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Double-Peakon Solutions of Two Four-Component Camassa-Holm Type Equations

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Abstract

This paper is contributed to study two new integrable four-component systems reduced from a multi-component generation of Camassa-Holm equation. Some double peakon solutions of both systems are described in an explicit formula by the method of variation of constant for ordinary differential equations. These double peakon solutions are established in weak sense. The dynamic behaviors of the obtained double peakon solutions are illustrated by some figures.

Keywords

Peakon Solution, Dynamic Behavior, Four-Component CH Type Equation

1. Introduction

In 1993, Camassa and Holm derived the celebrated Camassa-Holm (CH) equation [1], which is formally integrable, since it admits Lax pair formalism [1], bi-Hamiltonian structure [2] as well as infinitely many conservation laws [2]. One of the remarkable properties of the equation is that it possesses peakon wave solutions. Subsequently, a large amount of literature was devoted to find new integrable models with peakon solutions, such as Degasperis-Procesi (DP) equation, the Fokas-Olver-Rosenau-Qiao (FORQ) equation, the Novikov's cubic nonlinear equation and some other completely integrable peakon systems. It is a natural idea to continue studying multi-component generalizations of peakon equations. One of the most popular two-component integrable systems, which admits Lax Pair and infinitely conservation laws, has multi-peakon solitons [3]. A three-component model with peakon solutions has been studied by Geng and Xue [4].

Very recently, in Ref. [5], another multi-component system of Camassa-Holm equation, which admited Lax pair and infinitely many conservation laws, denoted by CH(N,H) with $2N$ components and an arbitrary smooth function H of $u_j, v_j (1 \le j \le N)$ and their derivatives, was derived and studied

$$m_{j,t} = (m_j H)_x + m_j H + \frac{1}{(N+1)^2} \sum_{i=1}^{N} [m_i (u_j - u_{j,x})(v_i + v_{i,x}) + m_j (u_i - u_{i,x})(v_i + v_{i,x})],$$

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$$n_{j,t}=(n_jH)_x-n_jH-\frac{1}{(N+1)^2}\sum_{i=1}^{N}[n_i(u_i-u_{i,x})(v_j+v_{j,x})+n_j(u_i-u_{i,x})(v_i+v_{i,x})],$$

$$m_j=u_j-u_{j,xx},\ n_j=v_j-v_{j,xx},\ 1\le j\le N. \tag{1}$$

Particularly, in the case of $H=0, N=2$, Equation (1) becomes the following system

$$\begin{aligned}
m_{1t}&=\frac{1}{9}\{m_1[2(u_1-u_{1,x})(v_1+v_{1,x})+(u_2-u_{2,x})(v_2+v_{2,x})]+m_2(u_1-u_{1,x})(v_2+v_{2,x})\},\\
m_{2t}&=\frac{1}{9}\{m_1(u_2-u_{2,x})(v_1+v_{1,x})+m_2[(u_1-u_{1,x})(v_1+v_{1,x})+2(u_2-u_{2,x})(v_2+v_{2,x})]\},\\
n_{1t}&=-\frac{1}{9}\{n_1[2(u_1-u_{1,x})(v_1+v_{1,x})+(u_2-u_{2,x})(v_2+v_{2,x})]+n_2(u_2-u_{2,x})(v_1+v_{1,x})\},\\
n_{2t}&=-\frac{1}{9}\{n_1(u_1-u_{1,x})(v_2+v_{2,x})+n_2[(u_1-u_{1,x})(v_1+v_{1,x})+2(u_2-u_{2,x})(v_2+v_{2,x})]\},\\
m_1&=u_1-u_{1,xx},\ m_2=u_2-u_{2,xx},\ n_1=v_1-v_{1,xx},\ n_2=v_2-v_{2,xx},
\end{aligned} \tag{2}$$

and in the case of $H=-\frac{1}{9}[(u_1-u_{1,x})(v_1+v_{1,x})+(u_2-u_{2,x})(v_2+v_{2,x})], N=2$, it is reduced into

$$\begin{aligned}
m_{1t}&=(m_1H)_x+\frac{1}{9}m_1(u_1-u_{1,x})(v_1+v_{1,x})+\frac{1}{9}m_2(u_1-u_{1,x})(v_2+v_{2,x}),\\
m_{2t}&=(m_2H)_x+\frac{1}{9}m_1(u_2-u_{2,x})(v_1+v_{1,x})+\frac{1}{9}m_2(u_2-u_{2,x})(v_2+v_{2,x}),\\
n_{1t}&=(n_1H)_x-\frac{1}{9}n_1(u_1-u_{1,x})(v_1+v_{1,x})-\frac{1}{9}n_2(u_2-u_{2,x})(v_1+v_{1,x}),\\
n_{2t}&=(n_2H)_x-\frac{1}{9}n_1(u_1-u_{1,x})(v_2+v_{2,x})-\frac{1}{9}n_2(u_2-u_{2,x})(v_2+v_{2,x}),\\
m_1&=u_1-u_{1,xx},\ m_2=u_2-u_{2,xx},\ n_1=v_1-v_{1,xx},\ n_2=v_2-v_{2,xx}.
\end{aligned} \tag{3}$$

Xia and Qiao have presented bi-Hamiltonian structures and single peakon solutions [5] of Equations (2) and (3). According to the work in [5], we will investigate the double peakon solutions of Equations (2) and (3) in this paper. Section 2 is devoted to look for double peakon solutions of Equations (2) and (3). Further, we discuss the dynamic behaviors of the obtained peakon solutions by some figures. Some conclusions and open problems are addressed in Section 3.

2. Two-Peakon Solutions

2.1. Two-Peakon Solutions to Equation (2)

We assume that the system Equation (2) admits two peakon solitons of the form

$$\begin{aligned}
u_1&=p_1e^{-|x-q_1|}+p_2e^{-|x-q_2|},\ u_2=r_1e^{-|x-q_1|}+r_2e^{-|x-q_2|},\\
v_1&=s_1e^{-|x-q_1|}+s_2e^{-|x-q_2|},\ v_2=w_1e^{-|x-q_1|}+w_2e^{-|x-q_2|},
\end{aligned} \tag{4}$$

where $p_i(t)$, $r_i(t)$, $s_i(t)$, $w_i(t)$ and $q_i(t)$ $(i=1,2)$, are functions of t to be determined. Moreover, we can obtain their derivatives in the weak sense as follows

$$\begin{aligned}
u_{1,x}&=-p_1sgn(x-q_1)e^{-|x-q_1|}-p_2sgn(x-q_2)e^{-|x-q_2|},\\
u_{2,x}&=-r_1sgn(x-q_1)e^{-|x-q_1|}-r_2sgn(x-q_2)e^{-|x-q_2|},\\
v_{1,x}&=-s_1sgn(x-q_1)e^{-|x-q_1|}-s_2sgn(x-q_2)e^{-|x-q_2|},\\
v_{2,x}&=-w_1sgn(x-q_1)e^{-|x-q_1|}-w_2sgn(x-q_2)e^{-|x-q_2|},\\
m_1&=2p_1\delta(x-q_1)+2p_2\delta(x-q_2),\ m_2=2r_1\delta(x-q_1)+2r_2\delta(x-q_2),
\end{aligned} \tag{5}$$

$$n_1 = 2s_1\delta(x-q_1)+2s_2\delta(x-q_2),\ n_2 = 2w_1\delta(x-q_1)+2w_2\delta(x-q_2).$$

Without loss of generality, we assume that $q_1 > q_2$. Substituting Equations (4) and (5) into Equation (2) and in the distribution sense, we can obtain the following double peakon differential equations

$$\begin{aligned}
&q_{1,t} = q_{2,t} = 0,\\
&p_{1,t} = \frac{4}{27}p_1\Delta_{11} + \frac{2}{9}p_1\Delta_{21}e^{q_2-q_1} + \frac{2}{9}w_1e^{q_2-q_1}\Delta,\quad p_{2,t} = \frac{4}{27}p_2\Delta_{22} + \frac{2}{9}p_2\Delta_{21}e^{q_2-q_1},\\
&r_{1,t} = \frac{4}{27}r_1\Delta_{11} + \frac{2}{9}r_1\Delta_{21}e^{q_2-q_1} - \frac{2}{9}s_1e^{q_2-q_1}\Delta,\quad r_{2,t} = \frac{4}{27}r_2\Delta_{22} + \frac{2}{9}r_2\Delta_{21}e^{q_2-q_1},\\
&s_{1,t} = -\frac{4}{27}s_1\Delta_{11} - \frac{2}{9}s_1\Delta_{21}e^{q_2-q_1},\quad s_{2,t} = -\frac{4}{27}s_2\Delta_{22} - \frac{2}{9}s_2\Delta_{21}e^{q_2-q_1} - \frac{2}{9}r_2e^{q_2-q_1}\Lambda,\\
&w_{1,t} = -\frac{4}{27}w_1\Delta_{11} - \frac{2}{9}w_1\Delta_{21}e^{q_2-q_1},\quad w_{2,t} = -\frac{4}{27}w_2\Delta_{22} - \frac{2}{9}w_2\Delta_{21}e^{q_2-q_1} + \frac{2}{9}p_2e^{q_2-q_1}\Lambda,
\end{aligned} \tag{6}$$

where $\Delta_{11} = p_1s_1 + r_1w_1$, $\Delta_{22} = p_2s_2 + r_2w_2$, $\Delta_{21} = 2(p_2s_1 + r_2w_1)$, $\Delta = r_1p_2 - p_1r_2$ and $\Lambda = s_1w_2 - s_2w_1$. Δ_{11} and Δ_{22} taking derivative with respect to t ,we can obtain

$$\Delta_{11,t} = 0,\quad \Delta_{22,t} = 0, \tag{7}$$

Thus we have

$$\Delta_{11} = A_1,\quad \Delta_{22} = B_1, \tag{8}$$

where A_1 and B_1 are arbitrary integration constants. In the following, we assume that $A_1 = B_1$.

Δ_{21} taking derivative with respect to t and combining with Equation (6), we see

$$\Delta_{21,t} = 0. \tag{9}$$

Thus, we can get

$$\Delta_{21} = C_1, \tag{10}$$

where C_1 is arbitrary constant.

Combining the Equation (6) with Equations (8) and (10), Equation (6) is reduced to

$$\begin{aligned}
&q_1 = m\quad q_2 = n,\\
&p_{1,t} = \frac{4}{27}p_1A_1 + \frac{2}{9}p_1C_1e^{n-m} + \frac{2}{9}w_1e^{n-m}\Delta,\quad p_{2,t} = \frac{4}{27}p_2A_1 + \frac{2}{9}p_2C_1e^{n-m},\\
&r_{1,t} = \frac{4}{27}r_1A_1 + \frac{2}{9}r_1C_1e^{n-m} - \frac{2}{9}s_1e^{n-m}\Delta,\quad r_{2,t} = \frac{4}{27}r_2A_1 + \frac{2}{9}r_2C_1e^{n-m},\\
&s_{1,t} = -\frac{4}{27}s_1A_1 - \frac{2}{9}s_1C_1e^{n-m},\quad s_{2,t} = -\frac{4}{27}s_2A_1 - \frac{2}{9}s_2C_1e^{n-m} - \frac{2}{9}r_2e^{n-m}\Lambda,\\
&w_{1,t} = -\frac{4}{27}w_1A_1 - \frac{2}{9}w_1C_1e^{n-m},\quad w_{2,t} = -\frac{4}{27}w_2A_1 - \frac{2}{9}w_2C_1e^{n-m} + \frac{2}{9}p_2e^{n-m}\Lambda,
\end{aligned} \tag{11}$$

where m, n are integration constants. According to Equation (11), we can arrive at

$$\begin{aligned}
&p_2 = C_2e^{(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t},\quad r_2 = C_3e^{(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t},\\
&s_1 = C_4e^{-(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t},\quad w_1 = C_5e^{-(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t},
\end{aligned} \tag{12}$$

where C_2, C_3, C_4 and C_5 denote integration constants. Moreover, with the help of Equation (11), we easily obtain that

$$\begin{aligned}
&\Delta_t = (\frac{8}{27}A_1 + \frac{4}{9}C_1e^{n-m})\Delta - \frac{2}{9}(C_3C_5 + C_2C_4)e^{n-m}\Delta,\\
&\Lambda_t = -(\frac{8}{27}A_1 + \frac{4}{9}C_1e^{n-m})\Lambda + \frac{2}{9}(C_3C_5 + C_2C_4)e^{n-m}\Lambda.
\end{aligned} \tag{13}$$

Solving the differential equations of Equation (13), we get

$$\Delta = C_6 e^{(\frac{8}{27}A_1+\frac{4}{9}C_1e^{n-m}-\frac{2}{9}C_2C_4e^{n-m}-\frac{2}{9}C_3C_5e^{n-m})t},$$
$$\Lambda = C_7 e^{-(\frac{8}{27}A_1+\frac{4}{9}C_1e^{n-m}-\frac{2}{9}C_2C_4e^{n-m}-\frac{2}{9}C_3C_5e^{n-m})t}. \tag{14}$$

Using the method of variation of constant, we can have

$$p_1 = [-\frac{C_5C_6}{C_2C_4+C_3C_5}e^{-\frac{2}{9}(C_2C_4+C_3C_5)e^{n-m}t} + C_8]e^{(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t},$$
$$r_1 = [\frac{C_4C_6}{C_2C_4+C_3C_5}e^{-\frac{2}{9}(C_2C_4+C_3C_5)e^{n-m}t} + C_9]e^{(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t},$$
$$s_2 = [-\frac{C_3C_7}{C_2C_4+C_3C_5}e^{\frac{2}{9}(C_2C_4+C_3C_5)e^{n-m}t} + C_{10}]e^{-(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t},$$
$$w_2 = [\frac{C_2C_7}{C_2C_4+C_3C_5}e^{\frac{2}{9}(C_2C_4+C_3C_5)e^{n-m}t} + C_{11}]e^{-(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}. \tag{15}$$

Based on Equations (8) and (10), those constants have relations as

$$C_2C_4 + C_3C_5 = \frac{C_1}{2},\quad C_2C_{10} + C_3C_{11} = C_8C_4 + C_9C_5 = A_1. \tag{16}$$

Thus, we establish the double peakon solutions of the Equation (2)

$$u_1 = [-\frac{C_5C_6}{C_2C_4+C_3C_5}e^{-\frac{2}{9}(C_2C_4+C_3C_5)e^{n-m}t} + C_8]e^{(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}e^{-|x-m|}$$
$$+ C_2e^{(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}e^{-|x-n|},$$
$$u_2 = [\frac{C_4C_6}{C_2C_4+C_3C_5}e^{-\frac{2}{9}(C_2C_4+C_3C_5)e^{n-m}t} + C_9]e^{(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}e^{-|x-m|}$$
$$+ C_3e^{(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}e^{-|x-n|},$$
$$v_1 = C_4e^{-(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}e^{-|x-m|}$$
$$+[-\frac{C_3C_7}{C_2C_4+C_3C_5}e^{\frac{2}{9}(C_2C_4+C_3C_5)e^{n-m}t} + C_{10}]e^{-(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}e^{-|x-n|},$$
$$v_2 = C_5e^{-(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}e^{-|x-m|}$$
$$+[\frac{C_2C_7}{C_2C_4+C_3C_5}e^{\frac{2}{9}(C_2C_4+C_3C_5)e^{n-m}t} + C_{11}]e^{-(\frac{4}{27}A_1+\frac{2}{9}C_1e^{n-m})t}e^{-|x-n|}, \tag{17}$$

where C_i and A_1 $(i = 1\cdots 11)$ satisfy Equation (16).

Figure 1 show the profiles of the double peakon solutions Equation (17). The amplitudes of the peakons grow or decay exponentially with time *t*. All peak positions don't change along with the time *t* and the collision between the two-peakon waves will never happen.

2.2. Double Peakon Solutions to Equation (3)

By means of the similar calculation as those in the Section 2.1, taking $q_1 > q_2$, we arrive at the differential equations as follows

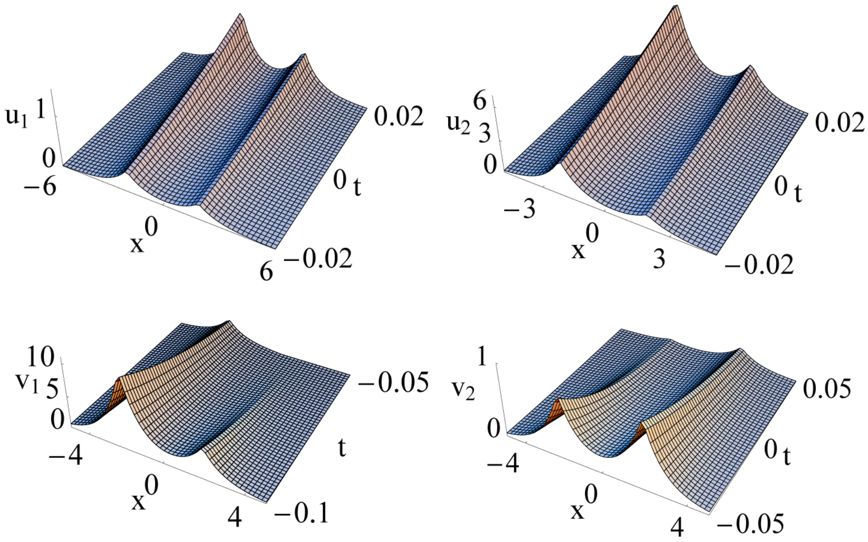

Figure 1. 3D graphs of the double peakon solutions for u_i, v_i, $i = 1, 2$, defined by Equation (17) with $C_1 = 2, C_2 = 1/2, C_3 = 2, C_4 = 1, C_5 = 1/4, C_6 = 1, C_7 = 1, C_8 = 1, C_9 = 4, C_{10} = 1, C_{11} = 0, A_1 = 2, m = -2, n = 2$.

$$
\begin{aligned}
&q_{1,t} = \frac{2}{27}a_1 + \frac{2}{9}(p_2 s_1 + r_2 w_1)e^{-|q_1-q_2|}, \quad q_{2,t} = \frac{2}{27}b_1 + \frac{2}{9}(p_2 s_1 + r_2 w_1)e^{-|q_1-q_2|},\\
&p_{1,t} = \frac{2}{27}p_1 a_1 + \frac{2}{9}p_2 a_1 e^{-|q_1-q_2|}, \quad p_{2,t} = \frac{2}{27}p_2 b_1 + \frac{2}{9}p_2(p_2 s_1 + r_2 w_1)e^{-|q_1-q_2|},\\
&r_{1,t} = \frac{2}{27}r_1 a_1 + \frac{2}{9}r_2 a_1 e^{-|q_1-q_2|}, \quad r_{2,t} = \frac{2}{27}r_2 b_1 + \frac{2}{9}r_2(p_2 s_1 + r_2 w_1)e^{-|q_1-q_2|},\\
&s_{1,t} = -\frac{2}{27}s_1 a_1 - \frac{2}{9}s_1(p_2 s_1 + r_2 w_1)e^{-|q_1-q_2|}, \quad s_{2,t} = -\frac{2}{27}s_2 b_1 - \frac{2}{9}s_1 b_1 e^{-|q_1-q_2|},\\
&w_{1,t} = -\frac{2}{27}w_1 a_1 - \frac{2}{9}w_1(p_2 s_1 + r_2 w_1)e^{-|q_1-q_2|}, \quad w_{2,t} = -\frac{2}{27}w_2 b_1 - \frac{2}{9}w_1 b_1 e^{-|q_1-q_2|},
\end{aligned}
\tag{18}
$$

where a_1 and b_1 are constants. From which, it is easy to see that we may have the following relations

$$
\begin{aligned}
&\frac{w_1}{s_1} = C_1, \quad \frac{p_2}{r_2} = C_2, \quad \frac{p_{2,t}}{p_2} = q_{2,t},\\
&\frac{s_{1,t}}{s_1} = -q_{1,t}, \quad q_1 - q_2 = \frac{2}{27}(a_1 - b_1)t + C_4.
\end{aligned}
\tag{19}
$$

Supposing that $a_1 = b_1$ and letting $\Omega_{21} = p_2 s_1 + r_2 w_1$, we can readily see $\Omega_{21,t} = 0, \Omega_{21} = C_3$. Thus, the t-dependent functions satisfy the following ordinary differential equations

$$
\begin{aligned}
&q_{1,t} = \frac{2}{27}a_1 + \frac{2}{9}C_3 e^{-C_4}, \qquad q_{2,t} = \frac{2}{27}a_1 + \frac{2}{9}C_3 e^{-C_4},\\
&p_{1,t} = \frac{2}{27}p_1 a_1 + \frac{2}{9}p_2 a_1 e^{-C_4}, \qquad p_{2,t} = \frac{2}{27}p_2 a_1 + \frac{2}{9}C_3 p_2 e^{-C_4},\\
&r_{1,t} = \frac{2}{27}r_1 a_1 + \frac{2}{9}r_2 a_1 e^{-C_4}, \qquad r_{2,t} = \frac{2}{27}r_2 a_1 + \frac{2}{9}r_2 C_3 e^{-C_4},\\
&s_{1,t} = -\frac{2}{27}s_1 a_1 - \frac{2}{9}s_1 C_3 e^{-C_4}, \qquad s_{2,t} = -\frac{2}{27}s_2 a_1 - \frac{2}{9}s_1 a_1 e^{-C_4},\\
&w_{1,t} = -\frac{2}{27}w_1 a_1 - \frac{2}{9}w_1 C_3 e^{-C_4}, \qquad w_{2,t} = -\frac{2}{27}w_2 a_1 - \frac{2}{9}w_1 a_1 e^{-C_4}.
\end{aligned}
\tag{20}
$$

Thus, we obtain the double peakon solutions of Equation (3)

$$
\begin{aligned}
u_1 &= [\frac{C_6 a_1}{C_3} e^{\frac{2}{9}C_3 e^{-C_4 t}} + C_8] e^{\frac{2}{27}a_1 t} e^{-|x-\frac{2}{27}a_1 t-\frac{2}{9}C_3 e^{-C_4 t}-C_5|} \\
&\quad + C_6 e^{(\frac{2}{27}a_1+\frac{2}{9}C_3 e^{-C_4})t} e^{-|x-\frac{2}{27}a_1 t-\frac{2}{9}C_3 e^{-C_4 t}-C_5+C_4|}, \\
u_2 &= [\frac{C_6 a_1}{C_2 C_3} e^{\frac{2}{9}C_3 e^{-C_4 t}} + C_9] e^{\frac{2}{27}a_1 t} e^{-|x-\frac{2}{27}a_1 t-\frac{2}{9}C_3 e^{-C_4 t}-C_5|} \\
&\quad + \frac{C_6}{C_2} e^{(\frac{2}{27}a_1+\frac{2}{9}C_3 e^{-C_4})t} e^{-|x-\frac{2}{27}a_1 t-\frac{2}{9}C_3 e^{-C_4 t}-C_5+C_4|}, \\
v_1 &= C_7 e^{-(\frac{2}{27}a_1+\frac{2}{9}C_3 e^{-C_4})t} e^{-|x-\frac{2}{27}a_1 t-\frac{2}{9}C_3 e^{-C_4 t}-C_5|} \\
&\quad + [\frac{C_7 a_1}{C_3} e^{-\frac{2}{9}C_3 e^{-C_4 t}} + C_{10}] e^{-\frac{2}{27}a_1 t} e^{-|x-\frac{2}{27}a_1 t-\frac{2}{9}C_3 e^{-C_4 t}-C_5+C_4|}, \\
v_2 &= C_7 C_1 e^{-(\frac{2}{27}a_1+\frac{2}{9}C_3 e^{-C_4})t} e^{-|x-\frac{2}{27}a_1 t-\frac{2}{9}C_3 e^{-C_4 t}-C_5|} \\
&\quad + [\frac{C_7 C_1 a_1}{C_3} e^{-\frac{2}{9}C_3 e^{-C_4 t}} + C_{11}] e^{-\frac{2}{27}a_1 t} e^{-|x-\frac{2}{27}a_1 t-\frac{2}{9}C_3 e^{-C_4 t}-C_5+C_4|},
\end{aligned}
\tag{21}
$$

where $C_2 = C_6$, $C_6 C_7 + C_7 C_1 = C_3$, $C_8 + C_1 C_9 = 0$ and $C_{11} + C_2 C_{10} = 0$.

3. Conclusion

We provide an approach to obtain the double peakon solutions for the four-component CH type Equations (2) and (3) in the case of $\Delta_{11} = \Delta_{22}$ and $a_1 = b_1$ respectively. However, its exact double peakon solutions without $\Delta_{11} = \Delta_{22}$ or $a_1 = b_1$ are expected to attract more endeavor to study.

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