ basis in doing so.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Wolk, B. (2017) On an Intrinsically Local Gauge Symmetric $SU(3)$ Field Theory for Quantum Chromodynamics. *Advances in Applied Clifford Algebras*, **27**, 3225-3234.