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# The Dynamics of an Impulsive Competitive System with Infinite Delay and Diffusion

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## Abstract

In this paper, we consider an impulsive competitive system with infinite delay and diffusion. Firstly, on basis of inequality estimation techniques and comparison theorem of impulsive differential equations, we obtain some sufficient conditions for the permanence and extinction of the system. Then, we establish sufficient conditions for the globally attractive of the system by constructing appropriate Lyapunov function. Besides, under different impulsive conditions, we discuss the effect of time delay and diffusion on dynamic behavior of the competitive system.

## Keywords

Diffusion, Infinite Delay, Permanence, Extinction, Attractive

## 1. Introduction

In practical world, owing to many natural and man-made factors (e.g., fire, drought, flooding, crop-dusting, deforestation, hunting, harvesting, etc.), the biological species or ecological environments usually undergo some discrete changes of relatively short duration at some fixed times. Such sudden changes can often be characterized mathematically in the form of impulses. With the development of impulsive differential equations, many experts have adequate mathematical models to investigate the dynamical behaviors of such ecosystems with impulsive effects [1] [2] [3] [4] [5]. On the other hand, the Lotka-Volterra competition systems are very important and significant mathematical models in a non-autonomous environment. Many interesting results of the competitive systems on the existence of positive periodic solutions, permanence, extinction, global stability had been studied extensively (see [6] [7] [8] [9] [10]). For example, Wang [10] investigated the following competitive system with impulsive effects:

$$\begin{cases} \left.\begin{aligned} u'(t) &= u(t)\big(b_1(t) - a_{11}(t)u(t) - a_{12}(t)v(t)\big) \\ v'(t) &= u(t)\big(b_2(t) - a_{21}(t)u(t) - a_{22}(t)v(t)\big) \end{aligned}\right\}, t \neq t_k \\ \left.\begin{aligned} u(t_k^+) &= (1+h_k)u(t_k) \\ v(t_k^+) &= (1+g_k)v(t_k) \end{aligned}\right\}, t = t_k, k = 1,2,\cdots. \end{cases}$$

The author obtained sufficient conditions on the uniform persistence and extinction of the system by applying the theorem of differential equations.

In ecological environment, because of the natural enemy, severe competition, deterioration of the patch environment, spatial heterogeneity and human activities, species dispersal in two or more patches becomes one of widespread phenomena of nature. It is an important subject to study the effects of dispersion on the dynamics of species living in patchy environments. Many works on population dynamics in patch environment have been investigated [11] [12] [13] [14]. Moreover, in real ecology environment, the existed number on the history will affect indirectly the number of the species at the moment. Therefore, in order to establish more realistic models, the past history of systems should be taken into account, which has led to the introduction of time-delays in differential equations. Such biological system with infinite delay can be found in [15] [16] [17].

Motivated by above arguments, we establish an impulsive competitive system with infinite delay and diffusion as follows:

$$\begin{cases} \left.\begin{aligned} x_1'(t) &= x_1(t)\Big(a_1(t) - b_1(t)x_1(t) - c_1(t)\int_{-\infty}^0 k_1(s)y(t+s)\,\mathrm{d}s\Big) \\ &\quad + D_{12}(t)\big(x_2(t) - x_1(t)\big) \\ x_2'(t) &= x_2(t)\big(a_2(t) - b_2(t)x_2(t)\big) + D_{21}(t)\big(x_1(t) - x_2(t)\big) \\ y'(t) &= y(t)\Big(a_3(t) - b_3(t)y(t) - c_2(t)\int_{-\infty}^0 k_2(s)x_1(t+s)\,\mathrm{d}s\Big) \end{aligned}\right\}, t \neq t_k \\ \left.\begin{aligned} x_1(t_k^+) &= h_{1k}x_1(t_k) \\ x_2(t_k^+) &= h_{2k}x_2(t_k) \\ y(t_k^+) &= g_k y(t_k) \end{aligned}\right\}, k = 1,2,\cdots \end{cases} \tag{1.1}$$

where $x_i(t)(i=1,2)$ and $y(t)$ represent the population densities at time $t$, respectively. Let $0 = t_0 < t_k < t_{k+1}, k = 1,2,\cdots$ and $t_k \to \infty$ as $k \to \infty$. Species $x_1$ competes with $y$ in patch 1, while $x_1$ can disperse between patch 1 and patch 2, and $y$ is confined to patch 1. $D_{12}(t), D_{21}(t)$ denotes the diffusion coefficients of species $x$. $h_{1k}, h_{2k}$ and $g_k$ are impulsive coefficients at time $t_k$, respectively.

We consider system (1.1) with the following initial conditions

$$\begin{aligned} &x_1(\theta) = \phi_1(\theta), \quad x_2(\theta) = \phi_2(\theta), \quad y(\theta) = \phi_3(\theta), \\ &\phi_i \in PC_+(R_-, R_+), i = 1,2,3, \quad R_- = (-\infty, 0], R_+ = [0, +\infty). \end{aligned} \tag{1.2}$$

where $PC_+ = \{\phi = (\phi_1, \phi_2, \phi_3) \in PC\}$, $\phi_i(\theta) \geq 0$ for all $\theta \in R_-$ and $\phi_i(0^+) > 0$

for $i=1,2,3$. $PC$ is the space of bounded function $\phi(s): R_{-} \to R^3$ which is continuous everywhere expect at the point $t=t_k \in I$ and $\phi(t_k^+), \phi(t_k^-)$ exists with $\phi(t_k^-)=\phi(t_k)$ and with norm $\|\phi\| = \sup_{\theta \in R_{-}} \|\phi_\theta\|$.

In this paper, for any continuous function $f(t)$, we denote

$$f(t) = \max_{i \in I} \{f_i(t)\}, \quad \bar{f}(t) = \min_{i \in I} \{f_i(t)\}.$$

Throughout this paper, we assume that the system (1.1) satisfies ((C1), (C2) see [12], (C3) see [17]):

(C1) all functions are positive, continuous and bounded defined on $R_{+}$, $0 < D_{12}(t), D_{21}(t) \le D$.

(C2) $h_{1k}, h_{2k}$ and $g_k$ are positive constants for all $k=1,2,\cdots$.

(C3) $k_i(s)(i=1,2)$ is a non-negative, piece-wise continuous function defined on $R_{-}$ and satisfy $\int_0^{+\infty} k_i(s)\,\mathrm{d}s = 1$.

Applying some inequality techniques, comparison theorem of impulsive differential equations and Lyapunov function, we study the dynamic behaviors of an impulsive competitive system with infinite delay and diffusion, included permanence, extinction and globally attractive. This paper is organized as follows. Section 2 contains some preliminaries and presents the proof of the lemma. In Section 3, we establish some sufficient conditions which guarantee the system is permanence. In finally section, we give some conditions on the extinction of the system. In Section 4, we study the globally attractive of system (1.1).

## 2. Preliminaries

We consider the following impulsive non-autonomous logistic model

$$\begin{cases} x'(t) = x(t)\left(\alpha(t) - \beta(t)x(t)\right), & t \ne t_k \\ x(t_k^+) = h_k x(t_k), & k=1,2,\cdots, \end{cases} \tag{2.1}$$

where $\alpha(t)$ and $\beta(t)$ are bounded and continuous functions defined on $R_{+}$, $\beta(t) \ge 0$ for all $t \in R_{+}$ and impulsive coefficients $h_k$ are positive constants for any $k=1,2,\cdots$. Then we have the following Lemma 2.1.

**Lemma 2.1.** Suppose that there is a constant $\sigma > 0$ such that

$$\liminf_{t\to\infty} \left( \int_t^{t+\sigma} \beta(s)\,\mathrm{d}s \right) > 0 \tag{2.2}$$

$$\liminf_{t\to\infty} \left( \int_t^{t+\sigma} \alpha(s)\,\mathrm{d}s + \sum_{t \le t_k < t+\sigma} \ln f_k \right) > 0 \tag{2.3}$$

and function

$$h(t,\mu) = \sum_{t \le t_k < t+\mu} \ln h_k \tag{2.4}$$

is bounded on $t \in R_{+}$ and $\mu \in [0,\sigma)$. Then we have

1) There exist constant $M > 0$ and $m > 0$ such that

$$m \le \liminf_{t\to\infty} x(t) \le \limsup_{t\to\infty} x(t) \le M \tag{2.5}$$

for any positive solution $x(t)$ of system (2.1).

2) If all conditions of (1) hold, further if

$$\limsup_{t\to\infty}\left(\int_t^{t+\sigma}\alpha_1(s)\mathrm{d}s+\sum_{t\le t_k<t+\sigma}\ln f_k\right)\le 0 \tag{2.6}$$

then we have $\lim_{t\to\infty}x(t)=0$ for any positive solution $x(t)$ of system (2.1).

The proof of this Lemma can be found in [18], here we omit it.

Next we consider the following impulsive periodic single species logistic system with diffusion

$$\begin{cases} x_i'(t)=x_i(t)(r_i(t)-a_i(t))+\sum_{j=1}^{n}D_{ij}(x_j(t)-x_i(t)), t\ne t_k, \\ x_i(t_k^+)=h_{ik}x_i(t_k), i=1,2,\cdots,n, k=1,2,\cdots, \end{cases} \tag{2.5}$$

Assume that $r_i(t),a_i(t),D_{ij}(t)(i,j\in I)$ are positive, continuous and bounded functions defined on $R_+$. $D_2\ge D_{ij}\ge D_1>0(i\ne j)$, $D_{ii}=0$ for all $i,j\in I(I=1,2,\cdots,n)$ and $t\in R_+$, $h_{ik}>0$ for all $i\in I$, $k=1,2,\cdots$, then we have the following conclusions.

**Lemma 2.2.** Suppose that there is a positive constant $\bar{\sigma}$ such that

$$\liminf_{t\to\infty}\left(\int_t^{t+\bar{\sigma}}\bar{a}(s)\mathrm{d}s\right)>0 \tag{2.6}$$

$$\liminf_{t\to\infty}\left(\int_t^{t+\bar{\sigma}}\bar{r}(t)-\sum_{j=1}^{n}D_{ij}(t)\mathrm{d}t+\sum_{t\le t_k<t+\bar{\sigma}}\ln\bar{h}_k\right)>0 \tag{2.7}$$

and function

$$h(t,\mu)=\sum_{t\le t_k<t+\mu}\ln h_k,\quad \bar{h}(t,\mu)=\sum_{t\le t_k<t+\mu}\ln\bar{h}_k \tag{2.8}$$

is bounded on $t\in R_+$ and $\mu\in[0,\bar{\sigma})$. Then we have

1) There are constants $M>0$ and $m>0$ such that

$$m\le\liminf_{t\to\infty}x_i(t)\le\limsup_{t\to\infty}x_i(t)\le M$$

for any positive solution $x_i(t)$ of system(2.1). $t\in R_+, i\in I$.

2) If all conditions of (1) hold, further if

$$\liminf_{t\to\infty}\left(\int_t^{t+\bar{\sigma}}\beta_1(t)\mathrm{d}t\right)>0, \tag{2.9}$$

where $\beta_1(t)=\min_{i\in I}\left\{a_i(t)-\sum_{j=1}^{n}\frac{D_{ij}(t)}{m}\right\}\ge 0$ for all $t\in R_+, i,j\in I$, then system (2.2) is globally attractive.

**Proof**: Firstly, we prove system (2.5) is permanent. Let $(x_1(t),\cdots,x_i(t))$ be any solution of system (2.5). Define the function $V_1(t)=\max_{i\in I}\{x_i(t)\}$, when $t\ne t_k$ calculating the upper-right derivative of $V_1(t)$, we have

$$D^+V_1(t)\le x_i(t)(r_i(t)-a_i(t)x_i(t))\le V_1(t)(r(t)-\bar{a}(t)V_1(t))$$

when $t=t_k$, we have

$$V_1(t_k^+)=\max_{i\in I}\{x_i(t_k^+)\}=\max_{i\in I}\{h_{ik}x_i(t_k)\}\le\max_{i\in I}\{h_{ik}\}\max_{i\in I}\{x_i(t_k)\}=h_kV_1(t_k)$$

Consider the following auxiliary system

$$\begin{cases}D^+w(t)=w(t)(r(t)-\bar{a}(t)w(t)),t\neq t_k\\ w(t_k^+)=h_kw(t_k),k=1,2,\cdots\end{cases}\tag{2.10}$$

with initial condition $w(0)=V_1(0)$. Obviously, from condition (2.6), (2.8) and Lemma 2.1, there exists a constant $M>0$ such that $\limsup_{t\to\infty}w(t)\le M$. Then according to comparison theorem of impulsive differential equations, we derive

$$\limsup_{t\to\infty}x_i(t)\le\limsup_{t\to\infty}V_1(t)\le\limsup_{t\to\infty}w(t)\le M\quad\text{for }i\in I.\tag{2.11}$$

Now we prove there is a constant $m>0$ such that $\liminf_{t\to\infty}x_i(t)\ge m$. Defined $V_2(t)=\min_{i\in I}\{x_i(t)\}$, calculating the right-lower derivative of $V_2(t)$ when $t\neq t_k$, similar to above conclusion, we can obtain

$$D^+V_2(t)\ge x_i(t)(r_i(t)-a_i(t)x_i(t))\ge V_2(t)(\bar{r}(t)-a(t)V_2(t))$$

when $t=t_k$, we have

$$V_2(t_k^+)=\min_{i=\{1,2\}}\{x_i(t_k^+)\}=\min_{i=\{1,2\}}\{h_{ik}x_i(t_k)\}\ge\min_{i=\{1,2\}}\{h_{ik}\}\min_{i=\{1,2\}}\{x_i(t_k)\}=\bar{h}_kV_2(t_k)$$

by comparison theorem of impulsive differential equations, we derive $V_2(t)\ge v(t)$ for all $t\in R_+$, where $v(t)$ is the solution of the auxiliary equation

$$\begin{cases}D^+v(t)=v(t)(\bar{r}(t)-a(t)v_2(t)),t\neq t_k\\ v(t_k^+)=\bar{h}_kv(t_k),k=1,2,\cdots,\end{cases}\tag{2.12}$$

with initial condition $V_2(0)=v(0)$. Clearly, from condition (2.7), (2.8) and Lemma 2.1, there is a constant $m>0$ such that $\liminf_{t\to\infty}v(t)\ge m$. Therefore we have

$$\liminf_{t\to\infty}x_i(t)\ge\liminf_{t\to\infty}V_2(t)\ge\liminf_{t\to\infty}v(t)\ge m\quad\text{for }i\in I.$$

Next we consider the globally attractive of system (2.5). Construct a Lyapunov function

$$V(t)=\sum_{i=1}^n\left|\ln x_i(t)-\ln\tilde{x}_i(t)\right|$$

when $t=t_k$, we always have

$$V(t_k)=\sum_{i=1}^n\left|\ln h_{ik}x_i(t_k)-\ln h_{ik}\tilde{x}_i(t_k)\right|=V(t_k^+)$$

then $V(t)$ is continuous for all $t\in R_+$. In addition, when $t\in R_+$ and $t\neq t_k$ we have

$$\frac{1}{B}\left|x_i(t)-\tilde{x}_i(t)\right|\le\left|\ln x_i(t)-\ln\tilde{x}_i(t)\right|\le\frac{1}{A}\left|x_i(t)-\tilde{x}_i(t)\right|\tag{2.13}$$

Moreover, calculating the derivative of $V(t)$, we also have

$$\begin{aligned}D^+V(t)&=\sum_{i=1}^n\operatorname{sgn}(x_1(t)-\tilde{x}_1(t))\left(\frac{x_i'(t)}{x_i(t)}-\frac{\tilde{x}_i'(t)}{\tilde{x}_i(t)}\right)\\&\le\sum_{i=1}^n\left(-a_i(t)\left|x_i(t)-\tilde{x}_i(t)\right|\right)+\sum_{i=1}^n\sum_{j=1}^n\bar{D}_{ij}(t)\end{aligned}\tag{2.14}$$

where $\bar{D}_{ij}(t)=\begin{cases} D_{ij}(t)\left(\frac{x_j(t)}{x_i(t)}-\frac{\tilde{x}_j(t)}{\tilde{x}_i(t)}\right), x_i(t)>\tilde{x}_i(t), \\ D_{ij}(t)\left(\frac{\tilde{x}_j(t)}{\tilde{x}_i(t)}-\frac{x_j(t)}{x_i(t)}\right), x_i(t)<\tilde{x}_i(t). \end{cases}$

For all $t\in R_+$, we think about under the following two cases:

Case 1: if $x_i(t)>\tilde{x}_i(t)$, then

$$\bar{D}_{ij}(t)\le\frac{D_{ij}(t)}{x_i(t)}\left(x_j(t)-\tilde{x}_j(t)\right)\le\frac{D_{ij}(t)}{A}\left|x_j(t)-\tilde{x}_j(t)\right|$$

Case 2: if $x_i(t)<\tilde{x}_i(t)$, then

$$\bar{D}_{ij}(t)\le\frac{D_{ij}(t)}{\tilde{x}_i(t)}\left(\tilde{x}_j(t)-x_j(t)\right)\le\frac{D_{ij}(t)}{A}\left|x_j(t)-\tilde{x}_j(t)\right|$$

From Case 1, Case 2 and (2.14), we can obtain

$$\begin{aligned} D^+V(t)&\le\sum_{i=1}^{n}\left(-a_i(t)\left|x_i(t)-\tilde{x}_i(t)\right|\right)+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{D_{ij}}{A}\left|x_j(t)-\tilde{x}_j(t)\right| \\ &\le-\sum_{i=1}^{n}\left(a_i(t)-\sum_{j=1}^{n}\frac{D_{ij}}{A}\right)\left|x_i(t)-\tilde{x}_i(t)\right| \\ &\le-\beta(t)AV(t) \end{aligned} \tag{2.15}$$

By (2.15) and condition (2.9), we have $V(t)\le V(0)\exp\left(-A\int_0^t\beta(s)\mathrm{d}s\right)\to 0$ as $t\to\infty$. Furthermore, by (2.13), we have $\lim_{t\to\infty}\left(x_i(t)-\tilde{x}_i(t)\right)=0$, that is, system (2.5) has globally attractive positive solution. This completes the proof of Lemma 2.2.

## 3. Permanence

Note that system (1.1) always has a positive solution for all $t\in R_+$ if it has a positive initial condition. Here we state and prove the permanent of system (1.1).

**Theorem 3.1.** There exists a constant $M>0$ such that $\limsup_{t\to\infty}x_i(t)\le M$ and $\limsup_{t\to\infty}y(t)\le M$ for any positive solution of system (1.1) if there exists a constant $\omega>0$ such that

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega}\bar{b}(s)\mathrm{d}s\right)>0 \tag{3.1}$$

$$\liminf_{t\to\infty}\left(\int_t^{t+\omega}b_3(s)\mathrm{d}s\right)>0 \tag{3.2}$$

and function

$$h(t,\mu)=\sum_{t\le t_k<t+v}\ln h_k,\quad g(t,\mu)=\sum_{t\le t_k<t+\mu}\ln g_k \tag{3.3}$$

are bounded on $t\in R_+$ and $\mu\in[0,\omega)$ for all $t\in R_+, k=1,2,\cdots$.

**Proof.** Firstly, we prove that there is a constant $M_1>0$ such that $\limsup_{t\to\infty}x_i(t)\le M_1\ (i=1,2)$. Define the function $V(t)=\max\left\{x_1(t),x_2(t)\right\}$, we have two cases when $t \neq t_k$.

1) If $V(t) = x_1(t)$, we have

$$\begin{aligned} D^+V(t) &= x_1(t)\Big(a_1(t) - b_1(t)x_1(t) - c_1(t)\int_{-\infty}^{0} k_1(s)y(t+s)\,ds\Big) \\ &\quad + D_{12}(t)\big(x_2(t) - x_1(t)\big) \\ &\le x_1(t)\big(a_1(t) - b_1(t)x_1(t)\big) \\ &\le V(t)\big(a(t) - \bar{b}(t)V(t)\big) \end{aligned} \tag{3.4}$$

2) If $V(t) = x_2(t)$, we have

$$\begin{aligned} D^+V(t) &= x_2(t)\big(a_2(t) - b_2(t)x_2(t)\big) + D_{21}(t)\big(x_1(t) - x_2(t)\big) \\ &\le x_2(t)\big(a_2(t) - b_2(t)x_2(t)\big) \\ &\le V(t)\big(a(t) - \bar{b}(t)V(t)\big) \end{aligned} \tag{3.5}$$

clearly, from (3.4) and (3.5), we get

$$D^+V(t) \le V(t)\big(a(t) - \bar{b}(t)V(t)\big) \quad \text{for all} \quad t \in R_+$$

On the other hand, when $t = t_k$, we have

$$V(t_k^+) = \max_{i=\{1,2\}}\{x_i(t_k^+)\} = \max_{i=\{1,2\}}\{h_{ik}x_i(t_k)\} \le \max_{i=\{1,2\}}\{h_{ik}\}\max_{i=\{1,2\}}\{x_i(t_k)\} = h_k V(t_k).$$

Consider the following auxiliary equation

$$\begin{cases} D^+v(t) = v(t)\big(a(t) - \bar{b}(t)v(t)\big), t \neq t_k \\ v(t_k^+) = h_k v(t_k), k = 1, 2, \cdots \end{cases} \tag{3.6}$$

with initial condition $v(0) = V(0)$. Since the condition of Lemma 2.1 holds from (3.1) and (3.3), using Lemma 2.1, we get that there exists a constant $M_1 > 0$ such that $\limsup_{t\to\infty} v(t) \le M_1$. Applying the comparison theorem of impulsive differential equation, we obtain $V(t) \le v(t)$ for all $t \in R_+$. Finally, we have

$$\limsup_{t\to\infty} x_i(t) \le \limsup_{t\to\infty} V(t) \le \limsup_{t\to\infty} v(t) \le M_1 \quad \text{for} \quad i = 1, 2 \tag{3.7}$$

Then we prove that there is a constant $M_2 > 0$ such that $\limsup_{t\to\infty} y(t) \le M_2$. From the third and sixth equations of system (1.1), we obtain

$$\begin{cases} y'(t) = y(t)\Big(a_3(t) - b_3(t)y(t) - c_2(t)\int_{-\infty}^{0} k_2(s)x_1(t-s)\,ds\Big) \\ \qquad \le y(t)\big(a_3(t) - b_3(t)y(t)\big), t \neq t_k \\ y(t_k^+) = g_k y(t_k), k = 1, 2, \cdots \end{cases}$$

considering the following subsystem

$$\begin{cases} u'(t) = u(t)\big(a_3(t) - b_3(t)u(t)\big), t \neq t_k \\ u(t_k^+) = g_k u(t), k = 1, 2, \cdots \end{cases} \tag{3.8}$$

with initial condition $u(0) = y(0)$. Obviously, the condition of Lemma 2.1 holds from (3.2) and (3.3), we obtain that there is a constant $M_2 > 0$ such that

$\limsup_{t\to\infty} u(t) \le M_2$. Similar to the prove process of the bounded of species *x*, we have $\limsup_{t\to\infty} y(t) \le M_2$.

Let $M = \max\{M_1, M_2\}$, evidently, we have

$$\limsup_{t\to\infty} x_i(t) \le M\ (i=1,2),\quad \limsup_{t\to\infty} y(t) \le M$$

The proof of Theorem 3.1 is completed.

**Theorem 3.2.** Assume that all conditions of Theorem 3.1 are satisfied. In addition, there is a constant $\bar{\omega} > 0$ such that

$$\liminf_{t\to\infty}\left(\int_t^{t+\bar{\omega}} \bar{a}(s)\,\mathrm{d}s + \sum_{t\le t_k<t+\bar{\omega}} \ln \bar{h}_k\right) > 0 \tag{3.9}$$

$$\liminf_{t\to\infty}\left(\int_t^{t+\bar{\omega}}\left(a_3(t) - c_2(t)\int_{-\tau}^{0} k_2(s)\left(x_1^*(t+s)\right)\mathrm{d}s\right)\mathrm{d}s + \sum_{t\le t_k<t+\bar{\omega}} \ln g_k\right) > 0 \tag{3.10}$$

$$\liminf_{t\to\infty}\left(\int_t^{t+\bar{\omega}} \beta_2(t)\,\mathrm{d}t\right) > 0 \tag{3.11}$$

and function

$$\bar{h}(t,\mu) = \sum_{t\le t_k<t+\mu} \ln \bar{h}_k,\quad g(t,\mu) = \sum_{t\le t_k<t+\mu} \ln g_k \tag{3.12}$$

are bounded on $t \in R_+$ and $\mu \in [0, \bar{\omega})$, where

$\beta_2(t) = \min_{i=1,2}\left\{a_i(t) - \sum_{j=1}^{2} \frac{D_{ij}(t)}{p}\right\} \ge 0\,(i \ne j)$ for all $t \in R_+$.

Then the system is permanent.

**Proof.** Above all, we must prove that there exists a constant $m > 0$ such that $\liminf_{t\to\infty} x_i(t) \ge m\,(i=1,2)$, $\liminf_{t\to\infty} y(t) \ge m$.

Firstly, we prove $\liminf_{t\to\infty} x_i(t) \ge m\,(i=1,2)$. Defined $V(t) = \min\{x_1(t), x_2(t)\}$ and

$$H = \sup\left\{\left|x_i(t+s), y(t+s)\right| : t \in R_+, s \in R_-, i=1,2\right\} \tag{3.13}$$

When $t \ne t_k$, consider the following two cases.

Case 1: If $V(t) = x_1(t)$, we can choose a constant $\tau > 0$ such that $H\int_{-\infty}^{-\tau} k_1(s)\,\mathrm{d}s < M$, then we have

$$\begin{aligned}
V'(t) &= x_1(t)\left(a_1(t) - b_1(t)x_1(t) - c_1(t)\int_{-\infty}^{0} k_1(s)y(t+s)\,\mathrm{d}s\right) \\
&\quad + D_{12}(t)\left(x_2(t) - x_1(t)\right) \\
&= x_1(t)\left(a_1(t) - b_1(t)x_1(t) - c_1(t)\left(\int_{-\infty}^{-\tau} k_1(s)y(t+s)\,\mathrm{d}s\right.\right. \\
&\quad \left.\left. + \int_{-\tau}^{0} k_1(s)y(t+s)\,\mathrm{d}s\right)\right) + D_{12}(t)\left(x_2(t) - x_1(t)\right) \\
&\ge x_1(t)\left(a_1(t) - b_1(t)x_1(t) - c_1(t)\left(H\int_{-\infty}^{-\tau} k_1(s)\,\mathrm{d}s + \int_{-\tau}^{0} k_1(s)y(t+s)\,\mathrm{d}s\right)\right) \\
&\ge x_1(t)\left(a_1(t) - 2c_1(t)M - b_1(t)x_1(t)\right) \\
&\ge V(t)\left(\bar{a}(t) - 2c_1(t)M - b(t)V(t)\right),
\end{aligned}$$

Case 2: If $V(t) = x_2(t)$, we have

$$
\begin{aligned}
V'(t) &= x_2(t)\left(a_2(t) - b_2(t)x_2(t)\right) + D_{21}(t)\left(x_1(t) - x_2(t)\right) \\
&\geq x_2(t)\left(a_2(t) - b_2(t)x_2(t)\right) \\
&\geq V(t)\left(\bar{a}(t) - b(t)V(t)\right).
\end{aligned}
$$

By Case 1 and Case 2, we derive $V(t) \geq V(t)\left(\bar{a}(t) - b(t)V(t)\right)$.

When $t = t_k$, we can obtain

$$V\left(t_k^+\right) = \min_{i=\{1,2\}}\left\{x_i\left(t_k^+\right)\right\} = \min_{i=\{1,2\}}\left\{h_{ik}x_i\left(t_k\right)\right\} \geq \min_{i=\{1,2\}}\left\{h_{ik}\right\} \min_{i=\{1,2\}}\left\{x_i\left(t_k\right)\right\} = \bar{h}_k V\left(t_k\right).$$

Research the following equation with impulsive

$$
\begin{cases}
v'(t) = v(t)\left(\bar{a}(t) - b(t)v(t)\right), t \neq t_k \\
v\left(t_k^+\right) = \bar{h}_k v(t), k = 1, 2, \cdots
\end{cases}
\tag{3.14}
$$

By condition (3.9) and (3.12), we know that condition of Lemma 2.1 is satisfied. Consequently, there exists a constant $m_1 > 0$ such that

$$\liminf_{t\to\infty} x_i(t) \geq \liminf_{t\to\infty} V(t) \geq \liminf_{t\to\infty} v(t) \geq m_1 \quad \text{for} \quad i = 1, 2. \tag{3.15}$$

Then we investigate the following system:

$$
\begin{cases}
\left.\begin{aligned}
u_1'(t) &= u_1(t)\left(a_1(t) - b_1(t)u_1(t)\right) + D_{12}(t)\left(u_2(t) - u_1(t)\right) \\
u_2'(t) &= u_2(t)\left(a_2(t) - b_2(t)u_2(t)\right) + D_{21}(t)\left(u_1(t) - u_2(t)\right)
\end{aligned}\right\}, t \neq t_k \\
\left.\begin{aligned}
u_1\left(t_k^+\right) &= h_{1k}u_1\left(t_k\right) \\
u_2\left(t_k^+\right) &= h_{2k}u_2\left(t_k\right)
\end{aligned}\right\}, k = 1, 2, \cdots,
\end{cases}
\tag{3.16}
$$

From Lemma 2.2 and condition (3.1), (3.10), (3.12) and (3.13), we can know that there are positive constants $p$ and $P$ such that

$$p \leq x_i^*(t) \leq P$$

where $x_i^*(t)(i = 1, 2)$ is globally attractive for the system (3.16). In addition, we assume that $\left(u_1(t), u_2(t)\right)$ is a positive solution of system (3.16) with initial condition $u_i(0) = x_i(0)$. Evidently, we obtain that there exists a constant $\varepsilon_0 > 0$ small enough such that

$$x_i^*(t) - \varepsilon_0 \leq u_i(t) \leq x_i^*(t) + \varepsilon_0 \tag{3.17}$$

Similar to the discussion in [18], we obtain that condition (3.10) is independent of the choice of $x_i^*(t)$.

From condition (3.10), there are constant $\varepsilon_0 > 0$ small enough and $T > 0$ large enough such that

$$
\begin{aligned}
&\int_t^{t+\bar{\omega}} \left(a_3(t) - b_3(t)\varepsilon_0 - 2c_2(t)\varepsilon_0 - c_2(t)\int_{-\tau}^0 k_2(s)\left(x_1^*(t+s)\right)\mathrm{d}s\right)\mathrm{d}s \\
&+ \sum_{t \leq t_k < t+\bar{\omega}} \ln g_k > \varepsilon_0
\end{aligned}
\tag{3.18}
$$

for all $t \geq T$. By (3.12), we can get a positive constant $G$ such that

$$\left|g(t, \mu)\right| = \left|\sum_{t \leq t_k < t+\mu} \ln g_k\right| \leq G. \tag{3.19}$$

From system (1.1), we consider the following subsystem,

$$\begin{cases} \left.\begin{aligned} x_1'(t) &= x_1(t)\left(a_1(t)-b_1(t)x_1(t)-c_1(t)\int_{-\infty}^{0}k_1(s)y(t+s)\mathrm{d}s\right) \\ &\quad + D_{12}(t)\left(x_2(t)-x_1(t)\right) \\ x_2'(t) &= x_2(t)\left(a_2(t)-b_2(t)x_2(t)\right)+D_{21}(t)\left(x_1(t)-x_2(t)\right) \end{aligned}\right\}, t\neq t_k \\ \left.\begin{aligned} x_1\left(t_k^+\right) &= h_{1k}x_1\left(t_k\right) \\ x_2\left(t_k^+\right) &= h_{2k}x_2\left(t_k\right) \end{aligned}\right\}, k=1,2,\cdots. \end{cases}$$

By (3.16), (3.17) and comparison theorem of impulsive differential equations, we get that

$$x_1(t)\le u_1(t)\le x_1^*(t)+\varepsilon_0 \quad \text{for all} \quad t\ge T_1\ge T. \tag{3.20}$$

Next we prove there is a constant $m_2>0$ such that $\liminf_{t\to\infty} y(t)\ge m_2$.

In the beginning, we prove $\limsup_{t\to\infty} y(t)\ge\varepsilon_0$. Suppose that the proposition is not true, we have $\limsup_{t\to\infty} y(t)<\varepsilon_0$, that is $y(t)<\varepsilon_0$ for all $t\ge T_2>T_1$. Furthermore, we can choose a constant $\tau>0$ such that

$$H\int_{-\infty}^{-\tau}k_2(s)\mathrm{d}s\le\varepsilon_0 \tag{3.21}$$

Consequently, we have

$$\begin{aligned} &y'(t) \\ &\ge y(t)\left(a_3(t)-b_3(t)y(t)-c_2(t)\left(H\int_{-\infty}^{-\tau}k_2(s)\mathrm{d}s+\int_{-\tau}^{0}k_2(s)\left(x_1^*(t+s)+\varepsilon_0\right)\mathrm{d}s\right)\right) \\ &\ge y(t)\left(a_3(t)-b_3(t)\varepsilon_0-2c_2(t)\varepsilon_0-c_2(t)\int_{-\tau}^{0}k_2(s)\left(x_1^*(t+s)\right)\mathrm{d}s\right) \end{aligned} \tag{3.22}$$

For any $t\ge T_2+\tau$ and $t\neq t_k$, we can choose an integer $l\ge0$ such that $t=T_2+\tau+l\varpi+v$, where $v\in[0,\varpi)$ is a constant. Integrating (3.22) from $T_2+\tau$ to *t*, due to (3.18) and (3.19), we derive

$$\begin{aligned} y(t) &\ge y(T_2+\tau)\exp\left(\int_{T_2+\tau}^{t}\left(a_3(t)-b_3(t)y(t)\right.\right. \\ &\qquad \left.\left. -c_2(t)\int_{-\infty}^{0}k_2(s)x_1(t+s)\mathrm{d}s\right)\mathrm{d}s+\sum_{T_2+\tau\le t_k<t}\ln g_k\right) \\ &\ge y(T_2+\tau)\exp\left(\int_{T_2+\tau}^{t}\left(a_3(t)-b_3(t)\varepsilon_0-2c_2(t)\varepsilon_0\right.\right. \\ &\qquad \left.\left. -c_2(t)\int_{-\tau}^{0}k_2(s)x_1^*(t+s)\mathrm{d}s\right)+\sum_{T_2+\tau\le t_k<t}\ln g_k\right) \\ &\ge y(T_2+\tau)\exp\left(\left(\int_{T_2+\tau}^{T_2+\tau+\varpi}+\cdots+\int_{T_2+\tau+(l-1)\omega}^{T_2+\tau+l\varpi}+\int_{T_2+\tau+l\omega}^{t}\right)\left(a_3(t)-b_3(t)\varepsilon_0\right.\right. \\ &\qquad \left. -2c_2(t)\varepsilon_0-c_2(t)\int_{-\tau}^{0}k_2(s)x_1^*(t+s)\mathrm{d}s\right) \\ &\qquad \left. +\sum_{T_2+\tau\le t_k<T_2+\tau+\varpi}+\cdots+\sum_{T_2+\tau+(l-1)\varpi\le t_k<T_2+\tau+l\varpi}+\sum_{T_2+\tau+l\varpi\le t_k<t}\ln g_k\right) \\ &\ge y(T_2+\tau)\exp\left(l\varepsilon_0-\beta\varpi-G\right), \end{aligned}$$

where $\beta = \sup_{t\in R_+}\left(\left|a_3(t)\right| + b_3(t)\varepsilon_0 + 2c_2(t)\varepsilon_0 + c_2(t)\int_{-\tau}^{0} k_2(s)x_1^*(t+s)\mathrm{d}s\right)$. Therefore, we can get $y(t)\to\infty$ as $t\to\infty$, which is contradiction with $\limsup_{t\to\infty} y(t) < \varepsilon_0$. Obviously, we have

$$\limsup_{t\to\infty} y(t) \ge \varepsilon_0 .$$

Then we prove there exists a constant $\alpha > 0$ such that $\liminf_{t\to\infty} y(t) \ge \alpha$. Assume that the proposition is not true, then there exists a sequence $\{\phi_m(t)\} \in PC_+, m = 1,2,\cdots$ such that $\liminf_{t\to\infty} y(t,\phi_k) \le \frac{\varepsilon_0}{k^2}$ for all $k = 1,2,\cdots$.

From (3.19), we have

$$\mathrm{e}^{-G} \le g_k \le \mathrm{e}^{G} . \tag{3.23}$$

we choose an integer $K > \mathrm{e}^G$ such that for all $k \ge K$ and any solution $y(t)$ of system (1.1) satisfied:

1) If $y(t_l) \ge \frac{\varepsilon_0}{k}$, then $y(t_l^+) = g_k y(t_l) \ge \mathrm{e}^{-G}\frac{\varepsilon_0}{k} \ge \frac{\varepsilon_0}{k^2}$, for some $l = 1,2,\cdots$,

2) If $y(t_l) \le \frac{\varepsilon_0}{k^2}$, then $y(t_l^+) = g_k y(t_l) \le \mathrm{e}^{G}\frac{\varepsilon_0}{k^2} \ge \frac{\varepsilon_0}{k}$, for some $l = 1,2,\cdots$.

From above inequality, there exist two time sequences $\{s_q^{(k)}\}$ and $\{t_q^{(k)}\}$ such that for each $k = K+1, K+2,\cdots$, we have

$$0 < s_1^{(k)} < t_1^{(k)} < s_2^{(k)} < t_2^{(k)} < \cdots < s_q^{(k)} < t_q^{(k)} < \cdots$$

and

$$\{s_q^{(k)}\}\to\infty, \{t_q^{(k)}\}\to\infty \quad \text{as} \quad t\to\infty \tag{3.24}$$

$$y(s_q^{(k)},\phi_k) \ge \frac{\varepsilon_0}{k}, \frac{\varepsilon_0}{k^2} < y(s_q^{(k)+},\phi_k) \le \frac{\varepsilon_0}{k}, \tag{3.25}$$

$$\frac{\varepsilon_0}{k^2} \le y(t_q^{(k)},\phi_k) < \frac{\varepsilon_0}{k}, y(t_q^{(k)+},\phi_k) \le \frac{\varepsilon_0}{k^2}, \tag{3.26}$$

$$\frac{\varepsilon_0}{k^2} \le y(t,\phi_k) \le \frac{\varepsilon_0}{k} \quad \text{for all} \quad t\in(s_q^{(k)}, t_q^{(k)}). \tag{3.27}$$

Let $H^{(k)} = \sup\{|x_i(t+s,\phi_k), y(t+s,\phi_k)| : t\in R_+, s\in R_-, i=1,2\}$ for each $k = K+1, K+2,\cdots$, we choose a constant $\tau^{(k)} > 0$ such that

$$\int_{-\infty}^{-\tau^{(k)}} k(s)x_1(t+s,\phi_k)\mathrm{d}s \le H^{(k)}\int_{-\infty}^{-\tau^{(k)}} k(s)\mathrm{d}s < \varepsilon_0 , \tag{3.28}$$

by (3.20), for each $k = K+1, K+2,\cdots$, there is a $T_1^{(k)} > T$ such that

$$x_1(t,\phi_k) \le x_1^*(t) + \varepsilon_0 \quad \text{for all} \quad t \ge T_1^{(k)}. \tag{3.29}$$

Clearly, from (3.22), there is an $N_1^{(k)} > 0$ such that $s_q^{(k)} > T_1^{(k)} + \tau^{(k)}$ for $q \ge N_1^{(k)}$. Hence for any $t\in[s_q^k, t_q^k]$ and $t \ne t_l$, $q \ge N_1^{(k)}$. By (3.27) and (3.28), we can obtain

$$
\begin{aligned}
y'(t,\phi_k) &= y(t,\phi_k)\left(a_3(t)-b_3(t)y(t,\phi_k)-c_2(t)\int_{-\infty}^{0}k_2(s)x_1(t+s,\phi_k)\mathrm{d}s\right) \\
&\geq y(t,\phi_k)\left(a_3(t)-b_3(t)y(t,\phi_k)-c_2(t)\left(\int_{-\infty}^{-\tau^{(k)}}k_2(s)x_1(t+s,\phi_k)\mathrm{d}s\right.\right. \\
&\quad \left.\left.+\int_{-\tau^{(k)}}^{0}k_2(s)\left(x_1^*(t+s)+\varepsilon_0\right)\mathrm{d}s\right)\right) \\
&\geq y(t,\phi_k)\left(a_3(t)-b_3(t)\varepsilon_0-c_2(t)\left(H^{(k)}\int_{-\infty}^{-\tau^{(k)}}k_2(s)\mathrm{d}s\right.\right. \\
&\quad \left.\left.+\int_{-\tau^{(k)}}^{0}k_2(s)\left(x_1^*(t+s)+\varepsilon_0\right)\mathrm{d}s\right)\right) \\
&\geq y(t,\phi_k)\left(a_3(t)-b_3(t)\varepsilon_0-2c_2(t)\varepsilon_0-\int_{-\infty}^{0}k_2(s)x_1^*(t+s)\mathrm{d}s\right),
\end{aligned}
$$

choose an integer $l_q^{(k)}$ such that $t_q^{(k)}=s_q^k+l_q^k\varpi$. Integrating above inequality from $s_q^{(k)}$ to $t_q^{(k)}$, we obtain

$$
\begin{aligned}
y\left(t_q^{(k)+},\phi_k\right) &\geq y\left(s_q^{(k)},\phi_k\right)\exp\int_{s_q^{(k)}}^{t_q^{(k)}}\left(a_3(t)-b_3(t)\varepsilon_0-2c_2(t)\varepsilon_0\right. \\
&\quad \left.-\int_{-\tau^{(k)}}^{0}k_2(s)x_1^*(t+s)\mathrm{d}s+\sum_{s_q^{(k)}\leq t_k\leq t_q^{(k)}}\ln g_k\right) \\
&\geq y\left(s_q^{(k)},\phi_k\right)\exp\int_{s_q^{(k)}}^{t_q^{(k)}}(-\beta-2G)
\end{aligned}
$$

Consequently, from (3.25) and (3.26), we have

$$
t_q^{(k)}-s_q^{(k)}\geq\frac{\ln k}{\beta+2G} \tag{3.30}
$$

For any $t\geq s_q^{(k)}$ and $q\geq N_1^{(k)}$, we have

$$
\int_{-\infty}^{T_1^{(k)}}k_2(u-t)y(u,\phi_k)\mathrm{d}u\leq H^{(k)}\int_{-\infty}^{T_1^{(k)}-t}k_2(s)\mathrm{d}s \tag{3.31}
$$

and

$$
\int_{T_1^{(k)}}^{s_q^{(k)}}k_2(u-t)y(u,\phi_k)\mathrm{d}u\leq H\int_{-\infty}^{s_q^{(k)}-t}k_2(s)\mathrm{d}s. \tag{3.32}
$$

For each $k=K+1,K+2,\cdots$, there exist an $N_2^k\geq N_1^k$ and a constant $L>0$ such that

$$
H^{(k)}\int_{-\infty}^{T_1^{(k)}-s_q^{(k)}}k_2(s)\mathrm{d}s\leq\frac{1}{2}\varepsilon_0 \quad \text{for all} \quad q\geq N_2^{(k)} \tag{3.33}
$$

and

$$
H\int_{-\infty}^{-L}k_2(s)\mathrm{d}s\leq\frac{1}{2}\varepsilon_0 \tag{3.34}
$$

We can choose an integer $r_q^{(k)}\geq 0$ such that $t_q^{(k)}=s_q^{(k)}+L+r_q^{(k)}\varpi+w_q^{(k)}$, where $w_q^{(k)}\in[0,\varpi)$ is a constant. By (3.30), there exists a large enough $K_1\geq K$ such that

$$
r_q^{(k)}\varepsilon_0-\beta\varpi-2G\geq\varepsilon_0. \tag{3.35}
$$

For all $k \ge K_1, q \ge N_2^{(k)}$, $t \in \left[s_q^{(k)} + L, t_q^{(k)}\right]$ and $t \ne t_l$, by (3.27) and (3.31)-(3.34), we have

$$
\begin{aligned}
&y'(t,\phi_k) \\
&\ge y(t,\phi_k)\left(a_3(t) - b_3(t)y(t,\phi_k) - c_2(t)\left(\int_{-\infty}^{T_1^{(k)}} + \int_{T_1^k}^{s_q^{(k)}} + \int_{s_q^{(k)}}^{t} k_2(u-t)x_1(u,\phi_k)\mathrm{d}u\right)\right) \\
&\ge y(t,\phi_k)\left(a_3(t) - b_3(t)\frac{\varepsilon_0}{k} - c_2(t)\left(H^{(k)}\int_{-\infty}^{T_1^{(k)}-t} k_2(s)\mathrm{d}s\right.\right. \\
&\quad \left.\left. + H\int_{-\infty}^{s_q^{(k)}-t} k_2(s)\mathrm{d}s + \int_{s_q^{(k)}}^{t} k_2(u-t)\left(x_1^*(u) + \varepsilon_0\right)\mathrm{d}u\right)\right) \\
&\qquad \ge y(t,\phi_k)\left(a_3(t) - b_3(t)\varepsilon_0 - c_2(t)\left(\frac{\varepsilon_0}{2} + \frac{\varepsilon_0}{2} + \varepsilon_0 + \int_{-t}^{0} k(s)x_1^*(t+s)\mathrm{d}s\right)\right) \\
&\qquad = y(t,\phi_k)\left(a_3(t) - b_3(t)\varepsilon_0 - 2c_2(t)\varepsilon_0 - c_2(t)\int_{-t}^{0} k_2(s)x_1^*(t+s)\mathrm{d}s\right).
\end{aligned}
$$

Integrating above system from $s_q^{(k)} + L$ to $t_q^{(k)}$, we derive

$$
\begin{aligned}
&\frac{\varepsilon_0}{k^2} \ge y\left(t_q^{(k)+},\phi_k\right) \\
&\quad \ge y\left(s_q^{(k)} + L,\phi_k\right)\exp\left(\int_{s_q^{(k)}+L}^{s_q^{(k)}+L+\varpi} + \cdots + \int_{s_q^{(k)}+L+\left(r_q^{(k)}-1\right)\varpi}^{s_q^{(k)}+L+r_q^{(k)}\varpi} + \int_{s_q^{(k)}+L+r_q^{(k)}\omega}^{t_q^{(k)}}\left(a_3(t) - b_3(t)\varepsilon_0\right.\right. \\
&\qquad \left. - 2c_2(t)\varepsilon_0 - c(t)\int_{-t}^{0} k_2(s)\left(x_1^*(t+s)\right)\mathrm{d}s\right)\mathrm{d}s \\
&\qquad \left. + \left(\sum_{s_q^{(k)}+L \le t \le s_q^{(k)}+L+\varpi} + \cdots + \sum_{s_q^{(k)}+L+\left(r_q^{(k)}-1\right)\varpi \le t \le s_q^{(k)}+L+r_q^{(k)}\varpi} + \sum_{s_q^{(k)}+L+r_q^{(k)}\varpi \le t \le t_q^{(k)}} \ln g_k\right)\right) \\
&\quad \ge \frac{\varepsilon_0}{k^2}\exp\left(r_q^{(k)}\varepsilon_0 - \beta\varpi - 2G\right) \ge \frac{\varepsilon_0}{k^2}\exp(\varepsilon_0) > \frac{\varepsilon_0}{k^2},
\end{aligned}
$$

which is a contradiction. This contradiction shows that there exists a constant $m_2 > 0$ such that $\liminf_{t\to\infty} y(t) \ge m_2$. Hence, choose a constant $m = \min\{m_1, m_2\}$, then we finally have

$$
\liminf_{t\to\infty} x_i(t) \ge m\,(i = 1,2),\ \liminf_{t\to\infty} y(t) \ge m
$$

Therefore Theorem 3.2 holds. This completes the proof.

## 4. Extinction

In this section, we investigate the extinction of system (1.1). We note that, under conditions of Theorem 3.2, system (1.1) is always permanent.

**Theorem 4.1.** Assume that there is a constant $\eta \ge 0$ such that

$$
\limsup_{t\to\infty}\left(\int_t^{t+\eta} \gamma(s)\mathrm{d}s + \sum_{t \le t_k < t+\eta} \ln h_k\right) \le 0 \tag{4.1}
$$

$$
\limsup_{t\to\infty}\left(\int_t^{t+\eta} \left(a_3(s) - mc_2(s)\right)\mathrm{d}s + \sum_{t \le t_k < t+\eta} \ln g_k\right) \le 0 \tag{4.2}
$$

and function

$$h(t,\mu)=\sum_{t\le t_k<t+\mu}\ln h_k \tag{4.3}$$

$$g(t,\mu)=\sum_{t\le t_k<t+\mu}\ln g_k \tag{4.4}$$

are bounded function on $t\in R_+$ and $v\in[0,\eta)$, where $\gamma(t)=a_1(t)+a_2(t)+D(t)$, then we can obtain

$$\lim_{t\to\infty}x_i(t)=0,\quad \lim_{t\to\infty}y(t)=0,\quad (i=1,2) \tag{4.5}$$

for any positive solution $(x_1(t),x_2(t),y(t))$ of system (1.1).

**Proof.** Firstly, we prove the extinction of species *x*. Define $V(t)=x_1(t)+x_2(t)$. When $t\ne t_k$, calculating the right-upper derivative of $V(t)$, we have

$$\begin{aligned}
D^+V(t)&=x_1'(t)+x_2'(t)\\
&=x_1(t)\left(a_1(t)-b_1(t)x_1(t)-c_1(t)\int_{-\infty}^{0}k_1(s)y(t+s)\,\mathrm{d}s\right)\\
&\quad+D_{12}(t)(x_2(t)-x_1(t))+x_2(t)(a_2(t)-b_2(t)x_2(t))\\
&\quad+D_{21}(t)(x_1(t)-x_2(t))\\
&\le x_1(t)(a_1(t)-b_1(t)x_1(t)-D_{12}(t)+D_{21}(t))\\
&\quad+x_2(t)(a_2(t)-b_2(t)x_2(t)-D_{21}(t)+D_{12}(t))\\
&\le(x_1(t)+x_2(t))(a_1(t)-b_1(t)x_1(t)+D(t))\\
&\quad+(x_1(t)+x_2(t))(a_2(t)-b_2(t)x_2(t)+D(t))\\
&\le(x_1(t)+x_2(t))\left(a_1(t)+a_2(t)+2D(t)-2\bar{b}(t)(x_1(t)+x_2(t))\right)\\
&=V(t)\left(\gamma(t)-2\bar{b}(t)V(t)\right),
\end{aligned}$$

when $t=t_k$, we get

$$V(t_k^+)=x_1(t_k^+)+x_2(t_k^+)=h_{1k}x_1(t_k)+h_{2k}x_2(t_k)\le\max\{h_{1k},h_{2k}\}V(t_k)=h_kV(t_k)$$

By the comparison theory of impulsive differential equations, we have $V(t)\le v(t)$ for all $t\ge0$, where $v(t)$ is a solution of the auxiliary equation

$$\begin{cases}v'(t)=v(t)\left(\gamma(t)-2\bar{b}(t)v(t)\right),t\ne t_k\\ v(t_k^+)=h_kv(t_k),k=1,2,\cdots,\end{cases} \tag{4.6}$$

with the initial condition $v(0)=V(0)$. Since system (4.6) satisfies all conditions of Lemma 2.1 from conditions (4.1) and (4.3), we obtain $\lim_{t\to\infty}v(t)=0$ for any positive solution of system (4.6). Then we get

$$\lim_{t\to\infty}x_i(t)\le\lim_{t\to\infty}V(t)\le\lim_{t\to\infty}v(t)=0,(i=1,2). \tag{4.7}$$

In the following, we prove the extinction of species *y*. From third equation and sixth equation of system (1.1), we have

$$\begin{cases}y'(t)=y(t)\left(a_3(t)-b_3(t)y(t)-c_2(t)\int_{-\infty}^{0}k_2(s)x_1(t-s)\,\mathrm{d}s\right),t\ne t_k\\ y(t_k^+)=g_ky(t_k),k=1,2,\cdots\end{cases} \tag{4.8}$$

By Theorem 3.2, there exist $m>0$ and $T_0>0$ such that $x_1(t)>m$ for all

$t \geq T_0$. Obviously, when $t \neq t_k$, for any $t \geq T_0$, we obtain

$$y'(t) = y(t)\left(a_3(t) - b_3(t)y(t) - c_2(t)\int_{-\infty}^{0} k_2(s)x_1(t-s)\mathrm{d}s\right)$$
$$\leq y(t)\left(a_3(t) - b_3(t)y(t) - mc_2(t)\right)$$

Then we consider the following auxiliary system

$$\begin{cases} w'(t) = w(t)\left(a_3(t) - b_3(t)w(t) - mc_2(t)\right), t \neq t_k \\ w(t_k^+) = g_k w(t_k), k = 1, 2, \cdots \end{cases} \tag{4.9}$$

with the initial condition $w(T_0) = y(T_0)$. Since all condition of Lemma 2.1 holds from condition (4.2) and (4.4), we can obtain $\lim_{t\to\infty} w(t) = 0$ for any positive solution of system (4.9). Clearly, we have

$$\lim_{t\to\infty} y(t) \leq \lim_{t\to\infty} w(t) = 0. \tag{4.10}$$

From (4.7) and (4.10), we finally obtain (4.5) holds. This completes the proof of Theorem 4.1.

## 5. Globally Attractive

In this section, by constructing appropriate Lyapunov function, we establish the sufficient conditions on the globally attractive of system (1.1).

**Theorem 5.1.** Assume that all conditions of Theorem3.2 hold, further, there exists a constant $\lambda > 0$ such that

$$\liminf_{t\to\infty}\left(\int_t^{t+\lambda} \psi(s)\mathrm{d}s\right) > 0, \tag{5.1}$$

where

$$\psi(t) = \min_{t\in R^+}\left\{b_1(t) - \frac{D_{21}(t)}{m_1} - \int_0^{+\infty} k_2(s)c_2(t-s)\mathrm{d}s,\right.$$
$$\left. b_2(t) - \frac{D_{12}(t)}{m_1}, b_3(t) - \int_0^{+\infty} k_1(s)c_1(t-s)\mathrm{d}s\right\}$$

and $\psi(t) \geq 0$. Then system (1.1) is globally attractive, that is, for any two positive solutions $\left(x_1(t), x_2(t), y(t)\right)$ and $\left(\tilde{x}_1(t), \tilde{x}_2(t), \tilde{y}(t)\right)$ of system (1.1), the following limit hold.

$$\lim_{t\to\infty}\left|x_1(t) - \tilde{x}_1(t)\right| = 0, \lim_{t\to\infty}\left|x_2(t) - \tilde{x}_2(t)\right| = 0, \lim_{t\to\infty}\left|y(t) - \tilde{y}(t)\right| = 0. \tag{5.2}$$

**Proof.** For any two positive solutions $\left(x_1(t), x_2(t), y(t)\right), \left(\tilde{x}_1(t), \tilde{x}_2(t), \tilde{y}(t)\right)$, by Theorem 3.2, we obtain that there exist constants $m_1, M_1$ such that

$$m_1 \leq x_i(t), \tilde{x}_i(t), y(t), \tilde{y}(t) \leq M_1, i = 1, 2. \tag{5.3}$$

Then we have for any $t \in R_+$ and $t \neq t_k$,

$$\frac{1}{m_1}\left|x_i(t) - \tilde{x}_i(t)\right| \leq \left|\ln x_i(t) - \ln \tilde{x}_i(t)\right| \leq \frac{1}{M_1}\left|x_i(t) - \tilde{x}_i(t)\right| (i = 1, 2) \tag{5.4}$$

$$\frac{1}{m_1}\left|y(t) - \tilde{y}(t)\right| \leq \left|\ln y(t) - \ln \tilde{y}(t)\right| \leq \frac{1}{M_1}\left|y(t) - \tilde{y}(t)\right| \tag{5.5}$$

Define a Lyapunov function

$$V_1(t)=\left|\ln x_1(t)-\ln \tilde{x}_1(t)\right|+\left|\ln x_2(t)-\ln \tilde{x}_2(t)\right|+\left|\ln y(t)-\ln \tilde{y}(t)\right|$$

for any impulsive time $t_k$, we have

$$V_1(t_k)=\sum_{i=1}^{2}\left|\ln h_{ik}x_i(t_k)-\ln h_{ik}\tilde{x}_i(t_k)\right|+\left|\ln g_k y(t_k)-\ln g_k\tilde{y}(t_k)\right|=V_1(t_k^+),$$

$V_1(t)$ is continuous for all $t\in R_+$. For any $t\in R_+$ and $t\neq t_k$, calculating the derivative of $V_1(t)$, then we get

$$\begin{aligned}
D^+V_1(t)&=\operatorname{sgn}(x_1(t)-\tilde{x}_1(t))\left(\frac{x_1'(t)}{x_1(t)}-\frac{\tilde{x}_1'(t)}{\tilde{x}_1(t)}\right)\\
&\quad+\operatorname{sgn}(x_2(t)-\tilde{x}_2(t))\left(\frac{x_2'(t)}{x_2(t)}-\frac{\tilde{x}_2'(t)}{\tilde{x}_2(t)}\right)+\operatorname{sgn}(y(t)-\tilde{y}(t))\left(\frac{y'(t)}{y(t)}-\frac{\tilde{y}'(t)}{\tilde{y}(t)}\right)\\
&\le \operatorname{sgn}(x_1(t)-\tilde{x}_1(t))\Big[-b_1(t)(x_1(t)-\tilde{x}_1(t))\\
&\quad-c_1(t)\int_0^{+\infty}k_1(s)(y(t-s)-\tilde{y}(t-s))\mathrm{d}s+D_{12}(t)\left(\frac{x_2(t)}{x_1(t)}-\frac{\tilde{x}_2(t)}{\tilde{x}_1(t)}\right)\Big]\\
&\quad+\operatorname{sgn}(x_2(t)-\tilde{x}_2(t))\left[(-b_2(t)(x_1(t)-\tilde{x}_1(t)))+D_{21}(t)\left(\frac{x_1(t)}{x_2(t)}-\frac{\tilde{x}_1(t)}{\tilde{x}_2(t)}\right)\right]\\
&\quad+\operatorname{sgn}(y(t)-\tilde{y}(t))\Big[-b_3(t)(y(t)-\tilde{y}(t))\\
&\quad-c_2(t)\int_0^{+\infty}k_2(s)(x_1(t-s)-\tilde{x}_1(t-s))\mathrm{d}s\Big]
\end{aligned}\tag{5.6}$$

Let

$$\bar{D}_{12}(t)=\operatorname{sgn}(x_1(t)-\tilde{x}_1(t))D_{12}(t)\left(\frac{x_2(t)}{x_1(t)}-\frac{\tilde{x}_2(t)}{\tilde{x}_1(t)}\right),$$
$$\bar{D}_{21}(t)=\operatorname{sgn}(x_2(t)-\tilde{x}_2(t))D_{21}(t)\left(\frac{x_1(t)}{x_2(t)}-\frac{\tilde{x}_1(t)}{\tilde{x}_2(t)}\right).$$

for $\bar{D}_{12}(t),\bar{D}_{21}(t)$, we consider the following cases:

1) If $x_i(t)>\tilde{x}_i(t)\,(i=1,2)$ for all $t\ge 0$, then

$$\bar{D}_{12}(t)\le D_{12}(t)\left(\frac{x_2(t)}{x_1(t)}-\frac{\tilde{x}_2(t)}{\tilde{x}_1(t)}\right)\le\frac{D_{12}(t)}{x_1(t)}(x_2(t)-\tilde{x}_2(t))\le\frac{D_{12}(t)}{m_1}\left|x_2(t)-\tilde{x}_2(t)\right|,$$
$$\bar{D}_{21}(t)\le D_{21}(t)\left(\frac{x_1(t)}{x_2(t)}-\frac{\tilde{x}_1(t)}{\tilde{x}_2(t)}\right)\le\frac{D_{21}(t)}{x_2(t)}(x_1(t)-\tilde{x}_1(t))\le\frac{D_{21}(t)}{m_1}\left|x_1(t)-\tilde{x}_1(t)\right|.$$

2) If $x_i(t)<\tilde{x}_i(t)\,(i=1,2)$ for all $t\ge 0$, then

$$\bar{D}_{12}(t)\le D_{12}(t)\left(\frac{\tilde{x}_2(t)}{\tilde{x}_1(t)}-\frac{x_2(t)}{x_1(t)}\right)\le\frac{D_{12}(t)}{\tilde{x}_1(t)}(\tilde{x}_2(t)-x_2(t))\le\frac{D_{12}(t)}{m_1}\left|x_2(t)-\tilde{x}_2(t)\right|,$$
$$\bar{D}_{21}(t)\le D_{21}(t)\left(\frac{\tilde{x}_1(t)}{\tilde{x}_2(t)}-\frac{x_1(t)}{x_2(t)}\right)\le\frac{D_{21}(t)}{\tilde{x}_2(t)}(\tilde{x}_1(t)-x_1(t))\le\frac{D_{21}(t)}{m_1}\left|x_1(t)-\tilde{x}_1(t)\right|.$$

3) If $x_i(t)=\tilde{x}_i(t)\,(i=1,2)$ for all $t\in R_+$, similar to the arguments above, we

can get the same conclusion as (1) and (2). From (1), (2) and (3), we have

$$\begin{aligned} \bar{D}_{12}(t) &\le \frac{D_{12}(t)}{m_1}\left|x_2(t)-\tilde{x}_2(t)\right|, \\ \bar{D}_{21}(t) &\le \frac{D_{21}(t)}{m_1}\left|x_1(t)-\tilde{x}_1(t)\right|. \end{aligned} \tag{5.7}$$

Due to (5.6) and (5.7), we can obtain

$$\begin{aligned} D^+V_1(t) \le & -b_1(t)\left|x_1(t)-\tilde{x}_1(t)\right| + c_1(t)\int_0^{+\infty} k_1(s)\left|y(t-s)-\tilde{y}(t-s)\right|\mathrm{d}s \\ & + \frac{D_{12}(t)}{m_1}\left|x_2(t)-\tilde{x}_2(t)\right| + -b_2(t)\left|x_1(t)-\tilde{x}_1(t)\right| \\ & + \frac{D_{21}(t)}{m_1}\left|x_1(t)-\tilde{x}_1(t)\right| + -b_3(t)\left|y(t)-\tilde{y}(t)\right| \\ & + c_2(t)\int_0^{+\infty} k_2(s)\left|x_1(t-s)-\tilde{x}_1(t-s)\right|\mathrm{d}s. \end{aligned} \tag{5.8}$$

Moreover, we define

$$\begin{aligned} V_2(t) &= \int_0^{+\infty} k_1(s)\int_{t-s}^{t} c_1(u-s)\left|y(u)-\tilde{y}(u)\right|\mathrm{d}u\mathrm{d}s, \\ V_3(t) &= \int_0^{+\infty} k_2(s)\int_{t-s}^{t} c_2(u-s)\left|x_1(u)-\tilde{x}_1(u)\right|\mathrm{d}u\mathrm{d}s. \end{aligned}$$

Obviously, $V_2(t)$ and $V_3(t)$ are continuous for all $t \ge 0$ and $t \ne t_k$. Calculating the upper right derivative, we derive that

$$\begin{aligned} D^+V_2(t) &= \int_0^{+\infty} k_1(s)c_1(t-s)\left|y(t)-\tilde{y}(t)\right|\mathrm{d}s \\ &\quad - \int_0^{+\infty} k_1(s)c_1(t)\left|y(t-s)-\tilde{y}(t-s)\right|\mathrm{d}s, \\ D^+V_3(t) &= \int_0^{+\infty} k_2(s)c_2(t-s)\left|x_1(t)-\tilde{x}_1(t)\right|\mathrm{d}s \\ &\quad - \int_0^{+\infty} k_1(s)c_2(t)\left|x_1(t-s)-\tilde{x}_1(t-s)\right|\mathrm{d}s. \end{aligned} \tag{5.9}$$

Define $V(t) = V_1(t)+V_2(t)+V_3(t)$, then we can follows from (5.8) and (5.9) that

$$\begin{aligned} D^+V(t) &= D^+V_1(t)+D^+V_2(t)+D^+V_3(t) \\ &\le -b_1(t)\left|x_1(t)-\tilde{x}_1(t)\right| + \int_0^{+\infty} k_1(s)c_1(t-s)\left|y(t)-\tilde{y}(t)\right|\mathrm{d}s \\ &\quad + \frac{D_{12}(t)}{m_1}\left|x_2(t)-\tilde{x}_2(t)\right| - b_2(t)\left|x_2(t)-\tilde{x}_2(t)\right| \\ &\quad + \frac{D_{21}(t)}{m_1}\left|x_1(t)-\tilde{x}_1(t)\right| - b_3(t)\left|y(t)-\tilde{y}(t)\right| \\ &\quad + \int_0^{+\infty} k_2(s)c_2(t-s)\left|x_1(t)-\tilde{x}_1(t)\right|\mathrm{d}s \\ &\le -\left(b_1(t) - \frac{D_{21}(t)}{m_1} - \int_0^{+\infty} k_2(s)c_2(t-s)\mathrm{d}s\right)\left|x_1(t)-\tilde{x}_1(t)\right| \\ &\quad - \left(b_2(t) - \frac{D_{12}(t)}{m_1}\right)\left|x_2(t)-\tilde{x}_2(t)\right| \\ &\quad - \left(b_3(t) - \int_0^{+\infty} k_1(s)c_1(t-s)\mathrm{d}s\right)\left|y(t)-\tilde{y}(t)\right| \\ &\le -\psi(t)m_1V(t). \end{aligned}$$

Integrating above inequality, we further have

$V(t) \le V(0)\exp\left(-m_1\int_0^t \psi(s)\mathrm{d}s\right)$ for all $t \in R_+$. Then from (5.1) we have $\int_0^t \psi(s)\mathrm{d}s \to \infty$ as $t \to \infty$. Thus, we have $V(t) \to 0$ as $t \to \infty$. Finally, from (5.4) and (5.5) we know (5.2) holds. This completes the proof of Theorem 5.1.

## 6. Conclusions

In this paper, we investigated an impulsive competitive system with infinite delay and diffusion, in which $x_1$ can disperse between patch 1 and patch 2, but competitor $y$ is confined to patch 1. We also gave some sufficient conditions on permanence, extinction and global attractivity of system (1.1). From Theorem 3.1-Theorem 5.1, we can see that the impulse and dispersal have an influence on permanence, extinction and global attractivity. Moreover, we note that the infinite delay is harmless for the extinction, but it affects the permanence and global attractivity of system (1.1).

Further, we can observe that impulsive perturbations play an important role in the permanence and extinction from Theorem 3.1-Theorem 4.1. In ecological environment, many natural and man-made factors which can be described impulse in mathematical always lead to rapid decrease or increase of the population number. So we consider the following two cases.

Theorem 3.1 shows that if the density-coefficients $b_i(t)(i=1,2)$ are greater than zero and the impulsive coefficients $h_{ik}(i=1,2)$ are bounded, the species $x$ and $y$ are always ultimately bounded. In following discussion, we also assume that satisfies this condition.

**Discuss 1** On condition that the impulses lead to decrease of the number of species (such as fire, drought, hunting, harvesting, flooding deforestation), then the impulsive coefficients satisfy $0 < h_{ik} \le 1$ and $0 < g_k \le 1$ for all $(i=1,2,k=1,2,\cdots)$.

1) Theorem 3.2 shows that if the impulsive perturbations $h_{1k}, h_{2k}$ are relatively small compared to the intrinsic growth rate of *x*, the species $x$ can keep permanence; if the impulsive perturbations $g_k$ are relatively small, in addition, the delay, competition coefficients of $y$ and dispersal coefficients of $x$ relatively small make the intrinsic growth rate of $y$ to increase, then the species $y$ keeps permanence.

2) Theorem 4.1 shows that if the impulsive perturbations $h_{1k}, h_{2k}$ are relatively large compared to the intrinsic growth rate and dispersal coefficient of *x*, then the species $x$ tends to extinction; if the impulsive perturbations $g_k$ are relatively large and the intrinsic growth rate of $y$ is relatively small, the species $y$ tends to extinction.

**Discuss 2** On condition that impulses lead to increase of the number of species (such as feed, replenishment, input or other protective measures from human), that is the impulsive coefficients satisfy $h_{1k}, h_{2k}, g_k \ge 1$ for all $k = 1,2,\cdots$.

1) Theorem 3.2 shows that the species $x$ always keep permanence; if the delay, competition coefficients of $y$ and dispersal coefficients of $x$ are relatively small making the intrinsic growth rate of $y$ to increase; regardless of impulsive influence which is large or small, the species $y$ keeps permanence.

2) Theorem 4.1 shows that the species $x$ never tends to extinction; if the impulsive perturbations $g_k$ and the intrinsic growth rate are relatively small; besides, the competition coefficient is relatively large, then the species $y$ tends to extinction.

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