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Existence of Traveling Waves in Lattice Dynamical Systems

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Abstract

Existence of traveling wave solutions for some lattice differential equations is investigated. We prove that there exists $c_* > 0$ such that for each $c \geq c_*$, the systems under consideration admit monotonic nondecreasing traveling waves.

Keywords

Traveling Wave, Lattice Dynamical Systems, Schauder's Fixed Point Theorem

1. Introduction

Consider the following lattice differential equation

$$\begin{cases} \dot{u}_i = v\left(u_{i+1} - 2u_i + u_{i-1}\right) - f\left(u_i, (Bu)_i\right) + \alpha v_i, \quad i \in Z, \\ \dot{v}_i = -\sigma v_i + \beta u_i, \quad i \in Z, \end{cases} \tag{1.1}$$

where v, σ are positive constants, $\alpha\beta > 0$, f is a C^2-function, and $(Bu)_i = u_{i+1} - u_i$.

Lattice dynamical systems occur in a wide variety of applications, and a lot of studies have been done, e.g., see [1]-[4]. A pair of solutions $\{u_i\}_{i=-\infty}^{\infty}$, $\{v_i\}_{i=-\infty}^{\infty}$ of (1.1) is called a traveling wave solution with wave speed $c > 0$ if there exist functions $U, V : R \to R$ such that $u_i = U(i + ct)$, $v_i = V(i + ct)$ with $(U(-\infty), V(-\infty)) = (U_-, V_-)$ and $(U(+\infty), V(+\infty)) = (U_+, V_+)$. Let $\xi = i + ct$, note that (1.1) has a pair of traveling wave solutions if and only if U, V satisfy the functional differential equation

$$\begin{cases} c\dot{U}(\xi) = v\left(U(\xi+1) - 2U(\xi) + U(\xi-1)\right) - f\left(U(\xi), BU(\xi)\right) + \alpha V(\xi), \\ c\dot{V}(\xi) = -\sigma V(\xi) + \beta U(\xi). \end{cases} \tag{1.2}$$

Without loss of generality, we can impose (1.1) with asymptotic boundary conditions

$$\lim_{\xi \to -\infty} U(\xi) = 0, \quad \lim_{\xi \to +\infty} U(\xi) = k_1, \quad \lim_{\xi \to -\infty} V(\xi) = 0, \quad \lim_{\xi \to +\infty} V(\xi) = k_2. \tag{1.3}$$

By the property of equation, we can assume that $\alpha, \beta > 0$. In the following, we give some assumptions on

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nonlinear function f:

$$(A_1)\quad -f(k_1,0)+\alpha k_2=0,\quad f(0,0)=0,\quad -\sigma k_2+\beta k_1=0.$$

(A_2) There exists a positive-value continuous function $Q:R\to R$ such that

$$\max_{u_i,(Bu)_i\in[-r,r]}\left|f'_{u_i}\left(u_i,(Bu)_i\right)\right|+\max_{u_i,(Bu)_i\in[-r,r]}\left|f'_{(Bu)_i}\left(u_i,(Bu)_i\right)\right|\le Q(r),\quad Q(2k_1)<v.$$

$$(A_3)\quad -v<\frac{\partial f}{\partial x_2}(0,0)<0,\quad \frac{\partial f}{\partial x_1}(0,0)<-3v+\alpha\kappa-\frac{\beta}{\kappa},\quad \kappa=\frac{k_2}{k_1}.$$

$$(A_4)\quad \frac{\partial^2 f}{\partial x_i\partial x_j}(x_1,x_2)>0\ \text{ for any }\ (x_1,x_2)\in[0,k_1]\times[0,\omega],\quad i,j=1,2,$$

where $\omega=\left(e^{2\Lambda_*}-1\right)k_1$, Λ_* is given in Lemma 2.1.

$$(A_5)\quad -f\left(U(\xi),U(\xi+1)-U(\xi)\right)+\alpha V(\xi)\ne 0\ \text{ for any }\ (U,V)(\xi)\in(0,k_1)\times(0,k_2).$$

Select positive constants μ_1,μ_2 such that $\mu_1>2v+2Q(2k_1)$, $\mu_2>\sigma$, and define operators $H_1,H_2:C\left(R^2,R\right)\to C\left(R^2,R\right)$ by

$$H_1(U,V)(\xi)=\mu_1U(\xi)+v\left(U(\xi+1)-2U(\xi)+U(\xi-1)\right)-f\left(U(\xi),U(\xi+1)-U(\xi)\right)+\alpha V(\xi)$$

$$H_2(U,V)(\xi)=\mu_2V(\xi)-\sigma V(\xi)+\beta U(\xi).\tag{1.4}$$

Then, (1.2) can be rewritten as

$$c\dot U(\xi)=-\mu_1U(\xi)+H_1(U,V)(\xi),\quad c\dot V(\xi)=-\mu_2V(\xi)+H_2(U,V)(\xi).\tag{1.5}$$

Define the operators $F_i:C\left(R^2,R\right)\to C\left(R^2,R\right)$ by

$$F_i(U,V)(\xi)=\frac{1}{c}e^{-\frac{\mu_i}{c}\xi}\int_{-\infty}^{\xi}e^{\frac{\mu_i}{c}s}H_i(U,V)(s)ds,\quad i=1,2.$$

Note that F_i satisfy $cF_i'(U,V)(\xi)=-\mu_iF_i(U,V)(\xi)+H_i(U,V)(\xi), i=1,2,$ and a fixed point of $F=(F_1,F_2)$ is a solution of (1.2). Denote $|\cdot|$ the Euclidean norm in R^2. Define

$$B_\mu\left(R,R^2\right)=\left\{\Phi\in C\left(R,R^2\right):\sup_{t\in R}\left|\Phi(t)\right|e^{-\mu|t|}<\infty\right\},\left\|\Phi\right\|_\mu=\sup_{t\in R}\left|\Phi(t)\right|e^{-\mu|t|},$$

where $0<\mu<\min\left\{\frac{\mu_1}{c},\mu_2\right\}$. Note that $\left(B_\mu\left(R,R^2\right),\|\cdot\|_\mu\right)$ is a Banach space.

Definition 1.1. If the continuous functions $\left(\bar U(\xi),\bar V(\xi)\right):R\to R^2$ are differentiable almost everywhere and satisfy

$$\begin{cases}c\bar U'(\xi)\ge v\left(\bar U(\xi+1)-2\bar U(\xi)+\bar U(\xi-1)\right)-f\left(\bar U(\xi),B\bar U(\xi)\right)+\alpha\bar V(\xi),\\ c\bar V'(\xi)\ge-\sigma\bar V(\xi)+\beta\bar U(\xi),\end{cases}\tag{1.6}$$

Then, $\left(\bar U(\xi),\bar V(\xi)\right)$ is called an upper solution of (1.2).

Similarity, we can define a lower solution of (1.2). The main result of this paper is

Theorem 1.1. Assume that $(A_1)-(A_5)$ hold. Then there exists $c_*>0$ such that for every $c\ge c_*$, (1.2) admits a traveling wave solution $\left(U(\xi),V(\xi)\right)$ connecting $(0,0)$ and (k_1,k_2). Moreover, each component of traveling wave solution is monotonically nondecreasing in $\xi\in R$, and for each $c\ge c_*$, $U(\xi)$, $V(\xi)$ also satisfy $\lim_{\xi\to-\infty}U(\xi)e^{-\Lambda_1(c)\xi}=1$, $0<\lim_{\xi\to-\infty}V(\xi)e^{-\Lambda_1(c)\xi}\le\kappa$, where $\lambda=\Lambda_1(c)$ is the smallest solution of the equation

$$c\lambda-[(v-\frac{\partial f}{\partial x_2}(0,0))e^{\lambda}+ve^{-\lambda}]+2v+\frac{\partial f}{\partial x_1}(0,0)-\frac{\partial f}{\partial x_2}(0,0)-\alpha\kappa=0.$$

2. Upper-Lower Solutions of (1.2)

Set $\Delta(c,\lambda) = c\lambda - [(v - \frac{\partial f}{\partial x_2}(0,0))e^{\lambda} + ve^{-\lambda}] + 2v + \frac{\partial f}{\partial x_1}(0,0) - \frac{\partial f}{\partial x_2}(0,0) - \alpha\kappa$.

Lemma 2.1. Assume that (A_3) holds. Then there exists a unique $c_* > 0$ such that (i) if $c > c_*$, then there exist two positive numbers $\Lambda_1(c)$ and $\Lambda_2(c)$ with $\Lambda_1(c) < \Lambda_2(c)$ such that $\Delta(c,\Lambda_1(c)) = \Delta(c,\Lambda_2(c)) = 0$, $\Delta(c,\cdot) > 0$ in $(\Lambda_1(c),\Lambda_2(c))$, and $\Delta(c,\cdot) < 0$ in $R \setminus [\Lambda_1(c),\Lambda_2(c)]$; (ii) if $c < c_*$, then $\Delta(c,\lambda) < 0$ for all $\lambda \ge 0$; (iii) if $c = c_*$, then $\Lambda_1(c) = \Lambda_2(c) = \Lambda_*$, and $\Delta(c_*,\Lambda_*) = 0$.

Proof. Using assumption (A_3), we can get the result directly. □

Lemma 2.2. Assume that (A_1), (A_3) and (A_4) hold. Let c_*, $\Lambda_1(c)$, and $\Lambda_2(c)$ be defined as in Lemma 2.1, and $c > c_*$ be any number. Then for every $\theta \in (1, \min\{\frac{\Lambda_2(c)}{\Lambda_1(c)}, 2\})$ and $0 < h < \kappa$, there exists $Q(c,\theta) \ge 1$ such that for any $q \ge Q(c,\theta)$,

$$\phi^+(\xi) := \min\left\{k_1, e^{\Lambda_1(c)\xi} + qe^{\theta\Lambda_1(c)\xi}\right\}, \psi^+(\xi) := \min\left\{k_2, \kappa\left(e^{\Lambda_1(c)\xi} + qe^{\theta\Lambda_1(c)\xi}\right)\right\}, \xi \in R,$$

and

$$\phi_-(\xi) := \max\left\{0, e^{\Lambda_1(c)\xi} - qe^{\theta\Lambda_1(c)\xi}\right\}, \psi_-(\xi) := \max\left\{0, h\left(e^{\Lambda_1(c)\xi} - qe^{\theta\Lambda_1(c)\xi}\right)\right\}, \xi \in R$$

are a pair of upper solutions and a pair of lower solutions of (1.2), respectively.

Proof. Let

$$N_1^c[\phi,\psi](\xi) := c\phi'(\xi) - v[\phi(\xi+1) - 2\phi(\xi) + \phi(\xi-1)] + f(\phi(\xi), \phi(\xi+1) - \phi(\xi)) - \alpha\psi(\xi), \quad (2.1)$$

$$N_2^c[\phi,\psi](\xi) := c\psi'(\xi) + \sigma\psi(\xi) - \beta\psi(\xi). \quad (2.2)$$

Since $\kappa = \frac{k_2}{k_1}$, there exists ξ_1 such that $\phi^+(\xi_1) = k_1$, $\psi^+(\xi_1) = k_2$. If $\xi \ge \xi_1$, then $\phi^+(\xi) = k_1$, $\psi^+(\xi) = k_2$. By (A_1), we get that

$$N_1^c[\phi^+,\psi^+](\xi) \ge f(k_1,0) - \alpha k_2 = 0, \quad N_2^c[\phi^+,\psi^+](\xi) \ge \sigma k_2 - \beta k_1 = 0.$$

If $\xi < \xi_1$, then $\phi^+(\xi) = e^{\Lambda_1(c)\xi} + qe^{\theta\Lambda_1(c)\xi}, \psi^+(\xi) = \kappa\left(e^{\Lambda_1(c)\xi} + qe^{\theta\Lambda_1(c)\xi}\right)$. By (A_1), $(A_3)-(A_4)$, and using Lemma 2.1, we get that

$$\begin{aligned} N_1^c[\phi^+,\psi^+](\xi) \ge{}& c\left(\Lambda_1(c)e^{\Lambda_1(c)\xi} + q\theta\Lambda_1(c)e^{\theta\Lambda_1(c)\xi}\right) - v\Big[e^{\Lambda_1(c)(\xi+1)} + qe^{\theta\Lambda_1(c)(\xi+1)} - 2e^{\Lambda_1(c)\xi} \\ &-2qe^{\theta\Lambda_1(c)\xi} + e^{\Lambda_1(c)(\xi-1)} + qe^{\theta\Lambda_1(c)(\xi-1)}\Big] + f\Big(e^{\Lambda_1(c)\xi} + qe^{\theta\Lambda_1(c)\xi}, \\ &e^{\Lambda_1(c)(\xi+1)} + qe^{\theta\Lambda_1(c)(\xi+1)} - e^{\Lambda_1(c)\xi} - qe^{\theta\Lambda_1(c)\xi}\Big) - \alpha\kappa\left(e^{\Lambda_1(c)\xi} + qe^{\theta\Lambda_1(\xi)}\right) \\ \ge{}& \Delta(c,\Lambda_1(c))e^{\Lambda_1(c)\xi} + \Delta(c,\theta\Lambda_1(c))qe^{\theta\Lambda_1(c)\xi} \ge 0. \end{aligned} \quad (2.3)$$

Lemma 2.1 and (A_3) yields

$$c\kappa\Lambda_1(c)\theta + \kappa\sigma - \beta > c\kappa\Lambda_1(c) + \kappa\sigma - \beta > 0. \quad (2.4)$$

Thus,

$$\begin{aligned} N_2^c[\phi^+,\psi^+](\xi) ={}& c\kappa\left(\Lambda_1(c)e^{\Lambda_1(c)\xi} + q\theta\Lambda_1(c)e^{\theta\Lambda_1(c)\xi}\right) \\ &+ \kappa\sigma\left(e^{\Lambda_1(c)\xi} + qe^{\theta\Lambda_1(c)\xi}\right) - \beta\left(e^{\Lambda_1(c)\xi} + qe^{\theta\Lambda_1(c)\xi}\right) \\ ={}& (c\kappa\Lambda_1(c) + \kappa\sigma - \beta)e^{\Lambda_1(c)\xi} + q(c\kappa\Lambda_1(c)\theta + \kappa\sigma - \beta)e^{\theta\Lambda_1(c)\xi} > 0. \end{aligned}$$

Therefore, $(\phi^+,\psi^+)(\xi)$ is an upper solution of (1.2). Similarly, we can prove that $(\phi_-,\psi_-)(\xi)$ is a lower solution. □

3. Existence of Traveling Wave

Let $K=(k_1,k_2)$, $C_{[0,K]}(R,R^2)=\{(U,V)\in C(R,R^2):0\le U(s)\le k_1, 0\le V(s)\le k_2, s\in R.\}$. We have the following result.

Lemma 3.1 Assume that (A_1) and (A_2) hold. Then

(i) $F_1(U_1,V_1)(\xi)\ge F_1(U_2,V_2)(\xi)$ and $F_2(U_1,V_1)(\xi)\ge F_2(U_2,V_2)(\xi)$ for $\xi\in R$ if $(U_1,V_1)(\xi),(U_2,V_2)(\xi)\in C_{[0,K]}(R,R^2)$ satisfy $U_1(\xi)\ge U_2(\xi)$, $V_1(\xi)\ge V_2(\xi)$ for $\xi\in R$;

(ii) $F_1(U,V)(\xi), F_2(U,V)(\xi)$ are nondecreasing in $\xi\in R$ if $(U,V)(\xi)\in C_{[0,K]}(R,R^2)$ is nondecreasing in $\xi\in R$.

Proof. If $(U_1,V_1)(\xi),(U_2,V_2)(\xi)\in C_{[0,K]}(R,R^2)$ such that $U_1(\xi)\ge U_2(\xi)$ and $V_1(\xi)\ge V_2(\xi)$ for $\xi\in R$, then by (A_2) we have

$$\begin{aligned}&\left|f(U_1(\xi),BU_1(\xi))-f(U_2(\xi),BU_2(\xi))\right|\\&=\Big|\int_0^1\big[f_U'(U_2+\theta(U_1-U_2),BU_2+\theta(BU_1-BU_2))(U_1-U_2)\\&\quad+f_{BU}'(U_2+\theta(U_1-U_2),BU_2+\theta(BU_1-BU_2))(BU_1-BU_2)\big]d\theta\Big|\\&\le 2M_1(U_1(\xi)-U_2(\xi))+M_1(U_1(\xi+1)-U_2(\xi+1)),\end{aligned}\tag{3.1}$$

where $M_1=Q(2k_1)$. Note that

$$\begin{aligned}&H_1(U_1,V_1)(\xi)-H_1(U_2,V_2)(\xi)\\&=(\mu_1-2v)(U_1(\xi)-U_2(\xi))+v\big[(U_1(\xi+1)-U_2(\xi+1))+(U_1(\xi-1)-U_2(\xi-1))\big]\\&\quad-\big[f(U_1(\xi),BU_1(\xi))-f(U_2(\xi),BU_2(\xi))\big]+\alpha(V_1(\xi)-V_2(\xi)).\end{aligned}\tag{3.2}$$

Thus, from (3.1)-(3.2), we have

$$\begin{aligned}&H_1(U_1,V_1)(\xi)-H_1(U_2,V_2)(\xi)\\&\ge(\mu_1-2v-2M_1)(U_1(\xi)-U_2(\xi))+(v-M_1)(U_1(\xi+1)-U_2(\xi+1))\\&\quad+v(U_1(\xi-1)-U_2(\xi-1))+\alpha(V_1(\xi)-V_2(\xi))\ge 0,\end{aligned}$$

which implies that $H_1(U_1,V_1)(\xi)\ge H_1(U_2,V_2)(\xi)$. A similar argument can be done for $H_2(U,V)(\xi)$. Thus, we can get the desired results. □

Lemma 3.2. Assume that (A_1) and (A_2) hold. Then $F=(F_1,F_2):B_\mu(R,R^2)\to B_\mu(R,R^2)$ is continuous with respect to the norm $\|\cdot\|_\mu$ with $0<\mu<\min\left\{\frac{\mu_1}{c},\mu_2\right\}$.

Proof. We first prove that $H_1,H_2:B_\mu(R,R^2)\to B_\mu(R,R^2)$ are continuous. Denote $\Phi_1=(U_1,V_1)$, $\Phi_2=(U_2,V_2)$. For any $\varepsilon>0$, choose $0<\delta<\frac{\varepsilon}{N}$, where $N=\max\{\mu_1-2v+2M_1+(2v+M_1)e^\mu+\alpha,\mu_2-\sigma+\beta\}$. If Φ_1 and Φ_2 satisfy $\|\Phi_1-\Phi_2\|_\mu=\sup_{\xi\in R}|\Phi_1(\xi)-\Phi_2(\xi)|e^{-\mu|\xi|}<\delta$, then by (3.1),

$$\begin{aligned}&\left|H_1(U_1,V_1)(\xi)-H_1(U_2,V_2)(\xi)\right|e^{-\mu|\xi|}\\&=\Big|(\mu_1-2v)(U_1(\xi)-U_2(\xi))+v\big[(U_1(\xi+1)-U_2(\xi+1))+(U_1(\xi-1)-U_2(\xi-1))\big]\\&\quad-(f(U_1(\xi),BU_1(\xi))-f(U_2(\xi),BU_2(\xi)))+\alpha(V_1(\xi)-V_2(\xi))\Big|e^{-\mu|\xi|}\\&\le\big[\mu_1-2v+2M_1+(2v+M_1)e^\mu+\alpha\big]\|\Phi_1(\xi)-\Phi_2(\xi)\|_\mu<\varepsilon.\end{aligned}\tag{3.3}$$

Similarly, $H_2(U_1,V_1)(\xi)$ is continuous.

By definition of F_1, we have

$$\left|F_1(U_1,V_1)(\xi)-F_1(U_2,V_2)(\xi)\right|=\frac{1}{c}e^{-\frac{\mu_1}{c}\xi}\left|\int_{-\infty}^{\xi}\left(H_1(U_1,V_1)-H_1(U_2,V_2)\right)(s)\,ds\right|$$
$$\le\frac{1}{c}\left\|H_1(U_1,V_1)(\xi)-H_1(U_2,V_2)(\xi)\right\|_{\mu}e^{-\frac{\mu_1}{c}\xi}\int_{-\infty}^{\xi}e^{\frac{\mu_1}{c}s+\mu|s|}ds. \tag{3.4}$$

If $\xi<0$, it follows that

$$\left|F_1(U_1,V_1)(\xi)-F_1(U_2,V_2)(\xi)\right|e^{-\mu|\xi|}\le\frac{1}{\mu_1-c\mu}\left\|H_1(U_1,V_1)(\xi)-H_1(U_2,V_2)(\xi)\right\|_{\mu}. \tag{3.5}$$

If $\xi\ge 0$, it follows that

$$\left|F_1(U_1,V_1)(\xi)-F_1(U_2,V_2)(\xi)\right|e^{-\mu|\xi|}$$
$$\le\left[\left(\frac{1}{\mu_1-c\mu}-\frac{1}{\mu_1+c\mu}\right)e^{-\frac{\mu_1+c\mu}{c}\xi}+\frac{1}{\mu_1+c\mu}\right]\left\|H_1(U_1,V_1)(\xi)-H_1(U_2,V_2)(\xi)\right\|_{\mu} \tag{3.6}$$
$$\le\frac{1}{\mu_1-c\mu}\left\|H_1(U_1,V_1)(\xi)-H_1(U_2,V_2)(\xi)\right\|_{\mu}.$$

Combining (3.5) and (3.6), we get that F_1 is continuous with respect to the norm $\|\cdot\|_{\mu}$. A Similar argument can be done for F_2. □

Define

$$\Gamma=\Gamma\left(\left[\phi_-,\psi_-\right],\left[\phi^+,\psi^+\right]\right)$$
$$:=\left\{(\phi,\psi)\in C\left(R,R^2\right)\left|\begin{array}{l}(i)\ \phi(\xi),\psi(\xi)\ are\ nondecreasing\ in\ R;\\(ii)\ \phi_-(\xi)\le\phi(\xi)\le\phi^+(\xi)\ and\ \psi_-(\xi)\le\psi(\xi)\le\psi^+(\xi)\\\qquad for\ all\ \xi\in R;\\(iii)\ \left|\phi(\xi_1)-\phi(\xi_2)\right|\le\frac{2\mu_1k_1}{c}|\xi_1-\xi_2|\ and\ \left|\psi(\xi_1)-\right.\\\qquad\left.\psi(\xi_2)\right|\le\frac{2\mu_2k_2}{c}|\xi_1-\xi_2|\ for\ all\ \xi_1,\xi_2\in R.\end{array}\right.\right\}.$$

It is easy to verify that Γ is nonempty, convex and compact in $B_{\mu}\left(R,R^2\right)$. As the proof of Claim 2 in the proof of Theorem A in [5], we have

Lemma 3.3. Assume that $(A_1)-(A_3)$ hold. Then $F(\Gamma)\subset\Gamma$.

Proof of Theorem 1.1. By the definition of Γ, Lemma 3.2-3.3 and Schauder's fixed point theorem, we get that there exists a fixed point $(\phi^*(\xi),\psi^*(\xi))\in\Gamma$. Note that $(\phi^*(\xi),\psi^*(\xi))$ is nondecreasing in $\xi\in R$, assumption (A_5) and Lemma 2.2 imply that $\lim_{\xi\to-\infty}(\phi^*(\xi),\psi^*(\xi))=(0,0)$, $\lim_{\xi\to+\infty}(\phi^*(\xi),\psi^*(\xi))=(k_1,k_2)$. Therefore, $(\phi^*(\xi),\psi^*(\xi))$ is a traveling wave solution of (1.1). □

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