Journal of Applied Mathematics and Physics, 2017, 5, 1370-1379
http://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

N-Rotating Loop-Soliton Solution of the Coupled Integrable Dispersionless Equation

Souleymanou Abbagari[1,2,3,4], Saliou Youssoufa[2,3,4], Hermann T. Tchokouansi[2,3,4], Victor K. Kuetche[2,3,4], Thomas B. Bouetou[2,3,4], Timoleon C. Kofane[2,4]

[1]Department of Basic Science, Law and Humanities, Institute of Mines and Petroleum Industries, University of Maroua, Maroua, Cameroon
[2]Laboratory of Mechanics, Materials and Structures, Department of Physics, University of Yaoundé I, Yaoundé, Cameroon
[3]National Advanced School of Engineering, University of Yaoundé I, Yaoundé, Cameroon
[4]Centre d'Excellence en Technologies de l'Information et de la Communication (CETIC), University of Yaoundé I, Yaoundé, Cameroon
Email: abbagaris@yahoo.fr

How to cite this paper: Abbagari, S., Youssoufa, S., Tchokouansi, H.T., Kuetche, V.K., Bouetou, T.B. and Kofane, T.C. (2017) N-Rotating Loop-Soliton Solution of the Coupled Integrable Dispersionless Equation. *Journal of Applied Mathematics and Physics*, **5**, 1370-1379.
https://doi.org/10.4236/jamp.2017.56113

Received: May 25, 2017
Accepted: June 26, 2017
Published: June 29, 2017

Abstract

In this paper, we investigate the Rotating N Loop-Soliton solution of the coupled integrable dispersionless equation (CIDE) that describes a current-fed string within an external magnetic field in 2D-space. Through a set of independent variable transformation, we derive the bilinear form of the CIDE Equation. Based on the Hirota's method, Perturbation technique and Symbolic computation, we present the analytic N-rotating loop soliton solution and proceed to some illustrations by presenting the cases of three- and four-soliton solutions.

Keywords

Coupled Integrable Dispersionless Equation, Bilinear Method, Soliton Solutions, Perturbation Technique, Symbolic Computation

1. Introduction

During the past several years, the study of coupled nonlinear evolution Equations has played an important role in explaining many interesting phenomena, like electromagnetic wave propagation in impurity media, water waves, pulse in biological chains and so on [1] [2] [3]. At the same time, the coupled integrable dispersionless system (CIDE) has attracted much interest in view of its wide range of application in various fields of mathematics, physics, applied mathematics, theory of quantum and theory of conformal maps on the complex plane [4] [5] [6] [7]. The CIDE, has first been presented by Konno and Oono in Ref.

[8] based on a Lie-group $\mathcal{G}=SU(2)$, and its generalization based on the Lie-group $\mathcal{G}=SL(2,\mathcal{R})$ are examples of such system [7] [9] [10], which have attracted great deal of interest because of its nice integrability structure and soliton solution. Based on this standpoint, the solitons show loop shapes in the three-dimensional Euclidean space. The angular momentum conservation law can be derived from the Equations of motion of the string such that we can expect rotating loop solitons.

So far, several successful methods have been developed to obtain explicit solution for soliton Equations, such as the Inverse Scattering Transformation (IST) [1] [11] [12], Bäcklund and Darboux Transformations [13] [14], the Hirota's method [15] [16], the Wronskian and Cassoratian techniques [17] [18], the Algebra geometric method [19] and so on. Among these methods, the Hirota's bilinear method has been proven to be an efficient and direct approach to construct soliton solutions to nonlinear evolution Equations via the bilinear forms from the dependent variables transformation.

In Ref [8], Konno and Oono have presented the well known CIDE

$$\begin{aligned} &q_{xt}+\frac{1}{2}(rs)_x=0,\\ &r_{xt}-q_x r=0,\\ &s_{xt}-q_x s=0, \end{aligned} \tag{1}$$

where Equation (1) describes the current-fed within an external magnetic field [20]. In Equation (1), *q*, *r*, and *s* are all functions of x and t, the subscripts denote partial derivatives with respect to the space-like and time-like variables respectively.

The aim of this work is to verify if the congestion, due to the displacement of a great number of soliton will modify the conservation properties observed for the case of two solitons. Indeed, we provide the explicit expression of the *N*-Rotating loop soliton solution to the CIDE for the general positive integer $N\geq 2$ and to illustrate our general result, we discuss particular cases of *N*. Thus the following paper is organized as follows. In section 2, we summarize the transformation of the CIDE Equation (1) into an Equation in bilinear form. In section 3, we give the full expression of the N-Rotating loop soliton solution and we illustrate our results by considering in detail the cases of $N=1,2,3,4$ and we end this work with a brief summary.

2. Hirota's Bilinearization of the CIDE

Let us consider the following setting [20] [21] [22]

$$\begin{aligned} &r=X+iY,\quad s=X-iY,\\ &q=Z,\quad \sigma=x+t,\quad \tau=x-t, \end{aligned} \tag{2}$$

which inserted into Equation (1) gives

$$\boldsymbol{r}_{\tau\tau}-\boldsymbol{r}_{\sigma\sigma}=(\boldsymbol{r}_\tau+\boldsymbol{r}_\sigma)\times(\boldsymbol{J}\times\boldsymbol{r}), \tag{3}$$

where $\boldsymbol{r}=(X,Y,Z)$ stands to be the vector position of the string, $\boldsymbol{J}=(1,0,1)$

is the constant electric current [23]. In Equation (3) the factor $\boldsymbol{r}_\tau + \boldsymbol{r}_\sigma$ can be interpreted as the Lorentz force acting on effective internal current, $\boldsymbol{r}_\sigma$ can be considered as an internal electric current and $\boldsymbol{r}_\tau$ is a correction term induced by the motion of string to $\boldsymbol{r}_\sigma$. Equation (3) can therefore represent a current-fed string interacting with the external magnetic field $\boldsymbol{B} = \boldsymbol{J} \times \boldsymbol{r}$ which satisfies the two Maxwell's Equations $rot\boldsymbol{B} = 2\boldsymbol{J}$ and $div\boldsymbol{B} = 0$. Using the boundary condition $\boldsymbol{r} \to (0,0,\sigma)$ for $|\sigma| \to \infty$, we bilinearize Equation (3) as

$$\mathrm{D}_x \mathrm{D}_t Q \cdot F = Q \cdot F, \quad \mathrm{D}_t^2 F \cdot F = \frac{1}{2} Q \cdot Q^*, \tag{4}$$

using the transformation

$$r = \frac{Q}{F}, \quad q = x - 2\partial_t \ln F, \tag{5}$$

where D denotes the Hirota's derivative [15] [16]. Now, expanding Q and F as series

$$\begin{aligned} &F = 1 + \epsilon^2 F_2 + \epsilon^4 F_4 + \cdots + \epsilon^{2i} F_{2i} + \cdots, \\ &Q = \epsilon Q_1 + \epsilon^3 Q_3 + \cdots + \epsilon^{2i+1} Q_{2i+1} + \cdots. \end{aligned} \tag{6}$$

Substituting the expansion into the above bilinear Equations, we find that there are only even order terms of ϵ in the first Equation while odd order terms in the second one. Arranging the coefficients at each order of ϵ, we have

$$\begin{aligned} &\epsilon : \mathrm{D}_x \mathrm{D}_t (1 \cdot Q_1) = Q_1, \\ &\epsilon^2 : \mathrm{D}_t^2 (1 \cdot F_2 + F_2 \cdot 1) = \frac{1}{2} Q_1 Q_1^*, \\ &\epsilon^3 : \mathrm{D}_x \mathrm{D}_t (1 \cdot Q_3 + Q_1 \cdot F_2) = Q_3 + Q_1 F_2, \\ &\epsilon^4 : \mathrm{D}_t^2 (1 \cdot F_4 + F_2 \cdot F_2 + F_4 \cdot 1) = \frac{1}{2} (Q_1 Q_3^* + Q_3 Q_1^*), \\ &\epsilon^5 : \mathrm{D}_x \mathrm{D}_t (1 \cdot Q_5 + Q_1 \cdot F_4 + Q_3 \cdot F_2) = Q_1 F_4 + Q_3 F_2 + Q_5, \\ &\epsilon^{2i} : \mathrm{D}_t^2 \left(\sum_{m=0}^{i} F_{2m} \cdot F_{2i-2m} \right) = \frac{1}{2} \left(\sum_{k=0}^{i-1} Q_{2k+1} Q_{2i-2k-1}^* \right), \\ &\epsilon^{2i+1} : \mathrm{D}_x \mathrm{D}_t \left(\sum_{l=0}^{i} Q_{2l+1} \cdot F_{2i-2l} \right) = \left(\sum_{l=0}^{i} Q_{2l+1} F_{2i-2l} \right). \end{aligned} \tag{7}$$

It is then possible to obtain at the required order the required number of soliton solutions by determining the full expansion of F and Q.

3. Rotating one and Two-Loop Soliton Solution

In this section, we derive the rotating solitons *i.e.*, solutions that the Z component of the angular momentum is a conserved quantity. In order to construct one-rotating soliton solution, we take

$$Q_1 = \exp(\eta_1), \tag{8}$$

where $\eta_1 = k_1 x + \omega_1 t + \gamma_1$. Substituting it into Equation (7), limiting our interest to the terms of ϵ^i, $i \le 2$, we obtain

$$\omega_1 k_1 = 1, \quad F_2 = A_{j_1^*}^{i_1} \exp\left(\eta_1 + \eta_1^*\right), \quad i_1 = j_1 = 1, \tag{9}$$

the first part of Equation (9) standing for the dispersion relation and the coefficient $A_{1^*}^1$ is giving by $A_{1^*}^1 = \dfrac{1}{4\left(\omega_1 + \omega_1^*\right)^2}$. This show that the expansion can be truncated as the finite sum

$$F = 1 + \frac{\epsilon^2 \exp\left(\eta_1 + \eta_1^*\right)}{4\left(\omega_1 + \omega_1^*\right)^2}, \quad Q = \epsilon \exp\left(\eta_1\right). \tag{10}$$

Absorbing the parameter ϵ into the phase constant γ_1 gives the one-rotating soliton solution of the CIDE as it is depicted in **Figure 1**.

Next, we choose the solution of Equation (7) while limiting our interest to the terms of ϵ^i, $i \le 4$ to be

$$Q_1 = A^1 \exp\left(\eta_1\right) + A^2 \exp\left(\eta_2\right), \tag{11}$$

where the phase $\eta_i = k_i x + \omega_i t + \gamma_i$ and the dispersion relation $k_i \omega_i = 1$ with $i = 1, 2$. From Equation (7) we have

$$\begin{aligned} F_2 &= A_{1^*}^1 \exp\left(\eta_1 + \eta_1^*\right) + A_{2^*}^1 \exp\left(\eta_1 + \eta_2^*\right) \\ &\quad + A_{1^*}^2 \exp\left(\eta_2 + \eta_1^*\right) + A_{2^*}^2 \exp\left(\eta_2 + \eta_2^*\right), \\ Q_3 &= A_{1^*}^{12} \exp\left(\eta_1 + \eta_2 + \eta_1^*\right) + A_{2^*}^{12} \exp\left(\eta_1 + \eta_2 + \eta_2^*\right) \\ F_4 &= A_{1^* 2^*}^{12} \exp\left(\eta_1 + \eta_2 + \eta_1^* + \eta_2^*\right), \end{aligned} \tag{12}$$

where

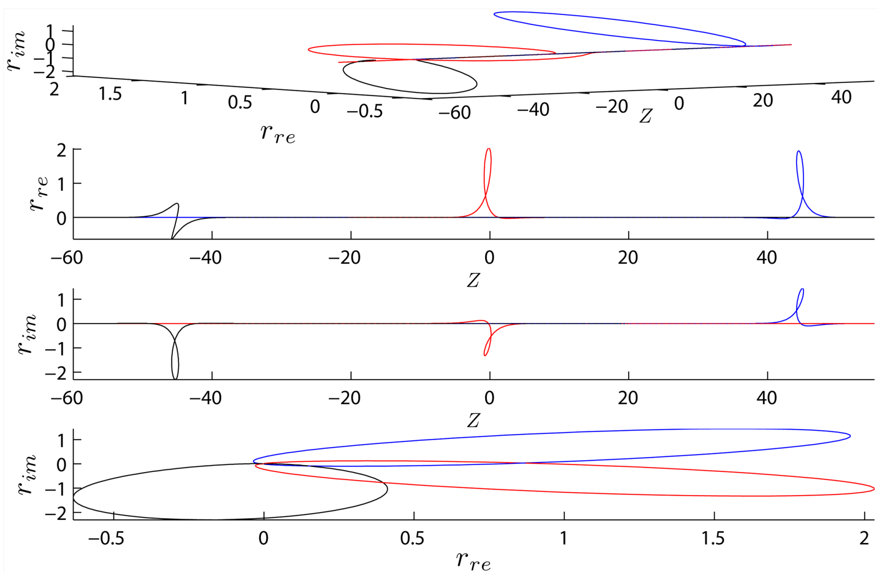

Figure 1. From left to right panels rotating one-loop soliton solution to the CIDE Equation (1): For left we depict at times $t = -30$ (blue color), $t = 0$ (red color) and $t = 30$ (black color) corresponding to three moving states, with $v_1 = 0.66$ and the computed angular velocities of such wave is $\Omega_1 = 0.40$, respectively.

$$
\begin{aligned}
&A^{i_1}=1,\quad (i_1=1,2),\\
&A^{i_1}_{j_1^*}=\frac{1}{4\left(\omega_{i_1}+\omega^*_{j_1}\right)^2},\quad \begin{pmatrix} i_1=1,2\\ j_1=1,2\end{pmatrix},\\
&A^{i_1i_2}_{j_1^*}=4A^{i_1}_{j_1^*}A^{i_2}_{j_1^*}\left(\omega_{i_1}-\omega_{i_2}\right)^2,\quad \begin{pmatrix} i_1=1;\, i_2=2\\ j_1=1,2\end{pmatrix},\\
&A^{i_1i_2}_{j_1^*j_2^*}=4^2A^{i_1}_{j_1^*}A^{i_1}_{j_2^*}A^{i_2}_{j_1^*}A^{i_2}_{j_2^*}\left(\omega_{i_1}-\omega_{i_2}\right)^2\left(\omega^*_{j_1}-\omega^*_{j_2}\right)^2,\quad \begin{pmatrix} i_1=1;\, i_2=2\\ j_1=1;\, j_2=2\end{pmatrix}.
\end{aligned}
\tag{13}
$$

According to the above analysis, the two-rotating soliton solution is obtained when we substitute Equations (11)-(13) into Equation (5) as it is depicted in Figure 2.

Generally we can conjecture the N-rotating soliton solution as

$$
\begin{aligned}
F&=1+\sum_{m=1}^{[N]}\sum_{\substack{{}_NC_m\\ {}_NC_m}}A^{i_1\cdots i_m}_{j_1^*\cdots j_m^*}\exp\left(\eta_{i_1}+\cdots+\eta_{i_m}+\eta^*_{j_1}+\cdots+\eta^*_{j_m}\right),\\
Q&=\sum_{m=0}^{[N-1]}\sum_{\substack{{}_NC_{m+1}\\ {}_NC_m}}A^{i_1\cdots i_{m+1}}_{j_1^*\cdots j_m^*}\exp\left(\eta_{i_1}+\cdots+\eta_{i_{m+1}}+\eta^*_{j_1}+\cdots+\eta^*_{j_m}\right),
\end{aligned}
\tag{14}
$$

where the phase $\eta_p=k_px+\omega_pt+\gamma_p$ and the dispersion relation $k_p\omega_p=1$ with $p=1,\cdots,N$.

$$
A^{i_1}=1,\quad A^{i_1}_{j_1^*}=\frac{1}{4\left(\omega_{i_1}+\omega^*_{j_1}\right)^2},\quad A^{i_1i_2}_{j_1^*}=4A^{i_1}_{j_1^*}A^{i_2}_{j_2^*}\left(\omega_{i_1}-\omega_{i_2}\right)^2,
\tag{15}
$$

$$
A^{i_1\cdots i_m}_{j_1^*\cdots j_n^*}=4^{\left[C_m^2+C_n^2\right]}\left(\prod_{\alpha<\beta}^{(m)}(\omega_\alpha-\omega_\beta)^2\right)\left(\prod_{\lambda<\gamma}^{(n)}\left(\omega^*_\lambda-\omega^*_\gamma\right)^2\right)\left(\prod_{\substack{\delta=i_1\cdots i_m\\ \nu=j_1\cdots j_n}}A^{\delta}_{\nu}\right),
\tag{16}
$$

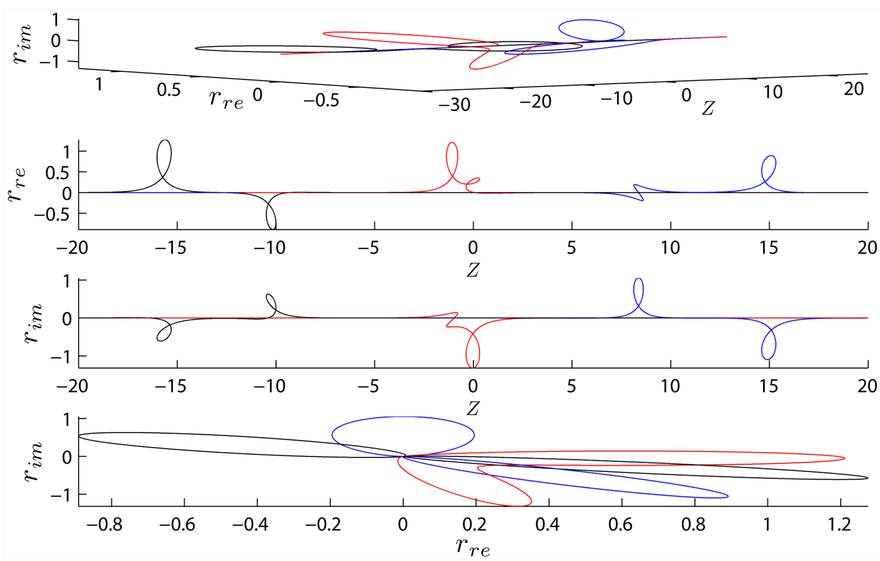

Figure 2. From left to right panels rotating two-loop soliton solution to the CIDE Equation (1): For right we depict at times $t=-30$ (blue color), $t=0$ (red color) and $t=30$ (black color) corresponding to three moving states, with $v_1=2$, $v_2=3.33$ and the computed angular velocities of such wave is $\Omega_1=0.10$ and $\Omega_2=0.15$, respectively.

where $[N]$ denotes the maximum integer which does not exceed N, ${}_N C_m$ indicate the summation over all possible combinations of m elements from N and (m) indicates the product of all possible combinations of m elements with $(\alpha < \beta)$. Using the real parameters, we write the phase into two parts as

$$\eta_n = \left(k_{n,re}x + \omega_{n,re}t + \gamma_{n,re}\right) + i\left(k_{n,im}x + \omega_{n,im}t + \gamma_{n,im}\right), \quad n = 1,\cdots,N, \tag{17}$$

where the real parts and imaginary parts of the parameters k_n and ω_n are obtained using the dispersion relation as

$$\begin{aligned} &k_{n,re} = \sqrt{\left(1 - v_n\Omega_n^2\right)v_n}, \quad \omega_{n,re} = \sqrt{\frac{1 - v_n\Omega_n^2}{v_n}}, \\ &\omega_{n,im} = \Omega_n, \quad k_{n,im} = -v_n\Omega_n, \end{aligned} \tag{18}$$

here, v_n and Ω_n are the phase velocity and the angular velocity of the soliton, which respect the following condition

$$v_n > 0, \quad \Omega_n \in \left]-1/\sqrt{v_n}; 1/\sqrt{v_n}\right[. \tag{19}$$

Now, let us consider two simple cases: $N = 3$ and $N = 4$.

- Case $N = 3$

We then write the following expressions of F and Q with all coefficients, where $\exp\left(\eta_{j0}^*\right) = 1$. This leads to the three-rotating soliton solution depicted in Figure 3.

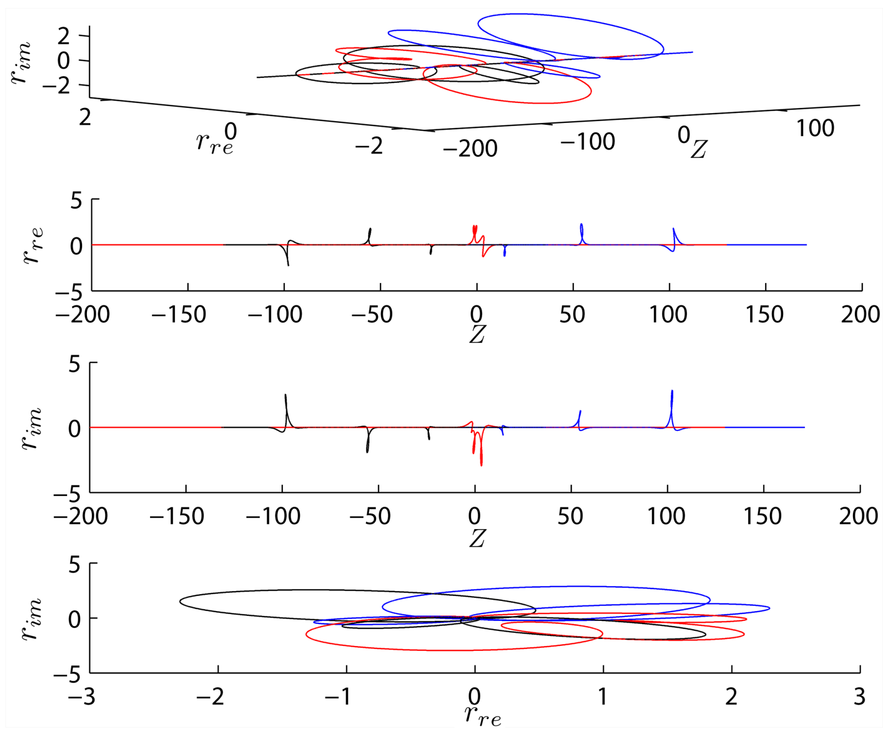

Figure 3. From left to right panels rotating three-loop soliton solution to the CIDE Equation (1): For left we depict at times $t = -30$ (blue color), $t = 0$ (red color) and $t = 30$ (black color) corresponding to three moving states, with $v_1 = 2$, $v_2 = 0.30$, $v_3 = 0.55$ and the computed angular velocities of such wave is $\Omega_1 = 0.25$, $\Omega_2 = 1.00$, $\Omega_3 = 0.50$, respectively.

$$
\begin{aligned}
F &= 1 + \sum_{\substack{{}_3C_1\\{}_3C_1}} A_{j_1^*}^{i_1} \exp\left(\eta_{i_1} + \eta_{j_1}^*\right) + \sum_{\substack{{}_3C_2\\{}_3C_2}} A_{j_1^* j_2^*}^{i_1 i_2} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{j_1}^* + \eta_{j_2}^*\right) \\
&\quad + \sum_{\substack{{}_3C_3\\{}_3C_3}} A_{j_1^* j_2^* j_3^*}^{i_1 i_2 i_3} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{i_3} + \eta_{j_1}^* + \eta_{j_2}^* + \eta_{j_3}^*\right), \\
Q &= \sum_{\substack{{}_3C_1\\{}_3C_0}} A_{j_0^*}^{i_1} \exp\left(\eta_{i_1} + \eta_{j_0}^*\right) + \sum_{\substack{{}_3C_2\\{}_3C_1}} A_{j_1^*}^{i_1 i_2} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{j_1}^*\right) \\
&\quad + \sum_{\substack{{}_3C_3\\{}_3C_2}} A_{j_1^* j_2^*}^{i_1 i_2 i_3} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{i_3} + \eta_{j_1}^* + \eta_{j_2}^*\right),
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
& A_{j_0^*}^{i_1} = A^{i_1} = 1, \quad \left(i_1 = 1,2,3\right), \\
& A_{j_1^*}^{i_1} = \frac{1}{4\left(\omega_{i_1} + \omega_{j_1}^*\right)^2}, \quad \begin{pmatrix} i_1 = 1,2,3 \\ j_1 = 1,2,3 \end{pmatrix}, \\
& A_{j_1^*}^{i_1 i_2} = 4 A_{j_1^*}^{i_1} A_{j_1^*}^{i_2} \left(\omega_{i_1} - \omega_{i_2}\right)^2, \quad \begin{pmatrix} i_1 = 1,2; i_2 = i_1 + 1 \\ j_1 = 1,2,3 \end{pmatrix} \\
& A_{j_1^* j_2^*}^{i_1 i_2} = 4^2 A_{j_1^*}^{i_1} A_{j_2^*}^{i_1} A_{j_1^*}^{i_2} A_{j_2^*}^{i_2} \left(\omega_{i_1} - \omega_{i_2}\right)^2 \left(\omega_{j_1}^* - \omega_{j_2}^*\right)^2, \quad \begin{pmatrix} i_1 = 1,2; i_2 = i_1 + 1 \\ j_1 = 1,2; j_2 = j_1 + 1 \end{pmatrix}, \\
& A_{j_1^* j_2^*}^{i_1 i_2 i_3} = 4^4 A_{j_1^*}^{i_1} A_{j_2^*}^{i_1} A_{j_1^*}^{i_2} A_{j_2^*}^{i_2} A_{j_1^*}^{i_3} A_{j_2^*}^{i_3} \left(\omega_{i_1} - \omega_{i_2}\right)^2 \left(\omega_{i_1} - \omega_{i_3}\right)^2 \\
& \qquad \times \left(\omega_{i_2} - \omega_{i_3}\right)^2 \left(\omega_{j_1}^* - \omega_{j_2}^*\right)^2, \quad \begin{pmatrix} i_1 = 1; i_2 = 2; i_3 = 3 \\ j_1 = 1,2; j_2 = j_1 + 1 \end{pmatrix}, \\
& A_{j_1^* j_2^* j_3^*}^{i_1 i_2 i_3} = 4^6 A_{j_1^*}^{i_1} A_{j_2^*}^{i_1} A_{j_3^*}^{i_1} A_{j_1^*}^{i_2} A_{j_2^*}^{i_2} A_{j_3^*}^{i_2} A_{j_1^*}^{i_3} A_{j_2^*}^{i_3} A_{j_3^*}^{i_3} \left(\omega_{i_1} - \omega_{i_2}\right)^2 \left(\omega_{i_1} - \omega_{i_3}\right)^2 \left(\omega_{i_2} - \omega_{i_3}\right)^2 \\
& \qquad \times \left(\omega_{j_1}^* - \omega_{j_2}^*\right)^2 \left(\omega_{j_1}^* - \omega_{j_3}^*\right)^2 \left(\omega_{j_2}^* - \omega_{j_3}^*\right)^2, \quad \begin{pmatrix} i_1 = 1; i_2 = 2; i_3 = 3 \\ j_1 = 1; j_2 = 2; j_3 = 3 \end{pmatrix}.
\end{aligned} \tag{21}
$$

- Case $N = 4$

In this case the four-rotating soliton solution is obtain by

$$
\begin{aligned}
F &= 1 + \sum_{\substack{{}_4C_1\\{}_4C_1}} A_{j_1^*}^{i_1} \exp\left(\eta_{i_1} + \eta_{j_1}^*\right) + \sum_{\substack{{}_4C_2\\{}_4C_2}} A_{j_1^* j_2^*}^{i_1 i_2} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{j_1}^* + \eta_{j_2}^*\right) \\
&\quad + \sum_{\substack{{}_4C_3\\{}_4C_3}} A_{j_1^* j_2^* j_3^*}^{i_1, i_2 i_3} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{i_3} + \eta_{j_1}^* + \eta_{j_2}^* + \eta_{j_3}^*\right) \\
&\quad + \sum_{\substack{{}_4C_4\\{}_4C_4}} A_{j_1^* j_2^* j_3^* j_4^*}^{i_1 i_2 i_3 i_4} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{i_3} + \eta_{i_4} + \eta_{j_1}^* + \eta_{j_2}^* + \eta_{j_3}^* + \eta_{j_4}^*\right), \\
Q &= \sum_{\substack{{}_4C_1\\{}_4C_0}} A_{j_0^*}^{i_1} \exp\left(\eta_{i_1} + \eta_{j_0}^*\right) + \sum_{\substack{{}_4C_2\\{}_4C_1}} A_{j_1^*}^{i_1 i_2} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{j_1}^*\right) \\
&\quad + \sum_{\substack{{}_4C_3\\{}_4C_2}} A_{j_1^* j_2^*}^{i_1 i_2 i_3} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{i_3} + \eta_{j_1}^* + \eta_{j_2}^*\right) \\
&\quad + \sum_{\substack{{}_4C_4\\{}_4C_3}} A_{j_1^* j_2^* j_3^*}^{i_1 i_2 i_3 i_4} \exp\left(\eta_{i_1} + \eta_{i_2} + \eta_{i_3} + \eta_{i_4} + \eta_{j_1}^* + \eta_{j_2}^* + \eta_{j_3}^*\right),
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
& A_{j_0^*}^{i_1} = A^{i_1} = 1, \quad \left(i_1 = 1,2,3,4\right), \\
& A_{j_1^*}^{i_1} = \frac{1}{4\left(\omega_{i_1} + \omega_{j_1}^*\right)^2}, \quad \begin{pmatrix} i_1 = 1,2,3,4 \\ j_1 = 1,2,3,4 \end{pmatrix},
\end{aligned}
$$

$$A^{i_1,i_2}_{j_1^*} = 4A^{i_1}_{j_1^*}A^{i_2}_{j_1^*}\left(\omega_{i_1}-\omega_{i_2}\right)^2, \quad \begin{pmatrix} i_1 = 1,2,3; \quad i_2 = i_1+1 \\ j_1 = 1,2,3,4 \end{pmatrix},$$

$$A^{i_1,i_2}_{j_1^*,j_2^*} = 4^2 A^{i_1}_{j_1^*}A^{i_1}_{j_2^*}A^{i_2}_{j_1^*}A^{i_2}_{j_2^*}\left(\omega_{i_1}-\omega_{i_2}\right)^2\left(\omega^*_{j_1}-\omega^*_{j_2}\right)^2, \quad \begin{pmatrix} i_1 = 1,2,3; i_2 = i_1+1 \\ j_1 = 1,2,3; \ j_2 = j_1+1 \end{pmatrix},$$

$$\begin{aligned} A^{i_1,i_2,i_3}_{j_1^*j_2^*} = {} & 4^4 A^{i_1}_{j_1^*}A^{i_1}_{j_2^*}A^{i_2}_{j_1^*}A^{i_2}_{j_2^*}A^{i_3}_{j_1^*}A^{i_3}_{j_2^*}\left(\omega_{i_1}-\omega_{i_2}\right)^2\left(\omega_{i_1}-\omega_{i_3}\right)^2 \\ & \times\left(\omega_{i_2}-\omega_{i_3}\right)^2\left(\omega^*_{j_1}-\omega^*_{j_2}\right)^2, \quad \begin{pmatrix} i_1 = 1,2; i_2 = i_1+1; i_3 = i_2+1 \\ j_1 = 1,2,3; \ j_2 = j_1+1 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} A^{i_1,i_2,i_3}_{j_1^*,j_2^*,j_3^*} = {} & 4^6 A^{i_1}_{j_1^*}A^{i_1}_{j_2^*}A^{i_1}_{j_3^*}A^{i_2}_{j_1^*}A^{i_2}_{j_2^*}A^{i_2}_{j_3^*}A^{i_3}_{j_1^*}A^{i_3}_{j_2^*}A^{i_3}_{j_3^*}\left(\omega_{i_1}-\omega_{i_2}\right)^2\left(\omega_{i_1}-\omega_{i_3}\right)^2\left(\omega_{i_2}-\omega_{i_3}\right)^2 \\ & \times\left(\omega^*_{j_1}-\omega^*_{j_2}\right)^2\left(\omega^*_{j_1}-\omega^*_{j_3}\right)^2\left(\omega^*_{j_2}-\omega^*_{j_3}\right)^2, \quad \begin{pmatrix} i_1 = 1,2; i_2 = i_1+1; i_3 = i_2+1 \\ j_1 = 1,2; \ j_2 = j_1+1; \ j_3 = j_2 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} A^{i_1,i_2,i_3,i_4}_{j_1^*,j_2^*,j_3^*} = {} & 4^9 A^{i_1}_{j_1^*}A^{i_1}_{j_2^*}A^{i_1}_{j_3^*}A^{i_2}_{j_1^*}A^{i_2}_{j_2^*}A^{i_2}_{j_3^*}A^{i_3}_{j_1^*}A^{i_3}_{j_2^*}A^{i_3}_{j_3^*}A^{i_4}_{j_1^*}A^{i_4}_{j_2^*}A^{i_4}_{j_3^*}\left(\omega_{i_1}-\omega_{i_2}\right)^2\left(\omega_{i_1}-\omega_{i_3}\right)^2 \\ & \times\left(\omega_{i_1}-\omega_{i_4}\right)^2\left(\omega_{i_2}-\omega_{i_3}\right)^2\left(\omega_{i_2}-\omega_{i_4}\right)^2\left(\omega_{i_3}-\omega_{i_4}\right)^2\left(\omega^*_{j_1}-\omega^*_{j_2}\right)^2 \\ & \times\left(\omega^*_{j_1}-\omega^*_{j_3}\right)^2\left(\omega^*_{j_2}-\omega^*_{j_3}\right)^2, \quad \begin{pmatrix} i_1 = 1; i_2 = 2; i_3 = 3; i_4 = 4 \\ j_1 = 1,2; \ j_2 = j_1+1; \ j_3 = j_2+1 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} A^{i_1,i_2,i_3,i_4}_{j_1^*,j_2^*,j_3^*,j_4^*} = {} & 4^{12} A^{i_1}_{j_1^*}A^{i_1}_{j_2^*}A^{i_1}_{j_3^*}A^{i_1}_{j_4^*}A^{i_2}_{j_1^*}A^{i_2}_{j_2^*}A^{i_2}_{j_3^*}A^{i_2}_{j_4^*}A^{i_3}_{j_1^*}A^{i_3}_{j_2^*}A^{i_3}_{j_3^*}A^{i_3}_{j_4^*}A^{i_4}_{j_1^*}A^{i_4}_{j_2^*}A^{i_4}_{j_3^*}A^{i_4}_{j_4^*} \\ & \times\left(\omega_{i_1}-\omega_{i_2}\right)^2\left(\omega_{i_1}-\omega_{i_3}\right)^2\left(\omega_{i_1}-\omega_{i_4}\right)^2\left(\omega_{i_2}-\omega_{i_3}\right)^2\left(\omega_{i_2}-\omega_{i_4}\right)^2 \\ & \times\left(\omega_{i_3}-\omega_{i_4}\right)^2\left(\omega^*_{j_1}-\omega^*_{j_2}\right)^2\left(\omega^*_{j_1}-\omega^*_{j_3}\right)^2\left(\omega^*_{j_1}-\omega^*_{j_4}\right)^2\left(\omega^*_{j_2}-\omega^*_{j_3}\right)^2 \\ & \times\left(\omega^*_{j_2}-\omega^*_{j_4}\right)^2\left(\omega^*_{j_3}-\omega^*_{j_4}\right)^2, \quad \begin{pmatrix} i_1 = 1; i_2 = 2; i_3 = 3; i_4 = 4 \\ j_1 = 1; \ j_2 = 2; \ j_3 = 3; \ j_4 = 4 \end{pmatrix}. \end{aligned} \tag{23}$$

Figure 4 gives the depiction of the four-rotating soliton solutions to the CIDE.

4. Summary and Discussion

In this work, we have investigated the CIDE under the view-point of Hirota's bilinearization. Investigating its one- and two-soliton solution, we have come to propose a generalization of such solution to explicit N-soliton solution of the same system. As a matter of illustration, we have provided explicit expressions of 3- and 4-soliton solutions to the CIDE, and have provided figures to enforce our results. In this figures it has appeared clearly that the solution exhibit particle character, since they interact elastically. Since the CIDE is of many physical implications, the N-soliton solution we have obtained is helpful in understanding the propagation of waves in some media such as the propagation electric field in optical fibers, since in Ref. [22] has provided the relation that link the CIDE and the short pulse system. In this work, we have not gone deeply in studying the interaction process between solitons. Such a study will help understand better the interaction process that occurs during the propagation of such waves in some media including optical fibers.

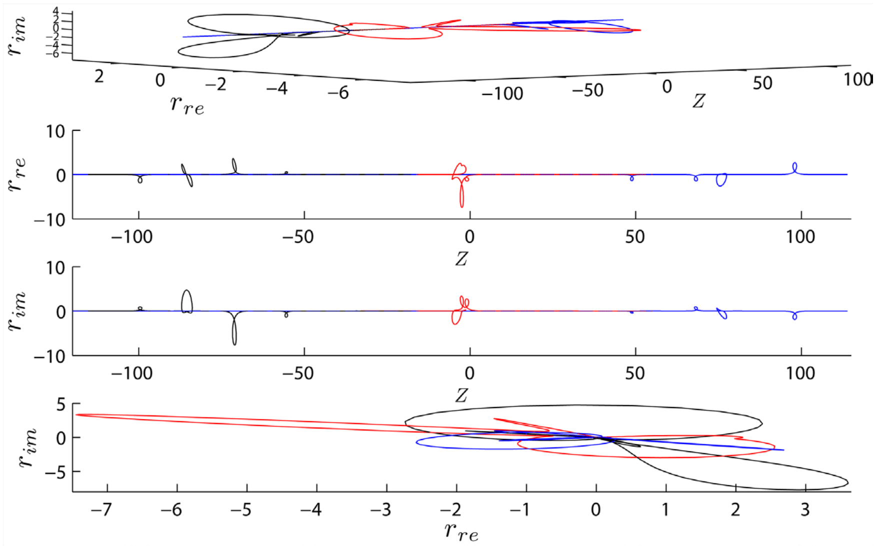

Figure 4. From left to right panels rotating four-loop soliton solution to the CIDE Equation (1): For right we depict at times $t=-90$ (blue color), $t=0$ (red color) and $t=90$ (black color) corresponding to three moving states, with $v_1=0.91$, $v_2=1.82$, $v_3=1.33$, $v_4=1.11$ and the computed angular velocities of such wave is $\Omega_1=0.015$ and $\Omega_2=0.050$, $\Omega_3=0.090$, $\Omega_4=0.990$ respectively.

Acknowledgements

The authors would like to express their sincere thanks to the anonymous referees for their critical comments and appropriate suggestions which have made this paper more precise and readable.

References

[1] Ablowitz, M.J. and Clarkson, P.A. (1991) Solitons, Nonlinear Evolution Equations and Inverse Scattering. Cambridge University Press, New York. https://doi.org/10.1017/CBO9780511623998

[2] Malomet, B.A. (1992) Bound Solitons in Coupled Nonlinear Schrdinger Equations. *Physical Review A*, **45**, 8321-8323. https://doi.org/10.1103/PhysRevA.45.R8321

[3] Nakkeeran, D.J. and Porsesian, K.J. (1995) Solitons in an Erbium-Doped Nonlinear Fibre Medium with Stimulated Inelastic Scattering. *Journal of Physics A*: *Mathematical and General*, **28**, 3817-3823. https://doi.org/10.1088/0305-4470/28/13/025

[4] Konopelchenko, B.G. and Magri, F. (2007) Coisotropic Deformations of Associative Algebras and Dispersionless Integrable Hierarchies. *Communications in Mathematical Physics*, **274**, 627-658. https://doi.org/10.1007/s00220-007-0295-2

[5] Kakuhata, H. and Konno, K. (1997) Canonical Formulation of a Generalized Coupled Dispersionless System. *Journal of Physics A*: *Mathematical and General*, **30L**, 401-407. https://doi.org/10.1088/0305-4470/30/12/002

[6] Aoyama, S. and Kodama, Y. (1994) Topological Conformal Field Theory with a Rational Potential and the Dispersionless KP Hierarchy. *Modern Physics Letters A*, **9**, 2481-2492. https://doi.org/10.1142/S0217732394002355

[7] Hassan, M.J. (2009) Darboux Transformation of the Generalized Coupled Dispersionless Integrable System. *Journal of Physics A*: *Mathematical and Theoretical*, **42**, 1-11.

[8] Konno, K. and Oono, H. (1994) New Coupled Integrable Dispersionless Equations. *Journal of the Physical Society of Japan*, **63**, 377-378. https://doi.org/10.1143/JPSJ.63.377

[9] Kakuhata, H. and Konno, K. (1996) A Generalization of Coupled Integrable, Dispersionless System. *Journal of the Physical Society of Japan*, **65**, 340-341. https://doi.org/10.1143/JPSJ.65.340

[10] Kotlyarov, V.P. (1994) On Equations Gauge Equation Uivalent to the Sine-Gordon and Pohlmeyer-Lund-Regge Equations. *Journal of the Physical Society of Japan*, **63**, 3535-3537. https://doi.org/10.1143/JPSJ.63.3535

[11] Novikov, S.P., Manakov, S.V., Pitaevskii, L.P. and Zakkarov, V.E. (1984) Theory of Solitons, The Inverse Scattering Method. Consultant Bureau, New York.

[12] Konno, K. and Kakuhata, H. (1996) Novel Solitonic Evolutions in a Coupled Integrable, Dispersionless System. *Journal of the Physical Society of Japan*, **65**, 713-721. https://doi.org/10.1143/JPSJ.65.713

[13] Levi, D. (1988) On a New Darboux Transformation for the Construction of Exact Solutions of the Schrodinger Equation. *Inverse Problem*, **4**, 165-172. https://doi.org/10.1088/0266-5611/4/1/014

[14] Matveev, V.B. and Salle, M.A. (1991) Darboux Transformations and Solitons. Springer, Berlin. https://doi.org/10.1007/978-3-662-00922-2

[15] Hirota, R. (2004) The Direct Method in Soliton Theory. Cambridge University Press, Cambridge. https://doi.org/10.1017/CBO9780511543043

[16] Hirota, R. and Ohtam, Y. (1991) Hierarchies of Coupled Soliton Equations. I. *Journal of the Physical Society of Japan*, **60**, 798-809. https://doi.org/10.1143/JPSJ.60.798

[17] Ma, W.X. and You, Y.C. (2005) Solving the Korteweg-de Vries Equation by Its Bilinear Form: Wronskian Solutions. *Transactions of the American Mathematical Society*, **357**, 1753-1778. https://doi.org/10.1090/S0002-9947-04-03726-2

[18] Ma, W.X. and Maruno, K.I. (2004) Complexiton Solutions of the Toda Lattice Equation. *Physica A*, **343**, 219-237. https://doi.org/10.1016/j.physa.2004.06.072

[19] Geng, X.G. (2003) Algebraic-Geometrical Solutions of Some Multidimensional Nonlinear Evolution Equations. *Journal of Physics A—Mathematical and General*, **36**, 2289-2303. https://doi.org/10.1088/0305-4470/36/9/307

[20] Kakuhata, H. and Konno, K. (1999) Loop Soliton Solutions of String Interacting with External Field. *Journal of the Physical Society of Japan*, **68**, 757-762. https://doi.org/10.1143/JPSJ.68.757

[21] Kakuhata, H. and Konno, K. (2002) Rotating Loop Soliton of the Coupled Dispersionless Equations. *Theoretical and Mathematical Physics*, **133**, 1675-1683. https://doi.org/10.1023/A:1021366309313

[22] Kuetche, V.K., Bouetou, T.B. and Kofane, T.C. (2008) On Exact N-Loop Soliton Solution to Nonlinear Coupled Dispersionless Evolution Equations. *Physics Letters A*, **372**, 665-669. https://doi.org/10.1016/j.physleta.2007.08.023

[23] Kuetche, V.K., Bouetou, T.B. and Kofane, T.C. (2007) Comment on Loop Soliton Solutions of String Interacting with External Field? *Journal of the Physical Society of Japan*, **76**, Article ID: 126001. https://doi.org/10.1143/JPSJ.76.126001