Journal of Applied Mathematics and Physics, 2014, 2, 1242-1256
Published Online December 2014 in SciRes. http://www.scirp.org/journal/jamp
http://dx.doi.org/10.4236/jamp.2014.213145

Reliability Modelling and Analysis of Redundant Systems Connected to Supporting External Device for Operation Attended by a Repairman and Repairable Service Station

Ibrahim Yusuf[1], Rabia Salihu Said[2], Fatima Salman Koki[2], Mansur Babagana[3]

[1]Department of Mathematical Sciences, Bayero University, Kano, Nigeria
[2]Department of Physics, Bayero University, Kano, Nigeria
[3]Department of Computer Science and Information Technology, Bayero University, Kano, Nigeria
Email: iyusuf.mth@buk.edu.ng, rssaidu.phy@buk.edu.ng, FatimaSK2775@gmail.com, mbabagana.cs@buk.edu.ng

Received 5 October 2014; revised 1 November 2014; accepted 24 November 2014

Abstract

In this paper, probabilistic models for three redundant configurations have been developed to analyze and compare some reliability characteristics. Each system is connected to a repairable supporting external device for operation. Repairable service station is provided for immediate repair of failed unit. Explicit expressions for mean time to system failure and steady-state availability for the three configurations are developed. Furthermore, we compare the three configurations based on their reliability characteristics and found that configuration II is more reliable and efficient than the remaining configurations.

Keywords

Availability, Supporting Device, Service Station, Redundancy

1. Introduction

High system reliability and availability play a vital role towards industrial growth as the profit is directly dependent on production volume which depends upon system performance. Thus the reliability and availability of a system may be enhanced by proper design, optimization at the design stage and by maintaining the same dur-

How to cite this paper: Yusuf, I., *et al.* (2014) Reliability Modelling and Analysis of Redundant Systems Connected to Supporting External Device for Operation Attended by a Repairman and Repairable Service Station. *Journal of Applied Mathematics and Physics*, **2**, 1242-1256. http://dx.doi.org/10.4236/jamp.2014.213145

ing its service life. Because of their prevalence in power plants, manufacturing systems, and industrial systems, many researchers have studied reliability and availability problem of different systems (see, for instance, Ref [1]-[8] and the references therein). In real-life situations we often encounter cases where the systems that cannot work without the help of external supporting devices connect to such systems. These external supporting devices are systems themselves that are bound to fail. Where such systems exist, a repairable service station is provided for the immediate repair of failed unit. Such systems are found in power plants, manufacturing systems, and industrial systems. Ref [9] [10] performed comparative analysis of some reliability characteristics between redundant systems requiring supporting units for their operation.

The problem considered in this paper is different from the work of Ref [9] [10]. The objectives of the present paper are three. The first is to develop the explicit expressions for the mean time to system failure (MTSF) and steady-state availability. The second objective is to perform a parametric investigation of various system parameters on mean time to system failure (MTSF) and steady-state availability and capture their impact on the mean time to system failure (MTSF) and steady-state availability. The third objective is to perform comparative analysis between the three configurations based on assumed numerical values in order to determine the optimal configuration.

2. Description of the Systems

We consider three redundant systems connected to an external supporting device for their operation as follows. The first system is a 2-out-of-3 system connected to a supporting device and has a repairable service station. The second is also a 2-out-of-3 system connected to supporting device and has two standby repairable service stations. The third system is a 3-out-of-4 system connected to a supporting device and has a repairable service station. We assume that switching is perfect and instantaneous. We also assume that two units cannot fail simultaneously. Whenever a unit fails with failure rate β_1, it is immediately sent to a service station for repair with service rate α_1. However, on the course of repairing failed unit, the service is bound to fail with failure rate of β_2 and service rate of α_2 and failed unit must wait whenever the service station is under repair for first and third system, while the standby service will continue repairing failed unit for the second system. The supporting device is a system that is prone to failure. Whenever the supporting device failed with rate β_3 it is attended by a repairman, the system stop working and must wait until the supporting device is repaired with rate α_3.

3. Mean Time to System Failure Models Formulation

3.1. MTSF Formulation for Configuration I

For configuration I, we define $P_i(t)$ to be the probability that the system at time $t \geq 0$ is in state S_i. Also let $P(t)$ be the probability row vector at time t, we have the following initial condition:

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)\right] = [1,0,0,0,0,0,0,0]$$

We obtain the following differential equations:

$$\begin{aligned}
\frac{\mathrm{d}P_0(t)}{\mathrm{d}t} &= -(\beta_1+\beta_2)P_0(t)+\alpha_1 P_1(t)+\alpha_2 P_2(t) \\
\frac{\mathrm{d}P_1(t)}{\mathrm{d}t} &= -\left(\alpha_1+\sum_{k=1}^{3}\beta_k\right)P_1(t)+\beta_1 P_0(t)+\alpha_3 P_3(t)+\alpha_1 P_4(t)+\alpha_2 P_5(t) \\
\frac{\mathrm{d}P_2(t)}{\mathrm{d}t} &= -\alpha_2 P_2(t)+\beta_2 P_0(t) \\
\frac{\mathrm{d}P_3(t)}{\mathrm{d}t} &= -(\alpha_3+\beta_1+\beta_2)P_3(t)+\beta_3 P_1(t)+\alpha_2 P_7(t) \\
\frac{\mathrm{d}P_4(t)}{\mathrm{d}t} &= -(\alpha_1+\beta_3)P_4(t)+\beta_1 P_1(t)+\alpha_3 P_6(t) \\
\frac{\mathrm{d}P_5(t)}{\mathrm{d}t} &= -(\alpha_2+\beta_3)P_5(t)+\beta_2 P_1(t)+\alpha_3 P_7(t) \\
\frac{\mathrm{d}P_6(t)}{\mathrm{d}t} &= -\alpha_3 P_6(t)+\beta_1 P_3(t)+\beta_3 P_4(t) \\
\frac{\mathrm{d}P_7(t)}{\mathrm{d}t} &= -(\alpha_2+\alpha_3)P_7(t)+\beta_2 P_3(t)+\beta_3 P_5(t)
\end{aligned} \tag{1}$$

This can be written in the matrix form as

$$\dot{P} = T_1 P \tag{2}$$

where

$$T_1 = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -\left(\alpha_1+\sum_{k=1}^{3}\beta_k\right) & 0 & \alpha_3 & \alpha_1 & \alpha_2 & 0 & 0 \\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & -(\alpha_3+\beta_1+\beta_2) & 0 & 0 & 0 & \alpha_2 \\ 0 & \beta_1 & 0 & 0 & -(\alpha_1+\beta_3) & 0 & \alpha_3 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -(\alpha_2+\beta_3) & 0 & \alpha_3 \\ 0 & 0 & 0 & \beta_1 & \beta_3 & 0 & -\alpha_3 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & \beta_3 & 0 & -(\alpha_2+\alpha_3) \end{bmatrix}$$

It is difficult to evaluate the transient solutions, the procedure to develop the explicit expression for $MTSF_1$ is to delete the rows and column of absorbing states of matrix T_1 and take the transpose to produce a new matrix, say M_1. Following Ref [11] [12], the expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(\text{absorbing})}\right] = MTSF_1 = P(0)\left(-Q_1^{-1}\right)\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{N_0}{D_0} \tag{3}$$

where

$$Q_1 = \begin{bmatrix} -(\beta_1+\beta_2) & \beta_1 & 0 \\ \alpha_1 & -\left(\alpha_1+\sum_{k=1}^{3}\beta_k\right) & \beta_3 \\ 0 & \alpha_3 & -(\alpha_3+\beta_1+\beta_2) \end{bmatrix}$$

$$N_0 = \alpha_1(\alpha_3+\beta_1+\beta_2) + 2\beta_1(\alpha_3+\beta_1+2\beta_2+\beta_3) + \beta_2(\alpha_3+\beta_2+\beta_3)$$

$$D_0 = \alpha_1\beta_1\beta_2 + \alpha_3\beta_1^2 + \beta_1^3 + 3\beta_1^2\beta_2 + 2\alpha_3\beta_1\beta_2 + 3\beta_1\beta_2^2 + \beta_1^2\beta_3 + 2\beta_1\beta_2\beta_3 + \alpha_1\alpha_3\beta_2 + \alpha_1\beta_2^2 + \alpha_3\beta_2^2 + \beta_2^3 + \beta_2^2\beta_3$$

3.2. MTSF Formulation for Configuration II

For configuration II, we define $P_i(t)$ to be the probability that the system at time $t \ge 0$ is in state S_i. Also let $P(t)$ be the probability row vector at time t, we have the following initial condition:

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \cdots, P_{10}(0)\right] = [1,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P} = T_2 P \tag{4}$$

where

$$T_2 = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -y_1 & 0 & \alpha_3 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & -y_2 & 0 & 0 & \alpha_2 & \alpha_1 & \alpha_3 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & -y_3 & 0 & 0 & \alpha_3 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & \beta_3 & 0 & 0 & -y_4 & 0 & \alpha_3 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -y_5 & 0 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_3 & 0 & -\alpha_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 \end{bmatrix}$$

and

$$y_1 = (\alpha_1 + \beta_1 + \beta_2 + \beta_3),\ y_2 = (\alpha_3 + \beta_1 + \beta_2 + \beta_3),\ y_3 = (\alpha_1 + \beta_3),\ y_4 = (\alpha_1 + \alpha_3 + \beta_3),\ y_5 = (\alpha_3 + \beta_1 + \beta_2)$$

Using the procedure described in Subsection 3.1, the expected time to reach an absorbing state is

$$E\left[T_{P(0)\to P(\text{absorbing})}\right] = MTSF_2 = P(0)\left(-Q_2^{-1}\right)\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \frac{N_1}{D_1} \tag{5}$$

where

$$Q_2 = \begin{bmatrix} -(\beta_1+\beta_2) & \beta_1 & 0 & 0 \\ \alpha_1 & -(\alpha_1+\beta_1+\beta_2+\beta_3) & \beta_3 & 0 \\ 0 & \alpha_3 & -(\alpha_3+\beta_1+\beta_2+\beta_3) & \beta_3 \\ 0 & 0 & \alpha_3 & -(\alpha_3+\beta_1+\beta_2) \end{bmatrix}$$

$$\begin{aligned} N_1 = {} & 4\beta_1\beta_2\beta_3 + 2\alpha_1\alpha_3\beta_1 + 2\alpha_1\alpha_3\beta_2 + 2\alpha_1\beta_1\beta_2 + \alpha_1\beta_1\beta_3 + \alpha_1\beta_2\beta_3 + 4\alpha_3\beta_1\beta_2 + \alpha_3\beta_1\beta_3 \\ & + \alpha_3\beta_2\beta_3 + \alpha_1\alpha_3^2 + \alpha_1\beta_1^2 + \alpha_1\beta_2^2 + \beta_1^3 + \beta_2^3 + \alpha_3^2\beta_1 + 2\alpha_3\beta_1^2 + 3\beta_1^2\beta_2 + 3\beta_1\beta_2^2 \\ & + 2\beta_1^2\beta_3 + \alpha_3^2\beta_2 + 2\alpha_3\beta_2^2 + 2\beta_2^2\beta_3 + \beta_1\beta_3^2 + \beta_2\beta_3^2 + \beta_1\left(\alpha_3^2 + 2\alpha_3\beta_1 + 2\alpha_3\beta_2\right. \\ & \left. + \beta_1^2 + 2\beta_1\beta_2 + \beta_2^2 + \beta_1\beta_3 + \beta_2\beta_3\right) + \beta_1\beta_3^2 + \beta_1\beta_3\left(\alpha_3 + \beta_1 + \beta_2\right) \end{aligned}$$

$$\begin{aligned} D_1 = {} & 6\beta_1^2\beta_2\beta_3 + 6\beta_1\beta_2^2\beta_3 + 2\alpha_1\alpha_3\beta_1\beta_2 + \alpha_1\beta_1\beta_2\beta_3 + 2\alpha_3\beta_1\beta_2\beta_3 + 4\beta_1\beta_2^3 + \alpha_3^2\beta_1^2 \\ & + 2\alpha_3\beta_1^3 + 4\beta_1^3\beta_2 + 6\beta_1^2\beta_2^2 + 2\beta_1^3\beta_3 + \beta_1^2\beta_3^2 + \alpha_1\beta_2^3 + \alpha_3^2\beta_2^2 + 2\alpha_3\beta_2^3 + 2\beta_2^3\beta_3 \\ & + \beta_3^2\beta_2^2 + \alpha_1\beta_1^2\beta_2 + 6\alpha_3\beta_1^2\beta_2 + \alpha_1\beta_1^2\beta_3 + 2\alpha_1\alpha_3\beta_2^2 + \alpha_1\beta_2^2\beta_3 + \alpha_3\beta_2^2\beta_3 \\ & + 2\alpha_1\beta_1\beta_2^2 + 2\alpha_3^2\beta_1\beta_2 + 6\alpha_1\beta_1\beta_2^2 + 2\beta_1\beta_2\beta_3^2 + \alpha_1\alpha_3^2\beta_2 + \beta_1^4 + \beta_2^4 \end{aligned}$$

3.3. MTSF Formulation for Configuration III

For configuration II, we define $P_i(t)$ to be the probability that the system at time $t \ge 0$ is in state S_i. Also let $P(t)$ be the probability row vector at time t, we have the following initial condition:

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)\right] = [1,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P} = T_3 P \tag{6}$$

where

$$T_3 = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_1+\beta_2+\beta_3) & 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 \\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & -(\alpha_1+\beta_3) & 0 & 0 & \alpha_3 & 0 \\ 0 & \beta_2 & 0 & 0 & -(\alpha_2+\beta_3) & 0 & 0 & \alpha_3 \\ 0 & \beta_3 & 0 & 0 & 0 & -(\alpha_3+\beta_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & 0 & \beta_3 & 0 & \beta_1 & -\alpha_3 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & \beta_2 & 0 & -(\alpha_2+\alpha_3) \end{bmatrix}$$

Using the procedure described in Subsection 3.1, the expected time to reach an absorbing state is

$$E\left[T_{P(0)\to P(\text{absorbing})}\right]=MTSF_3=P(0)\left(-Q_3^{-1}\right)\begin{bmatrix}1\\1\\1\end{bmatrix}=\frac{N_2}{D_2} \tag{7}$$

where

$$Q_3=\begin{bmatrix}-(\beta_1+\beta_2) & \beta_1 & 0\\ \alpha_1 & -(\alpha_1+\beta_1+\beta_2+\beta_3) & \beta_3\\ 0 & \alpha_3 & -(\alpha_3+\beta_1+\beta_2)\end{bmatrix}$$

$$N_2=\alpha_1(\alpha_3+\beta_1+\beta_2)+2\beta_1(\alpha_3+\beta_1+2\beta_2+\beta_3)+\beta_2(\alpha_3+\beta_2+\beta_3)$$

$$\begin{aligned}D_2={}&\alpha_1\beta_1\beta_2+\alpha_3\beta_1^2+\beta_1^3+3\beta_1^2\beta_2+2\alpha_3\beta_1\beta_2+3\beta_1\beta_2^2+\beta_1^2\beta_3\\&+2\beta_1\beta_2\beta_3+\alpha_1\alpha_3\beta_2+\alpha_1\beta_2^2+\alpha_3\beta_2^2+\beta_2^3+\beta_2^2\beta_3\end{aligned}$$

4. Availability Models Formulation

4.1. Availability Model Formulation for Configuration I

For the analysis of availability case of configuration I we use the same initial condition as in Subsection 3.1

$$P(0)=\left[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0)\right]=[1,0,0,0,0,0,0,0]$$

The differential equations above are expressed in the form

$$\begin{bmatrix}\dot P_0\\ \dot P_1\\ \dot P_2\\ \dot P_3\\ \dot P_4\\ \dot P_5\\ \dot P_6\\ \dot P_7\end{bmatrix}=\begin{bmatrix}-(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0\\ \beta_1 & -(\alpha_1+\beta_1+\beta_2+\beta_3) & 0 & \alpha_3 & \alpha_1 & \alpha_2 & 0 & 0\\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0\\ 0 & \beta_3 & 0 & -(\alpha_3+\beta_1+\beta_2) & 0 & 0 & 0 & \alpha_2\\ 0 & \beta_1 & 0 & 0 & -(\alpha_1+\beta_3) & 0 & \alpha_3 & 0\\ 0 & \beta_2 & 0 & 0 & 0 & -(\alpha_2+\beta_3) & 0 & \alpha_3\\ 0 & 0 & 0 & \beta_1 & \beta_3 & 0 & -\alpha_3 & 0\\ 0 & 0 & 0 & \beta_2 & 0 & \beta_3 & 0 & -(\alpha_2+\alpha_3)\end{bmatrix}\begin{bmatrix}P_0\\P_1\\P_2\\P_3\\P_4\\P_5\\P_6\\P_7\end{bmatrix}$$

The steady-state availability is given by

$$A_{V1}(\infty)=P_0(\infty)+P_1(\infty)+P_3(\infty) \tag{8}$$

In the steady-state, the derivatives of the state probabilities become zero and therefore Equation (2) become

$$T_1P=0 \tag{9}$$

which is in matrix form

$$\begin{bmatrix}-(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0\\ \beta_1 & -(\alpha_1+\beta_1+\beta_2+\beta_3) & 0 & \alpha_3 & \alpha_1 & \alpha_2 & 0 & 0\\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0\\ 0 & \beta_3 & 0 & -(\alpha_3+\beta_1+\beta_2) & 0 & 0 & 0 & \alpha_2\\ 0 & \beta_1 & 0 & 0 & -(\alpha_1+\beta_3) & 0 & \alpha_3 & 0\\ 0 & \beta_2 & 0 & 0 & 0 & -(\alpha_2+\beta_3) & 0 & \alpha_3\\ 0 & 0 & 0 & \beta_1 & \beta_3 & 0 & -\alpha_3 & 0\\ 0 & 0 & 0 & \beta_2 & 0 & \beta_3 & 0 & -(\alpha_2+\alpha_3)\end{bmatrix}\begin{bmatrix}P_0(\infty)\\P_1(\infty)\\P_2(\infty)\\P_3(\infty)\\P_4(\infty)\\P_5(\infty)\\P_6(\infty)\\P_7(\infty)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

Using the following normalizing condition

$$\sum_{J=0}^{7} P_J(\infty) = 1 \tag{10}$$

Substituting (10) in the last row of (9) to compute the steady-state probabilities, the expression for steady-state availability is given by

$$A_{V1}(\infty) = \frac{N_3}{D_3} \tag{11}$$

$$\begin{aligned} N_3 = {} & \alpha_1^2\alpha_2\alpha_3\left(\alpha_2^2\alpha_3 + \alpha_2^2\beta_1 + \alpha_2\alpha_3^2 + \alpha_2\alpha_3\beta_2 + \alpha_2\alpha_3\beta_1 + \alpha_2\alpha_3\beta_3 + \alpha_2\beta_1\beta_3 + \alpha_3^2\beta_2 + \alpha_3\beta_1\beta_2 + \alpha_3\beta_2^2\right) \\ & + \alpha_1\alpha_2\alpha_3\beta_1\left(\alpha_2^2\alpha_3 + \alpha_2^2\beta_1 + \alpha_2\alpha_3^2 + \alpha_2\alpha_3\beta_2 + \alpha_2\alpha_3\beta_1 + \alpha_2\alpha_3\beta_3 + \alpha_2\beta_1\beta_3 + \alpha_3^2\beta_2 + \alpha_3\beta_1\beta_2 + \alpha_3\beta_2^3\right) \\ & + \alpha_1\alpha_2\alpha_3\beta_1\left(\alpha_2^2\beta_3 + \alpha_2\alpha_3\beta_3 + \alpha_2\beta_2\beta_3 + \alpha_2\beta_3^2 - \alpha_2\beta_2^2 + \alpha_3\beta_2\beta_3\right) \end{aligned}$$

$$\begin{aligned} D_3 = {} & 2\alpha_2^2\alpha_3\beta_1^2\beta_2\beta_3 + 2\alpha_2\alpha_3^2\beta_1^2\beta_2\beta_3 + \alpha_2\alpha_3\beta_1^3\beta_2\beta_3 + 2\alpha_2\alpha_3\beta_1^2\beta_2^2\beta_3 + \alpha_1^2\alpha_2^2\alpha_3^3 + 3\alpha_2^2\alpha_3\beta_1^2\beta_3^2 \\ & + 2\alpha_2^2\alpha_3\beta_1^2\beta_3 + \alpha_1^2\alpha_2^3\alpha_3^2 + \alpha_2\alpha_3^2\beta_1\beta_2^3 + \alpha_2^2\alpha_3^2\beta_1^2 + 2\alpha_2\alpha_3^2\beta_1\beta_2^2\beta_3 + \alpha_2\alpha_3\beta_1\beta_2^3\beta_3 + \alpha_2^2\alpha_3^2\beta_1\beta_2^2 \\ & + \alpha_2^3\beta_1^3\beta_3 + \alpha_2^3\alpha_3\beta_1^3 + \alpha_2^3\beta_1^2\beta_3^2 + \alpha_2^3\alpha_3^2\beta_1^2 + \alpha_2^2\beta_1^3\beta_3^2 + \alpha_2^2\alpha_3^2\beta_1^3 + \alpha_2^2\beta_1^2\beta_3^2 + \alpha_2^2\beta_3^2\beta_2\beta_3^2 \\ & + \alpha_2^2\alpha_3^2\beta_1^2\beta_2 + \alpha_2\alpha_3^2\beta_1^3\beta_2 + \alpha_2\alpha_3^3\beta_1^2\beta_2 + \alpha_2^2\alpha_3\beta_1\beta_2^2\beta_3 + \alpha_2\alpha_3\beta_1\beta_2^2\beta_3^2 + \alpha_1^2\alpha_3^2\beta_2^3 + \alpha_1^2\alpha_3^3\beta_2^2 \\ & + \alpha_1\alpha_2^3\beta_1^2\beta_3 + \alpha_1\alpha_2^3\alpha_3\beta_1^2 + \alpha_1\alpha_2^3\alpha_3^2\beta_1 + \alpha_1\alpha_2^2\beta_1^2\beta_3^2 + \alpha_1\alpha_2^2\alpha_3^2\beta_1^2 + \alpha_1\alpha_2^2\alpha_3^3\beta_1 + \alpha_1^2\alpha_2^3\alpha_3\beta_1 \\ & + \alpha_1^2\alpha_2^2\alpha_3^2\beta_1 + \alpha_1^2\alpha_2^2\alpha_3^2\beta_3 + 2\alpha_1^2\alpha_2^2\alpha_3^2\beta_2 + \alpha_2\alpha_3^3\beta_1\beta_2^2 + 2\alpha_1^2\alpha_2\alpha_3^3\beta_2 + 2\alpha_1^2\alpha_2\alpha_3^2\beta_2^2 + \alpha_1^2\alpha_3^2\beta_1\beta_2^2 \\ & + \alpha_1\alpha_2^2\alpha_3\beta_1\beta_3 + \alpha_1\alpha_2^2\beta_1^2\beta_2\beta_3 + 2\alpha_1\alpha_2^2\alpha_3\beta_1^2\beta_3 + \alpha_1\alpha_2^2\alpha_3\beta_1^2\beta_2 + \alpha_1\alpha_2^2\alpha_3\beta_1\beta_3^2 + 2\alpha_1\alpha_2^2\alpha_3^2\beta_1\beta_3 \\ & + 2\alpha_1\alpha_2^2\alpha_3\beta\beta_2\beta_3 + 2\alpha_1\alpha_2^2\alpha_3^2\beta_1\beta_2 + 2\alpha_1\alpha_2\alpha_3\beta_1^2\beta_2\beta_3 + 2\alpha_2\alpha_3^2\beta_1^2\beta_2^2 + 2\alpha_1\alpha_2\alpha_3^2\beta_1^2\beta_2 + \alpha_1\alpha_2\alpha_3\beta_1\beta_2\beta_3^2 \\ & + 3\alpha_1\alpha_2\alpha_3^2\beta_1\beta_2\beta_3 + 2\alpha_1\alpha_2\alpha_3\beta_1\beta_2^2\beta_3 + 2\alpha_1\alpha_2\alpha_3^3\beta_1\beta_2 + 2\alpha_1\alpha_2\alpha_3^2\beta_1\beta_2^2 + \alpha_1^2\alpha_2^2\alpha_3\beta_1\beta_3 + \alpha_1^2\alpha_2^2\alpha_3\beta_1\beta_2 \\ & + \alpha_1^2\alpha_2\alpha_3\beta_1\beta_2\beta_3 + 2\alpha_1^2\alpha_2\alpha_3^2\beta_1\beta_2 + \alpha_1^2\alpha_2\alpha_3^2\beta_2\beta_3 + \alpha_2\alpha_3\beta_1^2\beta_2\beta_3^2 + 3\alpha_2^2\alpha_3^2\beta_1^2\beta_3 + 2\alpha_2^2\alpha_3\beta_1^3\beta_3 \end{aligned}$$

4.2. Availability Model Formulation for Configuration II

For the analysis of availability case of configuration II we use the same initial condition as in Subsection 3.2

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \cdots, P_{10}(0)\right] = [1,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \\ \dot{P}_9 \\ \dot{P}_{10} \end{bmatrix} = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -y_1 & 0 & \alpha_3 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & -y_2 & 0 & 0 & \alpha_2 & \alpha_1 & \alpha_3 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & -y_3 & 0 & 0 & \alpha_3 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & \beta_3 & 0 & 0 & -y_4 & 0 & \alpha_3 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -y_5 & 0 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_3 & 0 & -\alpha_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \end{bmatrix}$$

The steady-state availability is given by

$$A_{V2}(\infty) = P_0(\infty) + P_1(\infty) + P_3(\infty) + P_8(\infty) \tag{12}$$

In the steady-state, the derivatives of the state probabilities become zero and therefore Equation (4) become

$$T_2 P = 0 \tag{13}$$

which is in matrix form

$$\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -y_1 & 0 & \alpha_3 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & -y_2 & 0 & 0 & \alpha_2 & \alpha_1 & \alpha_3 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & -y_3 & 0 & 0 & \alpha_3 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & \beta_3 & 0 & 0 & -y_4 & 0 & \alpha_3 & 0 \\ 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -y_5 & 0 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_3 & 0 & -\alpha_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition

$$\sum_{J=0}^{10} P_J(\infty) = 1 \tag{14}$$

Substituting (14) in the last row of (13) to compute the steady-state probabilities, the expression for steady-state availability is given by

$$A_{V2}(\infty) = \frac{N_4}{D_4} \tag{15}$$

$$\begin{aligned} N_4 = {} & \alpha_1^2\alpha_2\alpha_3^2\left(\alpha_1\alpha_3+\alpha_1\beta_1+\alpha_3^2+\alpha_3\beta_3+2\alpha_3\beta_1+\beta_1^2+2\beta_1\beta_3\right) \\ & +\alpha_1\alpha_2\alpha_3^2\beta_1\left(\alpha_1\alpha_3+\alpha_1\beta_1+\alpha_3^2+\alpha_3\beta_3+2\alpha_3\beta_1+\beta_1^2+2\beta_1\beta_3\right) \\ & +\alpha_1\alpha_2\alpha_3\beta_1\beta_3\left(\alpha_1\alpha_3+\alpha_1\beta_1+\alpha_3^2+2\alpha_3\beta_1+\alpha_3\beta_3+\beta_1^2+\beta_1\beta_3\right) \\ & +\alpha_1\alpha_2\alpha_3\beta_1\beta_3^2\left(\alpha_1+\alpha_3+\beta_1+\beta_3\right) \end{aligned}$$

$$\begin{aligned} D_4 = {} & \alpha_1^3\alpha_3^3\beta_2+\alpha_1^2\alpha_2\beta_1^2\beta_3^2+2\alpha_1^2\alpha_2\alpha_3^2\beta_1^2+3\alpha_1^2\alpha_2\alpha_3^3\beta_1+\alpha_1^2\alpha_2\alpha_3^3\beta_3+\alpha_1^2\alpha_3^4\beta_2+2\alpha_1\alpha_2\beta_1^3\beta_3^2 \\ & +\alpha_2\beta_1^4\beta_3^2+\alpha_2\alpha_3^2\beta_1^4+2\alpha_2\beta_1^3\beta_3^3+2\alpha_2\alpha_3^3\beta_1^3+\alpha_2\beta_1^2\beta_3^4+\alpha_2\alpha_3^4\beta_1^2+\alpha_1^2\alpha_2\alpha_3^4+2\alpha_1\alpha_2\alpha_3^2\beta_1^3 \\ & +\alpha_1^3\alpha_2\alpha_3^2\beta_1+2\alpha_1\alpha_2\alpha_3^2\beta_1\beta_3^2+2\alpha_1\alpha_2\beta_1^2\beta_3^3+3\alpha_1\alpha_2\alpha_3^3\beta_1^2+\alpha_1\alpha_2\alpha_3^4\beta_1+\alpha_2\alpha_3\beta_1^4\beta_3+4\alpha_2\alpha_3\beta_1^3\beta_3^2 \\ & +4\alpha_2\alpha_3^2\beta_1^3\beta_3+2\alpha_2\alpha_3\beta_1^2\beta_3^3+\alpha_1^2\alpha_3^3\beta_2\beta_3+3\alpha_2\alpha_3^2\beta_1^2\beta_3^2+2\alpha_2\alpha_3^3\beta_1^2\beta_3+2\alpha_1\alpha_2\alpha_3\beta_1^3\beta_3+5\alpha_1\alpha_2\alpha_3\beta_1^2\beta_3^2 \\ & +5\alpha_1\alpha_2\alpha_3^2\beta_1^2\beta_3+\alpha_1\alpha_2\alpha_3\beta_1\beta_3^3+2\alpha_1\alpha_2\alpha_3^3\beta_1\beta_3+\alpha_1\alpha_3\beta_1^3\beta_2\beta_3+\alpha_1\alpha_3^2\beta_1^3\beta_2+2\alpha_1\alpha_3\beta_1^2\beta_2\beta_3^2 \\ & +4\alpha_1\alpha_3^2\beta_1^2\beta_2\beta_3+2\alpha_1\alpha_3^3\beta_1^2\beta_2+\alpha_1^3\alpha_2\alpha_3^3+\alpha_1\alpha_3\beta_1\beta_2\beta_3^3+\alpha_1^3\alpha_3^2\beta_1\beta_2+2\alpha_1\alpha_3^2\beta_1\beta_2\beta_3^2+2\alpha_1\alpha_3^3\beta_1\beta_2\beta_3 \\ & +\alpha_1\alpha_3^4\beta_1\beta_2+\alpha_1^2\alpha_2\alpha_3\beta_1^2\beta_3+\alpha_1^2\alpha_2\alpha_3\beta_1\beta_3^2+3\alpha_1^2\alpha_2\alpha_3^2\beta_1\beta_3+\alpha_1^2\alpha_3\beta_1^2\beta_2\beta_3+2\alpha_1^2\alpha_3^2\beta_1^2\beta_2 \\ & +\alpha_1^2\alpha_3\beta_1\beta_2\beta_3^2+3\alpha_1^2\alpha_3^3\beta_1\beta_2+3\alpha_1^2\alpha_3^2\beta_1\beta_2\beta_3 \end{aligned}$$

4.3. Availability Model Formulation for Configuration III

For the analysis of availability case of configuration III we use the same initial condition as in Subsection 3.3

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)\right] = [1,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$
\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \end{bmatrix} = \begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_1+\beta_2+\beta_3) & 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 \\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & -(\alpha_1+\beta_3) & 0 & 0 & \alpha_3 & 0 \\ 0 & \beta_2 & 0 & 0 & -(\alpha_2+\beta_3) & 0 & 0 & \alpha_3 \\ 0 & \beta_3 & 0 & 0 & 0 & -(\alpha_3+\beta_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & 0 & \beta_3 & 0 & \beta_1 & -\alpha_3 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & \beta_2 & 0 & -(\alpha_2+\alpha_3) \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix}
$$

The steady-state availability is given by

$$A_{V3}(\infty) = P_0(\infty) + P_1(\infty) + P_5(\infty) \tag{16}$$

In the steady-state, the derivatives of the state probabilities become zero and therefore Equation (6) become

$$T_3 P = 0 \tag{17}$$

which is in matrix form

$$
\begin{bmatrix} -(\beta_1+\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\alpha_1+\beta_1+\beta_2+\beta_3) & 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 \\ \beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_1 & 0 & -(\alpha_1+\beta_3) & 0 & 0 & \alpha_3 & 0 \\ 0 & \beta_2 & 0 & 0 & -(\alpha_2+\beta_3) & 0 & 0 & \alpha_3 \\ 0 & \beta_3 & 0 & 0 & 0 & -(\alpha_3+\beta_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & 0 & \beta_3 & 0 & \beta_1 & -\alpha_3 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & \beta_2 & 0 & -(\alpha_2+\alpha_3) \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Using the following normalizing condition

$$\sum_{J=0}^{7} P_J(\infty) = 1 \tag{18}$$

Substituting (18) in the last row of (17) to compute the steady-state probabilities, the expression for steady-state availability is given by

$$A_{V3}(\infty) = \frac{N_5}{D_5} \tag{19}$$

$$
\begin{aligned}
N_5 = {} & \alpha_1^2\alpha_2\alpha_3\left(\alpha_2\beta_1 + \alpha_2\alpha_3 + \alpha_3^2 + \alpha_3\beta_1 + \alpha_3\beta_3 + \alpha_3\beta_2 + \beta_1\beta_3\right) + \alpha_1\alpha_2\alpha_3\beta_1\beta_3\left(\alpha_2 + \alpha_3 + \beta_1 + \beta_3\right) \\
& + \alpha_1\alpha_2\alpha_3\beta_1\left(\alpha_2\beta_1 + \alpha_2\alpha_3 + \alpha_3^2 + \alpha_3\beta_1 + \alpha_3\beta_3 + \alpha_3\beta_2 + \beta_1\beta_3\right)
\end{aligned}
$$

$$
\begin{aligned}
D_5 = {} & \alpha_2\beta_1^3\beta_3^2 + \alpha_2\beta_1^2\beta_3^3 + \alpha_1^2\alpha_2\alpha_3^2\beta_1 + \alpha_1^2\alpha_2\alpha_3^3 + 2\alpha_1\alpha_2\alpha_3^2\beta_1\beta_3 + \alpha_1\alpha_2\beta_1^2\beta_3^2 + \alpha_1^2\alpha_3^2\beta_2^2 + \alpha_2\alpha_3^2\beta_1^2\beta_2 \\
& + \alpha_1\alpha_3\beta_1\beta_2^2\beta_3 + \alpha_2^2\alpha_3\beta_1^3 + \alpha_1\alpha_2^2\alpha_3\beta_1\beta_3 + 2\alpha_1\alpha_2\alpha_3\beta_1\beta_2\beta_3 + 2\alpha_1\alpha_2\alpha_3^2\beta_1\beta_2 + \alpha_1^2\alpha_2\alpha_3\beta_1\beta_2 + \alpha_1\alpha_2\alpha_3^3\beta_1 \\
& + \alpha_1\alpha_2\alpha_3^2\beta_1^2 + \alpha_1\alpha_2\beta_1^2\beta_2\beta_3 + \alpha_1^2\alpha_3^2\beta_1\beta_2 + \alpha_1\alpha_3^2\beta_1\beta_2^2 + \alpha_2\alpha_3^2\beta_1^3 + \alpha_2\alpha_3^3\beta_1^2 + \alpha_1\alpha_3^2\beta_1^2\beta_2 + \alpha_1\alpha_3^3\beta_1\beta_2 \\
& + \alpha_1^2\alpha_3\beta_1\beta_2\beta_3 + 2\alpha_1\alpha_3^2\beta_1\beta_2\beta_3 + 3\alpha_2\alpha_3^2\beta_1^2\beta_3 + 2\alpha_2\alpha_3\beta_1^3\beta_3 + 3\alpha_2\alpha_3\beta_1^2\beta_3^2 + \alpha_2^2\beta_1^3\beta_3 + 2\alpha_2^2\alpha_3\beta_1^2\beta_3 \\
& + \alpha_1\alpha_2^2\beta_1^2\beta_3 + \alpha_1\alpha_2^2\alpha_3\beta_1^2 + \alpha_1^2\alpha_2^2\alpha_3\beta_1 + \alpha_1\alpha_2^2\alpha_3^2\beta_1 + 2\alpha_1^2\alpha_2\alpha_3^2\beta_2 + \alpha_2^2\beta_1^2\beta_3^2 + \alpha_2^2\alpha_3^2\beta_1^2 + \alpha_1^2\alpha_2^2\alpha_3^2 \\
& + \alpha_1^2\alpha_2\alpha_3^2\beta_3 + \alpha_1^2\alpha_3^3\beta_2 + 2\alpha_2\alpha_3\beta_1^2\beta_2\beta_3 + \alpha_1\alpha_2\alpha_3\beta_1^2\beta_2 + \alpha_2\beta_1^2\beta_2\beta_3^2 + 2\alpha_1\alpha_2\alpha_3\beta_1^2\beta_3 + \alpha_1\alpha_2\alpha_3\beta_1\beta_3^2 \\
& + \alpha_1^2\alpha_2\alpha_3\beta_1\beta_3 + \alpha_1\alpha_3\beta_1^2\beta_2\beta_3 + \alpha_1^2\alpha_3^2\beta_2\beta_3 + \alpha_1\alpha_3\beta_1\beta_2\beta_3^2
\end{aligned}
$$

5. Comparison of the Three Configurations

In this section, we numerically compare the results for availability and MTSF for the developed models for the three configurations.

Case I:

We fix $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.4$, $\alpha_2 = 0.7$, $\alpha_3 = 0.9$ and vary α_1 between 0 to 1 for **Figure 1**, $\beta_2 = 0.4$, $\beta_3 = 0.4$, $\alpha_1 = 0.5$, $\alpha_2 = 0.7$, $\alpha_3 = 0.9$ and vary β_1 between 0 to 1 for **Figure 2**.

Case II:

We fix $\beta_1 = 0.02$, $\beta_2 = 0.04$, $\beta_3 = 0.04$, $\alpha_1 = 0.05$, $\alpha_3 = 0.09$ and vary α_2 between 0 to 1 for **Figure 3** and $\beta_1 = 0.02$, $\beta_3 = 0.04$, $\alpha_1 = 0.05$, $\alpha_3 = 0.3$, $\alpha_3 = 0.09$ and vary β_2 between 0 to 1 for **Figure 4**.

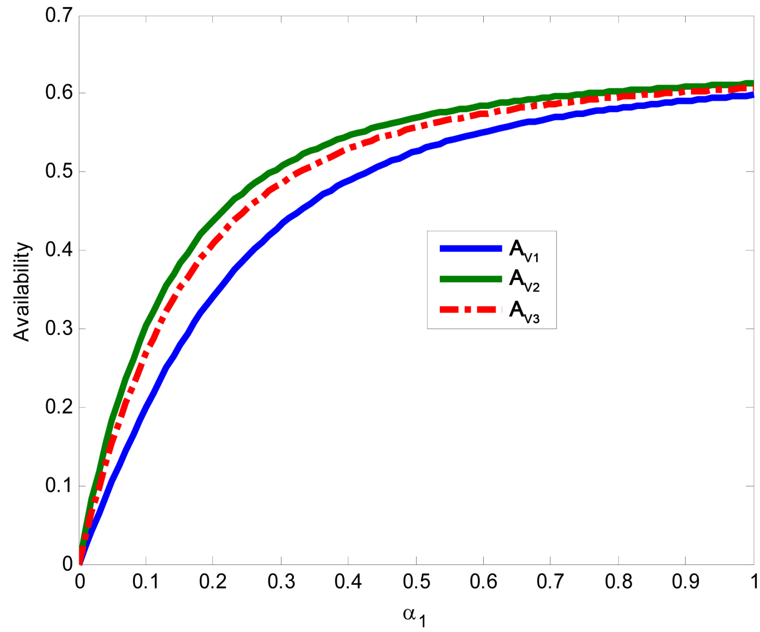

Figure 1. Availability against α_1.

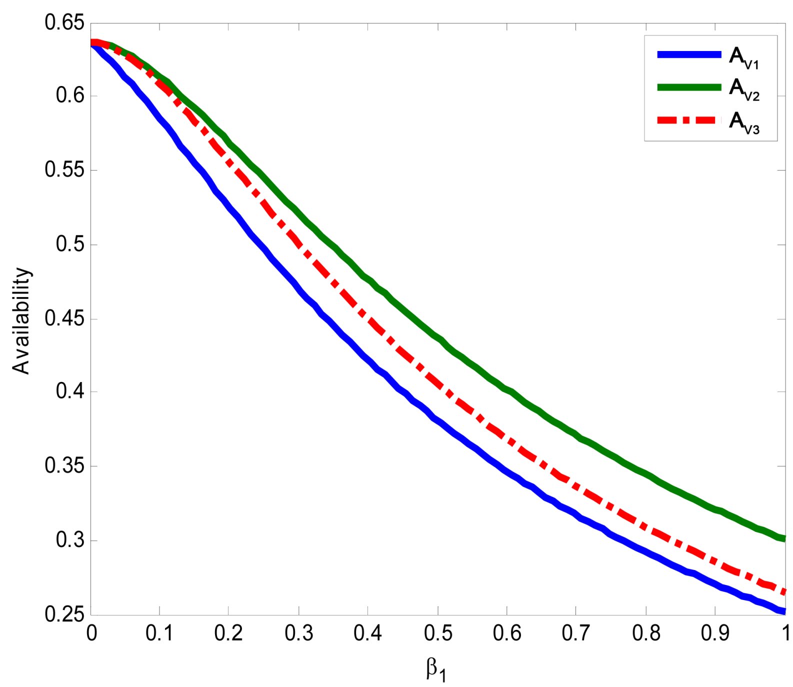

Figure 2. Availability against β_1.

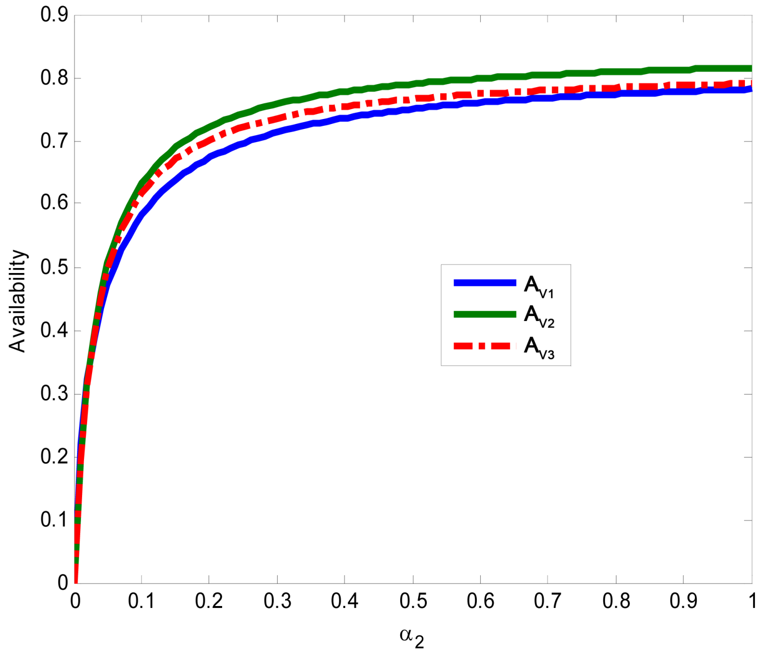

Figure 3. Availability against α_2.

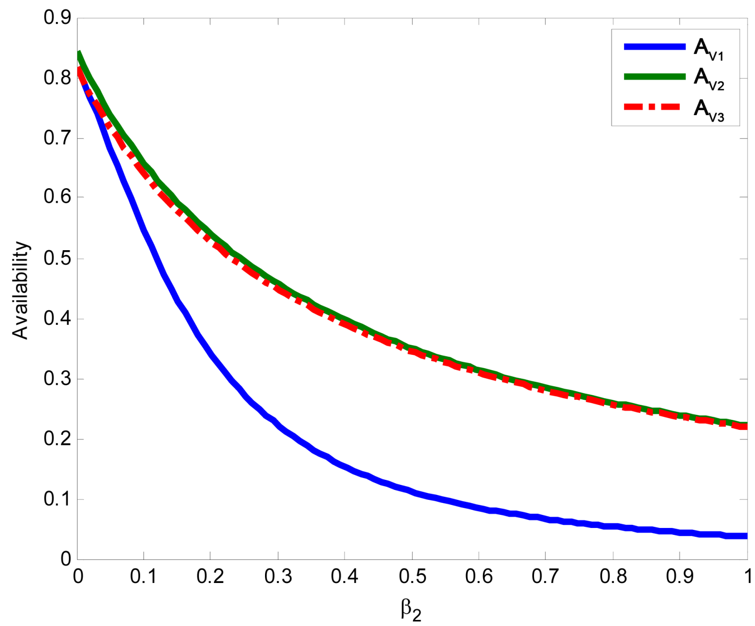

Figure 4. Availability against β_2.

Case III:

We fix $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.4$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$ and vary α_3 between 0 to 1 for **Figure 5** and $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\alpha_3 = 0.4$ and vary β_3 between 0 to 1 for **Figure 6**.

Case IV:

We fix $\beta_1 = 0.02$, $\beta_2 = 0.04$, $\beta_3 = 0.9$, $\alpha_3 = 0.8$ and vary α_1 between 0 to 1 for **Figure 7**, $\beta_1 = 0.02$, $\beta_2 = 0.04$, $\beta_3 = 0.9$, $\alpha_1 = 0.9$ and vary α_3 between 0 to 1 for **Figure 8** and $\beta_1 = 0.02$, $\beta_2 = 0.04$, $\alpha_1 = 0.9$, $\alpha_3 = 0.8$ and vary β_3 between 0 to 1 for **Figure 9**, and vary β_1 and β_2 for **Figures 9-11**.

From **Figure 1**, the availability results for the three systems being studied against the repair rate α_1. It is

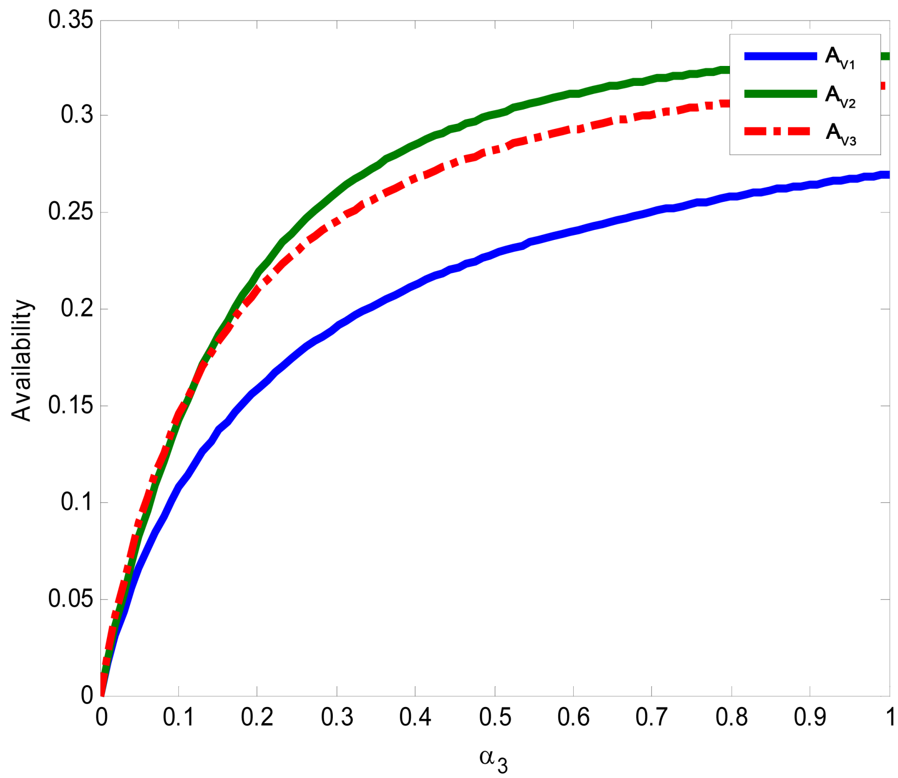

Figure 5. Availability against α_3.

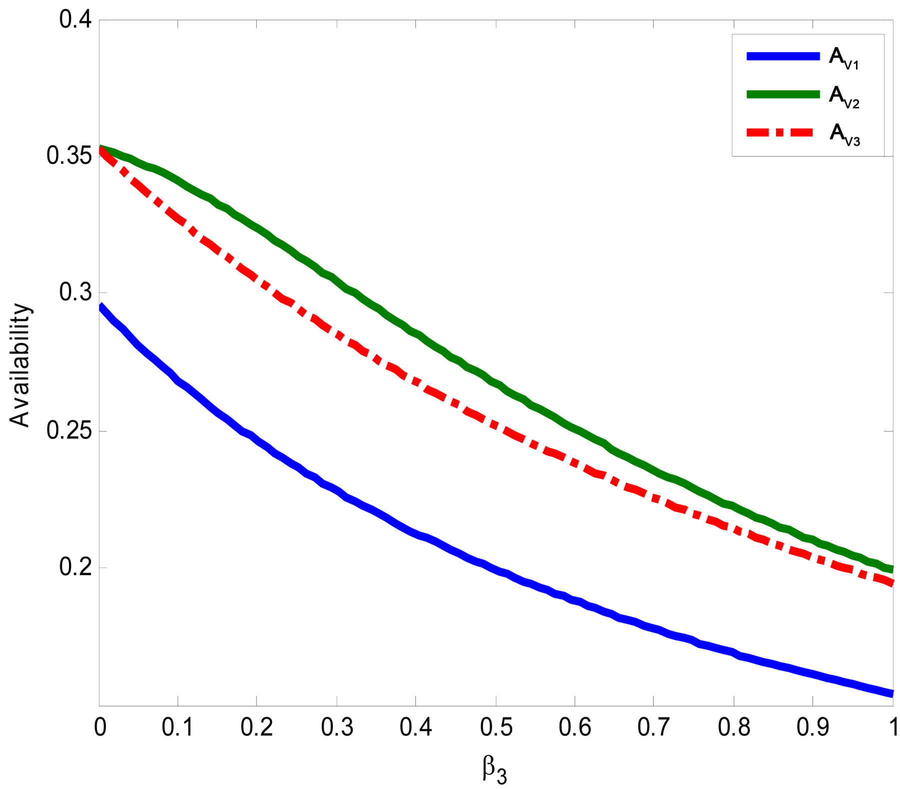

Figure 6. Availability against β_3.

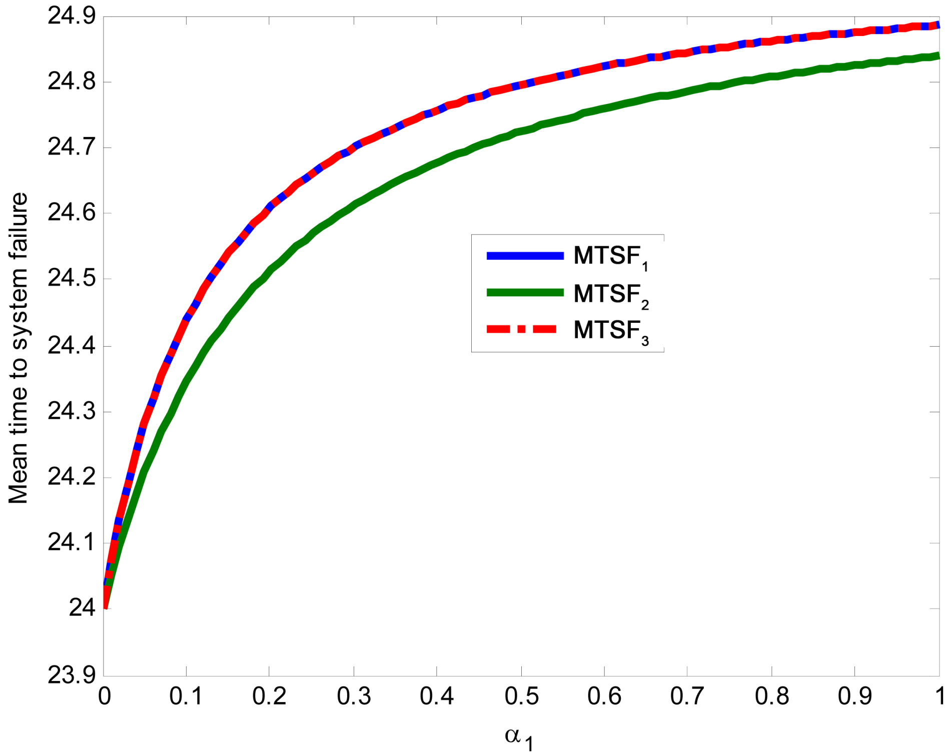

Figure 7. MTSF against α_1.

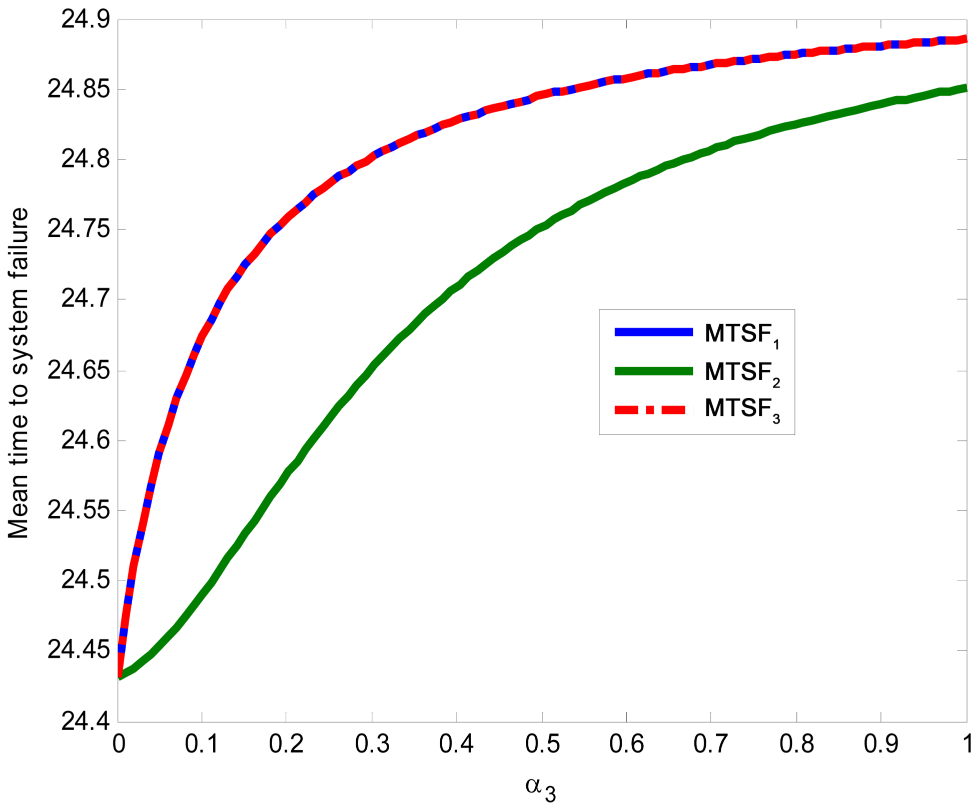

Figure 8. MTSF against α_3.

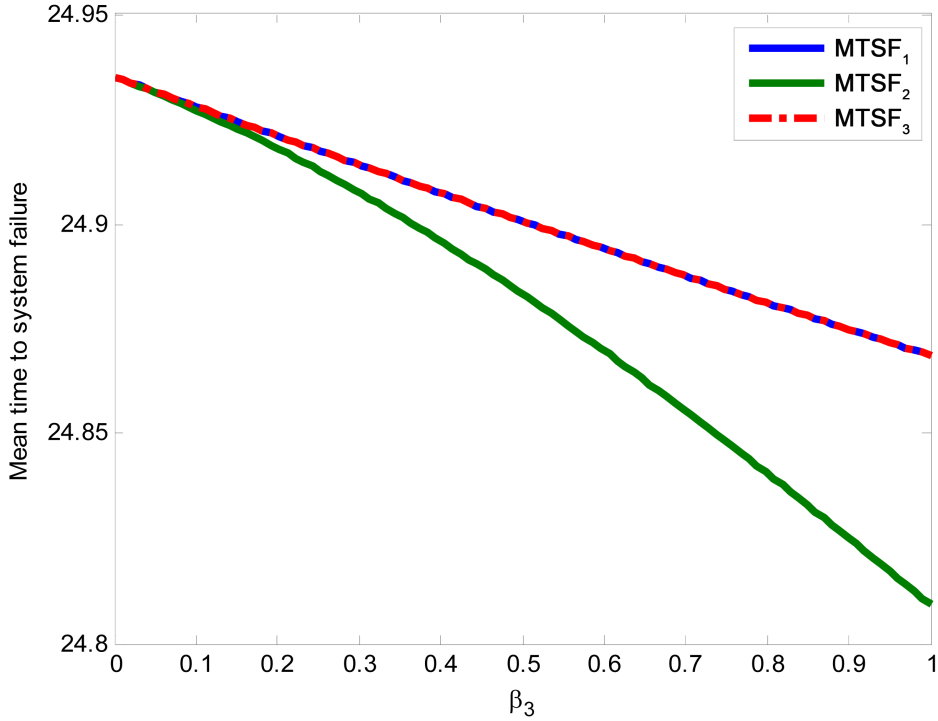

Figure 9. MTSF against β_3.

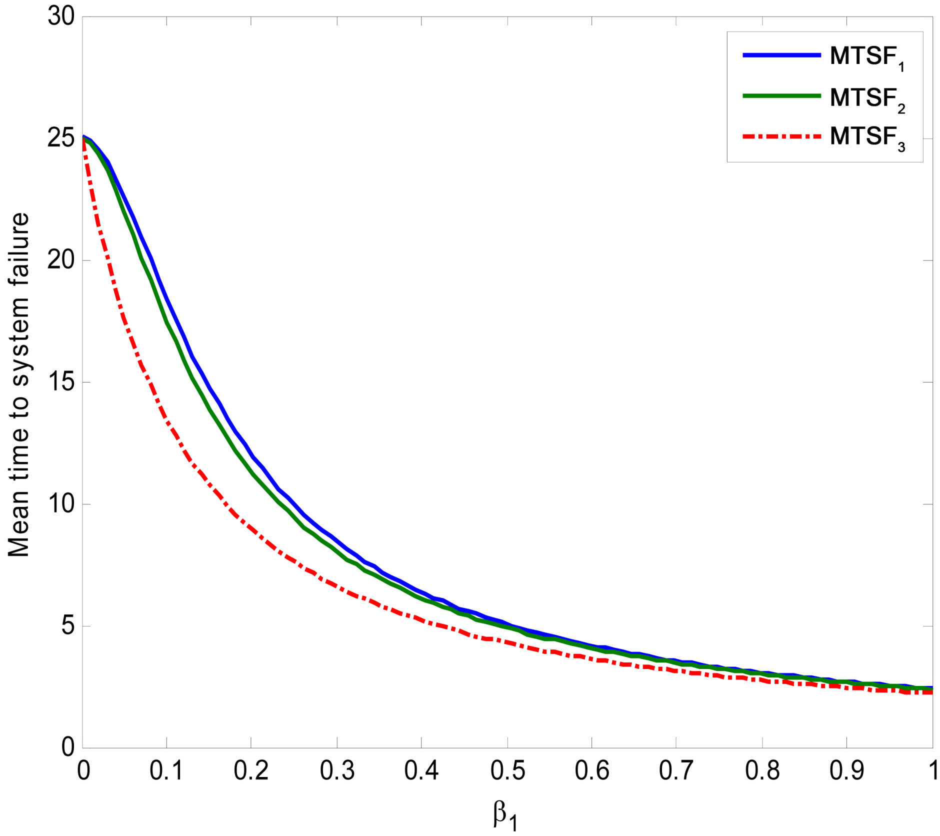

Figure 10. MTSF against β_1.

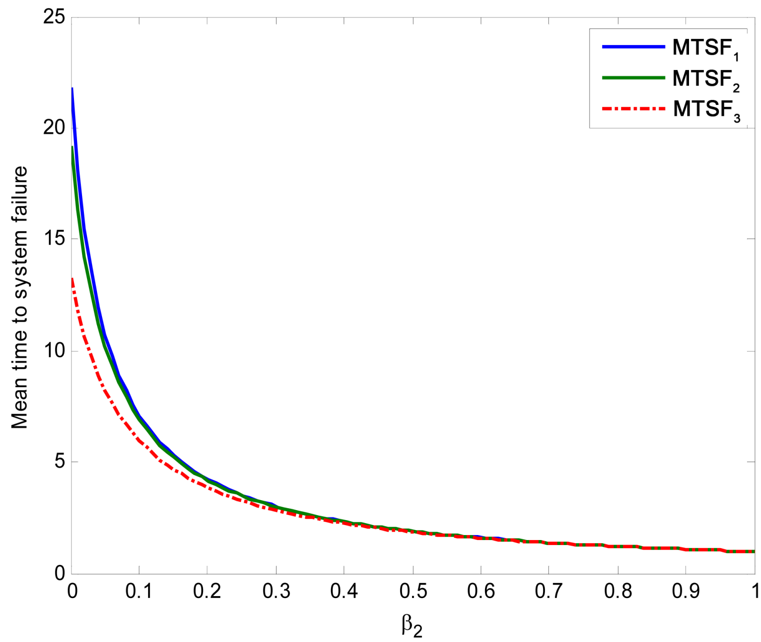

Figure 11. MTSF against β_2.

clear from the Figure that configuration II has higher availability with respect to α_1 as compared with the other two configurations. There is slight difference between the availability of configuration II and that of configuration III with respect to α_1. These tend to suggest that configuration II is better than the other configurations. **Figure 2** depicts the availability calculations for the three configurations against β_1. The observations that can be made here are much similar to those made on **Figure 1**. From **Figure 1** and **Figure 2**, it is clear that $A_{V2}(\infty) > A_{V3}(\infty) > A_{V1}(\infty)$.

However, one can say that the results from **Figure 3** show slight distinction between availability of three configurations with respect to α_2. The differences between availability of configuration II and the other two configurations slightly increase as α_2 increases. There is significant difference between the three configurations with respect to β_2 in **Figure 4**. It is evident from **Figure 4** that configurations II and III have higher availability than configuration I as β_2 increases. Thus, $A_{V2}(\infty) > A_{V3}(\infty) > A_{V1}(\infty)$ for **Figure 3** and $A_{V2}(\infty) \geq A_{V3}(\infty) > A_{V1}(\infty)$ for **Figure 4**.

Results from **Figure 5** and **Figure 6** show slight distinction between availability of three configurations with respect to α_3 and β_3. The differences between availability of configuration II and the other two configurations widen as α_3 and β_3 increases respectively. It is evident from **Figure 5** and **Figure 6** that configurations II has higher availability than configuration I and III as α_3 and β_2 increases. Thus, $A_{V2}(\infty) > A_{V3}(\infty) > A_{V1}(\infty)$.

Simulations of MTSF for the three configurations depicted in **Figures 7-9** show that MTSF increases as α_1 and α_3, and decreases as β_3 increases for any configuration. It is clear from these Figures that differences between MTSF of configuration I and III and configuration II widen as α_1, α_3 and β_3 increases respectively. It is evident from these Figures that configuration I and III have equal MTSF higher than configuration II as α_1, α_3 and β_3 increases. **Figure 10** and **Figure 11** show that the MTSF decreases as β_1 and β_2 for any configuration. It is evident from **Figure 10** and **Figure 11** that configurations I has higher MTSF than configuration II and III as β_1 and β_2 increases. Thus, from **Figures 7-9**, configuration I and II have equal MTSF. From **Figure 10** the optimal configuration is configuration I while in **Figure 11**, configuration I and II have equal MTSF.

6. Conclusion

In this paper, we studied the reliability characteristics of three dissimilar systems connected supporting device. We developed the explicit expressions for steady-state availability and mean time to system failure (MTSF) for each configuration and performed comparative analysis numerically to determine the optimal configuration. It is evident from **Figures 1-6** that configuration II is optimal configuration using steady-state availability while using MTSF, the optimal configuration depends on the values of α_1, α_3, β_1, β_2 and β_3. The present study will help the engineers and designers to develop sophisticated models and to design more critical system in interest of human kind.

References

[1] Khatab, A., Nahas, N. and Nourelfath, M. (2009) Availability of k-out-of-n: G Systems with Non Identical Components Subject to Repair Priorities. *Reliability Engineering & System Safety*, **94**, 142-151. http://dx.doi.org/10.1016/j.ress.2008.02.017

[2] Fathabadi, H.S. and Khodaei, M. (2012) Reliability Evaluation of Network Flows with Stochastic Capacity and Cost Constraint. *International Journal of Mathematics in Operational Research*, **4**, 439-452. http://dx.doi.org/10.1504/IJMOR.2012.048904

[3] Khalili-Damghani, K. and Amiri, M. (2012) Solving Binary-State Multi-Objective Reliability Redundancy Allocation Seriesparallel Problem Using Efficient Epsilon Constraint, Multi-Start Partial Bound Enumeration Algorithm, and DEA. *Reliability Engineering and System Safety*, **103**, 35-44. http://dx.doi.org/10.1016/j.ress.2012.03.006

[4] Khalili-Damghani, K., Abtahi, A.R., and Tavana, M. (2013) A New Multi-Objective Particle Swarm Optimization Method for Solving Reliability Redundancy Allocation Problems. *Reliability Engineering and System Safety*, **111**, 58-75. http://dx.doi.org/10.1016/j.ress.2012.10.009

[5] Wang, K.-H., Yen, T.-C. and Fang, Y.-C. (2012) Comparison of Availability between Two Systems with Warm Standby Units and Different Imperfect Coverage. *Quality Technology and Quantitative Management*, **9**, 265-282

[6] Srinivasan, S.K. and Subramanian, R. (2006) Reliability Analysis of a Three Unit Warm Standby Redundant System with Repair. *Annals of Operations Research*, **143**, 227-235. http://dx.doi.org/10.1007/s10479-006-7384-z

[7] Singh, V.V., Singh, B., Mangeyram and Goel, C.K. (2010) Availability Analysis of a System Having Three Units Super Priority, Priority and Ordinary under Peemmptive Resume Repair Policy. *International Journal of Reliability and Applications*, **11**, 41-53.

[8] Singh, V.V. and Rawal, D.K. (2011) Availability Analysis of a System Having Two Units in Series Configuration with Controller and Human Failure under Different Repair Policies. *International Journal of Scientific and Engineering Research*, **2**, 1-9.

[9] Yusuf, I. (2013) Comparison of Some Reliability Characteristics between Redundant Systems Requiring Supporting Units for Their Operation. *Journal of Mathematical and Computational Sciences*, **3**, 216-232.

[10] Yusuf, I. (2014) Comparative Analysis of Profit between Three Dissimilar Repairable Redundant Systems Using Supporting External Device for Operation. *Journal of Industrial Engineering International*, **10**, 2-9. http://dx.doi.org/10.1007/s40092-014-0077-3

[11] Wang, K.H. and Kuo, C.C. (2000) Cost and Probabilistic Analysis of Series Systems with Mixed Standby Components. *Applied Mathematical Modelling*, **24**, 957-967. http://dx.doi.org/10.1016/S0307-904X(00)00028-7

[12] Wang, K., Hsieh, C. and Liou, C. (2006) Cost Benefit Analysis of Series Systems with Cold Standby Components and a Repairable Service Station. *Journal of Quality Technology and Quantitative Management*, **3**, 77-92.