

Archimedes' Principle Revisited

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Abstract

Based on Newton's third law of motion, we present a different but quite general analysis of Archimedes' principle. This analysis explains the reduction in apparent weight of a submerged object in all cases, regardless of its position in the fluid. We also study the case in which the object rests on the bottom of the container where the net hydrostatic force on it is downward, and explain where in this case the reduction in the apparent weight comes from.

Keywords

Archimedes' Principle, Buoyancy, Apparent Weight

1. Introduction

Although the law of buoyancy was discovered by Archimedes over 2200 years ago, even today from time to time new articles appear in the literature inspecting its various aspects. More specifically, in the last two decade or so, more than a dozen papers have been published in different journals, ranging from pedagogical points of view [1] [2] to scrutinizing the original statements made by Archimedes [3] [4].

Archimedes' principle is one of the most essential laws of physics and fluid mechanics. Basically the principle states an object immersed in a fluid is buoyed up by a force equal to the weight of the fluid that it displaces. This principle, which is perhaps the most fundamental law in hydrostatics, explains many natural phenomena from both qualitative and quantitative points of view. The principle of isostasy, for example, which states that Earth's crust is in floating equilibrium with the denser mantle below [5] [6], is simply based on Archimedes' principle.

One of the applications of Archimedes' principle is in measurement of density of an irregularly shaped object. The simplest method is to use a graduated cylinder filled with water to a certain level. The object is then slowly lowered into

the cylinder until it becomes completely submerged. The increase in the level of water inside the cylinder is simply equal to the volume of the object. This method, however, requires that the diameter of the cylinder be at least as large as the diameter of the object, which reduces the accuracy of the measurement. In addition, this method certainly cannot be used to measure the volume of a large object such as a boulder. The problem, however, can be resolved by taking advantage of Archimedes' principle. A container partially filled with water is placed on a scale and the reading of the scale is recorded. The object is then hung from a string above the water, and slowly lowered into it until it is completely submerged, but without touching the bottom of the container (if the object is less dense than water, it can be pushed under water). The reading of the scale will increase by the mass of the displaced water (assuming that the scale measures mass), from which the volume of the object can be determined [7]. Alternatively, the object can be hung above water from a scale. As the object is lowered into water, the reading of the scale decreases by an amount equal to the mass of the displaced water. Thus, a boulder hanging from a spring or dial scale can be lowered into a large volume of water, such as a pond or a lake, and from the change of the reading of the scale, its volume can be determined.

Even though Archimedes' principle is over 2200 years old and despite its importance in hydrostatics, there are still some questions about it that have not yet been fully answered in the literature. For instance, debates are still going on regarding the interpretation of the principle when an object rests on the bottom of a fluid-filled container, where it experiences a net downward force by the fluid. It is therefore the objective of this article to derive the principle from a different point of view and answer some of the questions associated with the principle that have not been settled in the literature.

2. Derivations of Archimedes' Principle

A rigorous derivation of Archimedes' principle involves the concept of virtual work. In this method, the buoyant force is set equal to the negative of the gradient of the potential energy during an infinitesimal virtual displacement of the submerged object [8] [9]. This approach applies to objects of any shape; however, it has the limitation that the object must be completely surrounded by the fluid and that it should not be in contact with the container.

Alternatively, there are two simpler derivations of the principle [8]. One is based on the plausible argument that if the principle were not true, the subvolume of a fluid displaced by an object would not be in equilibrium. More specifically, the net fluid force on an arbitrarily shaped object would be the same as that on an equal volume of the fluid which was in equilibrium before it was displaced by the object [10] [11] [12] [13]. Therefore, the buoyant force is equal to the weight of the fluid displaced. This argument applies to any object of any shape regardless of its position in the container.

The second approach is based on the variation of hydrostatic pressure P_f as a function of depth of the fluid,

$$P_f = D_f gy \quad (1)$$

where D_f is density of the fluid, y is the depth, and g is the acceleration due to gravity. In this approach, an object of simple geometry such as a rectangular or cylindrical block is considered and the net fluid force due to the difference of hydrostatic pressure at the top and the bottom of the block is calculated [14]. This approach is simple; however, the proof for arbitrarily shaped objects is more involved as stated above [15], and it works only for objects that are completely surrounded by fluid. For objects resting on the bottom of the container, there is no fluid pressure in the contact area and the proof fails.

We now present a different quite general, yet simple derivation of Archimedes' principle that is valid regardless of the position of the object in the fluid. In this approach, the object can be completely surrounded by the fluid, be in contact with the walls of the container, rest on the bottom of the container, or even float in the fluid with only a fraction of its volume submerged.

3. A Different Approach to Archimedes' Principle

Consider a fluid of density D_f and an object of arbitrary shape of mass m and volume V , denser than the fluid. The fluid is in a container of cross-sectional area A and has a height H before the object enters it, as shown in **Figure 1(a)**. The object is supported by a string and, at this time, the tension in the string is mg .

Because fluid forces on the side walls of the container cancel, before the object enters the fluid, the net force F exerted by the fluid on the container is only due to the hydrostatic pressure at the bottom of the container, which is given by

$$F = D_f gHA. \quad (2)$$

Now we lower the object down into the fluid until it is submerged as shown in **Figure 1(b)**. This causes the height of the fluid in the container to increase by δH , where δH is given by

$$\delta H = \frac{V}{A} \quad (3)$$

Therefore, the net fluid force on the bottom of the container increases by δF , which is given by

$$\delta F = D_f g \delta HA = D_f gV \quad (4)$$

which is exactly equal to the weight of the fluid displaced. Therefore, when the object enters the fluid, the level of the fluid increases and the container experiences an additional downward fluid force equal to the weight of the fluid displaced by the object. This downward force can easily be detected by placing the container on a scale [7]. But then according to Newton's third law of motion, the container (through the fluid) exerts an equal upward force on the submerged object.

Therefore, regardless of its position, a submerged object experiences an

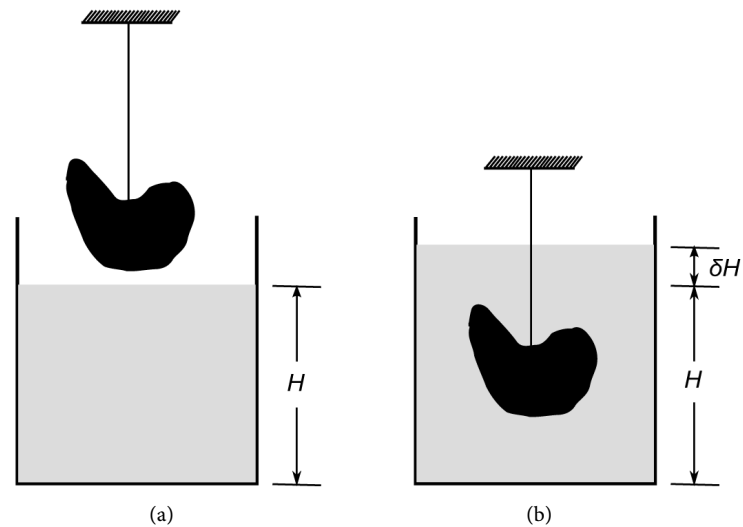


Figure 1. A fluid and an object of arbitrary shape which is denser than the fluid. (a) Before the object is lowered into the fluid, the height of the fluid is H ; (b) When the object is lowered into the fluid, the height of the fluid increases by δH .

upward force from the container-fluid system which is equal to the weight of the fluid displaced, $D_f gV$. Consequently, the tension in the string in **Figure 1(b)** and the reading of the scale in **Figure 2(b)** (to be explained later) would each be given by

$$F = mg - D_f gV \quad (5)$$

which is exactly the apparent weight of the object.

Note that the above analysis remains valid regardless of the position of the object in the fluid. Thus the object can be completely surrounded by the fluid, rest on the bottom of the container with no fluid under it, touch the walls of the container, or even float in the fluid with only a fraction of it submerged. In the case of a floating object, however, the volume V in the above equations should be taken to be the sub-volume of the object that is submerged.

A question that normally comes up during discussions of Archimedes' principle is that when an object in the form of a rectangular block rests on the bottom of a container with no fluid under it, where does the upward buoyant force come from? In fact, in this case because of the fluid pressure on top of the block, the net hydrostatic force on it would be *downward*, resulting in the apparent weight of the block to be greater than its true weight. But this conclusion is in complete contradiction with all observations since even in this case the apparent weight of the block is less than its true weight by the weight of the fluid displaced.

To resolve this contradiction, one may argue that in reality when a submerged object rests on the bottom of the container, there is almost always some fluid between the surfaces that appear to be in contact unless the surfaces are specially prepared and treated to prevent fluid seepage. This is because for ordinary flat surfaces, the actual area of contact is always much smaller than the apparent contact area [16] [17]. In fact the real contact area can be less than the apparent

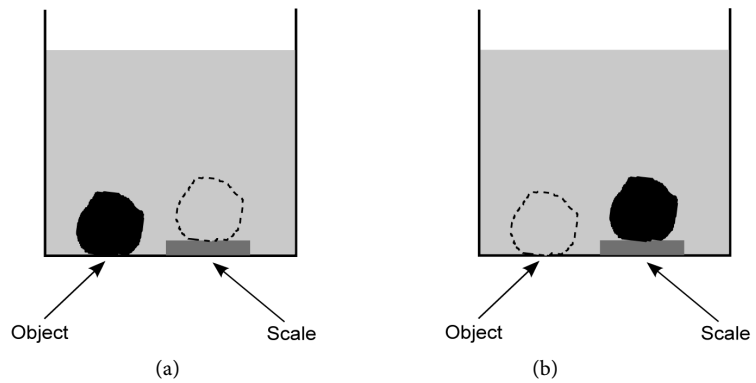


Figure 2. (a) An object denser than a fluid rests on the bottom of a container filled with the fluid. A weight scale is also located at the bottom of the container. The dashed region shows a volume of the fluid equal to the volume of the object; (b) The object has been moved to the top of the scale, displacing an equal volume of the fluid from that region.

macroscopic area by a factor of 10^4 [18]. However, if the surfaces are prepared properly to prevent fluid seepage between them, then obviously there is no fluid pressure there. Therefore, if a block rests on the bottom of the container with no fluid seepage between them, there would be no upward fluid force on the object and consequently there would be no buoyant force on it. Nonetheless, as explained below, experiments show that the even under these circumstances the apparent weight of the object is less than its true weight by the weight of the fluid displaced.

To resolve this apparent paradox, Jones and Gordon [19] designed an experiment to eliminate the upward fluid force on the bottom of a submerged object. They used an aluminum block resting on another aluminum block with highly flat contact surfaces. The surfaces were flat enough to prevent water from seeping between them but did not result in significant intermolecular forces between them [20]. They observed that the net fluid force on the object was in fact *downward*. Several years later, Bierman and Kincanon [3] re-examined this problem by using a submerged block in contact with the bottom of a container which had a hole in it, and studied the force needed to lift the block. Their experiment showed that this force increased linearly with the depth of the fluid, consistent with the laws of hydrostatics. Again, these experiments showed that the net fluid force on the object was indeed *downward*. Bierman and Kincanon concluded that in the statement of Archimedes' principle involving the buoyant force; it should be stressed that the submerged object must be surrounded by the fluid and not simply submerged.

What is missing in the interpretation of the experimental results of Jones and Gordon and of Bierman and Kincanon is that these experiments do not measure the apparent weight of the object. What they measure is the force needed to separate the object from the bottom of the container. This is similar to a suction cup sticking to a tabletop, where the net fluid (atmosphere) force on it is downward. The force needed to lift the suction cup straight up is much greater than the weight of the suction cup. To measure the weight of the suction cup, it must be

placed on a scale, regardless of whether air is driven out from under it or not.

One year later, Graf [4] argued that the reading of a scale located at the bottom of a container is the same when a submerged block is balanced on a thin pin and the pin bottom rests on the scale (where there is buoyant force) and when the block rests on the scale without any fluid seepage between them (where there is no buoyant force). He then concluded that when an object denser than a fluid is submerged in it, the apparent weight of the object is the same regardless of whether the submerged object rests on the bottom of the container or not. However, Graf did not explain where the upward force in the latter case comes from. In what follows, we address this issue and explain where in this case the reduction of the apparent weight comes from.

Consider an object of any shape of mass m and volume V resting at the bottom of a container filled with a fluid of density D_f , as shown in **Figure 2(a)**. A scale similar to that described by Graf [4] is also placed at the bottom of the container, and its tare function is used to zero its reading. The region enclosed by the dashed line contains a volume of the fluid that is equal to the volume of the object.

We now move the object and place it on the scale, as shown in **Figure 2(b)**. There may or may not be fluid seepage between the object and the scale, which is immaterial. As a result, the weight of the object mg is added to the scale but, at the same time, the weight of the fluid in the dashed region is removed from the top of the scale. Therefore, the reading of the scale S will be

$$S = mg - D_f gV \quad (6)$$

where the second term on the right hand side is the weight of the fluid in the dashed region. Consequently, the apparent weight of the object is less than its true weight by exactly the buoyant force on the object as if it was completely surrounded by the fluid. This analysis clearly shows where the reduction in the apparent weight in this case comes from; it comes from removal of a volume of fluid, equal to the volume of the object, from the region directly above the scale.

4. Discussion and Summary

In this article, we have looked at Archimedes' principle from a different, but quite general, perspective in the context of Newton's third law of motion. When an object enters a fluid in a container, the height of the fluid increases, resulting in a higher hydrostatic pressure and hence a higher downward force on the bottom of the container. Then according to Newton's third law, the container-fluid system exerts an equal upward force on the object resulting in the reduction of its apparent weight, regardless of the position of the object in the fluid. We have also shown where the reduction of the apparent weight of a submerged object comes from, when the object rests on the bottom of the container with no fluid seepage between them. The analysis presented here helps clarify why Archimedes' principle works the way it does, and why a submerged object appears to be lighter even when the net fluid force on it is downward.

Finally, we point out that Archimedes' principle does not consider surface tension. In fact, presence of surface tension results in violation of the principle [21]. Furthermore, Archimedes' principle breaks down in complex fluids [22].

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