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Whispering Gallery Modes Formed by Scattering of an Electromagnetic Plane Wave by Two Cylinders

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Abstract

We report the effect of scattering of electromagnetic plane waves by two cylinders on whispering gallery mode (WGM) formation in a cylinder. WGM can occur because of the presence of additional cylinder scatterers at specific location, while WGMs can only form in a single cylinder for specific cylinder radius and/or wavelength values, the matching accuracy required would be much greater than that required in our model for the additional cylinders locations. Analysis of the general solution to the problem showed that the effect can be explained by the interference of waves scattered by additional cylinders and incident on the main cylinder.

Keywords

Scattering, Wave, Cylinder, WGM

1. Introduction

The term “whispering gallery modes” (WGMs) was introduced by Lord Rayleigh to explain the effects of sound propagation a circular gallery [1]. The name itself reflects the fact that sound in enclosed spaces can propagate along such concave walls. Laser radiation that is attached to the perimeter of a miniature disk by multiple reflections from concave walls can be treated as an optical analogue of the “whispering gallery”. If the reflection surface is sufficiently smooth, the radiation then propagates with minimal losses. However, because the lateral surface is not smooth and has a nonzero radius of curvature, part of the wave then leaks out. When the radius of curvature increases, more radiation remains inside the

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disk. WGMs can thus be used to create electromagnetic cavities, as initially noted by R. Richtmyer [2]. This spherical form is the simplest form that can be used in fabrication of a resonator using WGMs. The interaction of spherical particles with electromagnetic waves has been studied theoretically for more than a century, dating back to the work of Mi [3], who considered the scattering of light by spherical particles, and Debye, who studied the scattering of waves on a sphere in the form of a series of refracted and reflected waves of various order [4]. However, despite the fact that this problem (the scattering of waves on an axisymmetric particle) is well known and has been studied for a long time, new and important results were obtained in 2004: the authors of Ref. [5] found and studied a narrow, high-intensity beam of light (called a photonic nanojet) that was generated at the shadow-side surfaces of dielectric cylinders that were illuminated by a plane wave. The renewed interest to cylinder scattering of plane waves led to a detailed study of the conditions required to produce a WGM. V.V. Kotlyar *et al.* [6] found that the WGM formed a focal spot outside the cylinder, and also determined the contributions of the cylinder eigenmodes to WGM formation. In general, fundaments of WGM propagation and its applications described in Ref. [7]. The effect of multiple cylinders scattering of an electromagnetic plane-wave on the formation of high field intensity areas studied in [8].

2. Modeling and Results

Our model consists of two cylinders. One of these (marked as A on **Figure 1**) is a basic cylinder, within which we consider WGM formation. The other cylinder (marked B on **Figure 1**) has an assistive function. The position of cylinder B is varied relative to the basic cylinder with the purpose of finding a location at which the intensity of the WGM increases. Our model, including all distances and notations, is presented in **Figure 1**. The wave is incident from the left side on the pair of cylinders. The wave propagation direction is along the axis of symmetry of the cylinders, and is selected as the x axis. The origin of coordinates corresponds to the center of main cylinder A. The propagation and scattering of the electromagnetic plane wave was studied using MATLAB toolbox that was developed in Ref. [9]. A transverse-electric (TE) polarized plane wave ($\lambda = 532$ nm)

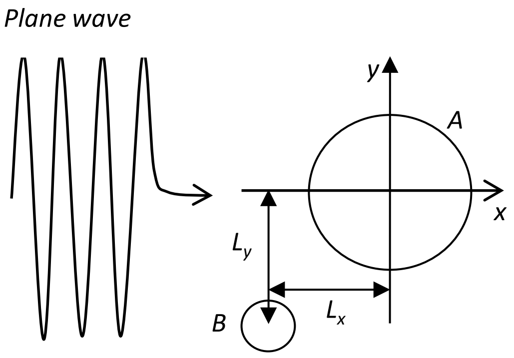
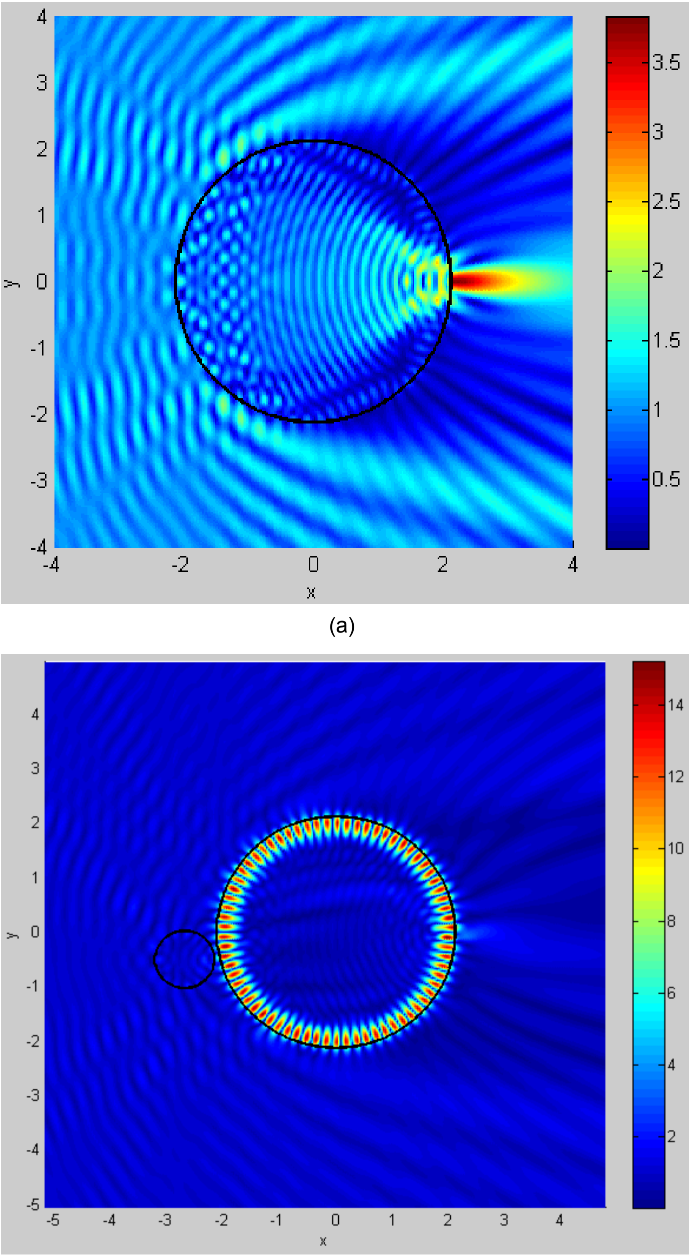


Figure 1. Geometry to simulate scattering of plane wave by two cylinders.

was used as the incident wave. The mesh grid size in the space was equal to 0.04 μm (0.075 of wavelength). The electric permittivity $\varepsilon = 1.59$ (quartz glass), and cylinder's A radius $R_A = 4\lambda$. For cylinder B (where the latter is introduced below), the radii $R_B = 0.25\ R_A$ were used. All distances below are measured in μm.

The calculated distribution of the absolute field intensity value is shown on **Figure 2(a)**. As shown, the scattering process leads to the formation of a photonic jet for the single cylinder A. Next we used additional cylinder B. An image for comparison of the single cylinder case with that of our model is shown in **Figure 2(b)** ($L_x = -2.7$, $L_y = 0.5$).



(a)

(b)

Figure 2. Distribution of the absolute value of total field scattered by one (a) and two (b) dielectric cylinders.

To determine how the positions of cylinder *B* affect the maximal absolute field value inside the cylinder *A*, we varied its positions by moving the centers within the ring defined in polar coordinates as interval of radii [$R_A + R_B$, $R_A + 1.6^* R_B$]. Each step in the ring was 0.06 $R_B/10$ along the radius, with an angle step of $(\pi/2)/75$. The resulting picture is shown in **Figure 3**. The positions of cylinder *B* that were used in **Figure 2** correspond with the data used for **Figure 3**.

3. Formation of WGM Due to Scattering on Neighboring Cylinder

We associate the near-surface area of high field intensity with WGM propagation, because the specific feature of WGMs is that the high intensity field in these modes is concentrated near the cavity walls. The absence of high field intensity areas inside a single scattering cylinder *A* indicates that WGM is caused by the presence of the additional cylinder *B*.

In general, the WGM are characterized by the specific value of the following relationship: cylinder radius/wavelength. As an example we consider simple expression $2\pi Rn/\lambda = T_{ml}$ [10], where T_{ml} is the *l*th root of the *m*th order Bessel function. This means that if the wavelength is known, then to determine the propagation of (*ml*) mode we must choose the cylinder radius based on the expression above. Additionally, when this mode makes a larger contribution to the field intensity, then the radius must be defined more accurately [6] [8]. For example, for a mode with $m = 15$, when the intensity is increased by eight times, then the accuracy of the matching radius is $10^{-4}\ \lambda$ [8]. It therefore follows from **Figure 3** that to attain the same increase in intensity, it is sufficient to determine the location within an accuracy of within a few percent of the wavelength value.

In the case of two or more cylinders, the equation for the derivation the WGM will contain a contribution caused by the presence of satellite cylinder. First of all, this means that now WGM are derived by many parameters (radii of the satellite cylinders, distance between cylinders, their mutual orientation, dielectric permittivity), but not only by a relation R/λ, as it was for single cylinder (it is confirmed by comparison of **Figure 2(a)** and **Figure 3**. Next, due to the presence of satellite cylinder WGM are formed by the interference of incident waves

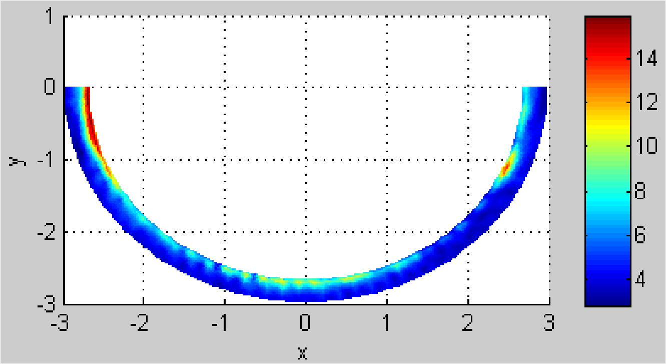


Figure 3. Distribution of the maximal value of total field inside cylinder A versus location of cylinder B.

and waves scattered by additional cylinder. The latter also follows from the general formulas describing multiple cylinders scattering. Now we analyze the general solution to the scattering problem on multiple cylinders. Our task in this analysis is reduced to an assessment of the contributions to the field inside cylinder A of the components that are associated with cylinder B. Let's consider the expression that corresponds to the solutions to Maxwell's equations for our model, which were given in [11]:

$$H_z^l = ik_l a_l \sum_{n=-\infty}^{\infty} (-i)^n e^{in\gamma_{lP}} J_n(k_l R_{lP}) A_{ln} \tag{1}$$

where $k_m = 2\pi/\lambda$, $a_l = R_A$, J_n – Bessel function of first kind, R_{lp} – distance between point P inside lth cylinder and its center, and

$$A_{ln} = \left(j_n(k_l a_l) m^2\right)^{-1} \cdot \left(\varepsilon_l e^{in\phi} j_n(k_m a_l) - a_{ln} H_n(k_m a_l) - j_n(k_m a_l) \sum_{\substack{j=1 \\ j \neq l}}^{N} \sum_{s=-\infty}^{s=\infty} (-i)^{s-n} H_{s-n}(k_m R_{lj}) e^{i(s-n)\gamma_{lj}} a_{js} \right) \tag{2}$$

The expansion coefficients (a_{jn}, b_{jn}) are related to the single cylinder scattering coefficients (a_{jn}^0, b_{jn}^0) and can be obtained by solving the following equation system [11]:

$$\sum_{l=1}^{N} \sum_{s=-\infty}^{\infty} \left(\delta_{lj} \delta_{ns} + (1 - \delta_{lj}) G_{ls}^{jn} a_{jn}^0 \right) a_{ls} = \eta_j e^{in\phi} a_{jn}^0 \tag{3}$$

where $G_{ls}^{jn} = (-i)^{s-n} H_{s-n}(k_m R_{lj}) \mathrm{e}^{i(s-n)\gamma_{lj}}$, $h_j = \exp\left(-ikm\left(x_j \cos(j) - y_j \sin(j)\right)\right)$, δ denotes the Kronecker delta.

Equation (3) allows us to conclude that in the presence of an additional cylinder the amplitude of a certain mode of the first cylinder contains contributions from all modes of the second cylinder. These contributions decrease as the distance between the cylinders increases. Therefore, the resonance is observed only when the location of the second cylinder is close to the edge of the first cylinder.

We now consider why the intensity is maximal in a narrow strip near the edge, but does not decrease gradually if the additional cylinder moves away from the main cylinder. To find a solution, we consider Equation (1) assuming asymptotic expressions for the Bessel and Hankel functions for large values of their arguments. Indeed, the arguments are much more than *1* for the parameters of our model (for example: $k_m a_l = 2\pi/\lambda\ R_A = 2\pi/\lambda * 4\lambda = 8\pi$; $k_l R_{lj} < 2\pi/\lambda * (R_A + R_B) > 8\pi$). Additionally, in Equation (1), we use only the second term of Equation (2) for A_{ln}, and only the first term in Equation (3) for a_{1n}. From the above, the contribution to the field of the 2-nd cylinder is described as follows:

$$\begin{aligned} H_z^l \propto & \sum_{n=-\infty}^{\infty} \exp\left(in\left(\gamma_{lP} - \pi/2\right) \exp\left(\pm i\left(k_l R_{lP} - \pi n/2 - \pi/4\right)\right)\right) \\ & \exp\left(-i\left(k_m R_{lj} - \pi/4\right)\right) \sum_{s=-\infty}^{\infty} a_{2s} \exp\left(i(s-n)\gamma_{lj}\right) \end{aligned} \tag{4}$$

Here we have deal with sum of waves with different phases and amplitudes. A well-known gain condition leads to the following equality

$$\pm(k_l R_{lP} - \pi/4) - k_m R_{lj} + \pi/4 = 2\pi p \qquad (5)$$

where p represents integer numbers. Analysis of this equation for different signs leads to the relationship $R_{lj} - (R_A + R_B) < \lambda$. This means that the maximum field intensity value occurs if the distance between the centers of the cylinders does not exceed the minimal possible value more than wavelength. **Figure 3** confirms the estimates given by Equations (4) and (5).

4. Conclusions

We have simulated the scattering of a plane wave by the pair of cylinders. It was found that WGMs can be formed inside the basic cylinder in the presence of an additional cylinder, without any special requirement for the wavelength or for the radii of the cylinders. However, additional cylinder must be located in specific positions near the edge of main cylinder. The accuracy required for these cylinder locations is much lower than the accuracy required for the setting of the resonant radius of the cylinder at which the WGMs can be observed. Analysis of the general solution to our model has shown that these effects can be explained by the interference of the waves that are scattered by the extra cylinder. Therefore, our work describes a new method for the formation of WGMs in a cylinder.

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