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Alternative Approaches of Convolution within Network Calculus

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Abstract

Network Calculus is a powerful mathematical theory for the performance evaluation of communication systems; among others it allows to determine worst-case performance measures. This is why it is often used to appoint Quality of Service guarantees in packet-switched systems like the internet. The main mathematical operation within this deterministic queuing theory is the min-plus convolution of two functions. For example the convolution of the arrival and service curve of a system which reflects the data's departure. Considering Quality of Service measures and performance evaluation, the convolution operation plays a considerable important role, similar to classical system theory. Up to the present day, in many cases it is not practical and simple to perform this operation. In this article we describe approaches to simplify the min-plus convolution and, accordingly, facilitate the corresponding calculations.

Keywords

Network Calculus, Min-Plus Convolution, Algebraic Laws, Convex Analysis

1. Introduction

Timeliness plays an important role regarding systems with real time requirements. This *Quality of Service* (QoS) requirement can be found in many kinds of embedded systems which permanently exchange data with their environment; like safety-critical automotive systems or real time networks. Since knowledge of mean values is not sufficient, analytical performance evaluation of such systems cannot be based on stochastic modeling as applied in traditional queuing theory. For these systems worst-case performance parameters like maximum delay of service times are required. In order to specify guarantees of performance figures—in terms of bounding values which are valid in any case—a mathematical tool *Network Calculus* (NC) as a novel system theory for deterministic queuing theory has been developed [1].

The development of this theory has been started by Cruz [2] [3] on the (σ, ρ) traffic description and his calculus for network delay. Further steps towards NC were taken by the work of Parekh and Gallagher [4] to determine the service curve of Generalized Processor Sharing (GPS) schedulers. The NC framework has been successfully applied for the analysis and dimensioning in various domains, including industrial automation networks [5] or automotive communications bus systems [6].

This article is structured as follows, in Section 2 we describe the basic modeling elements of NC theory. The

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subsequent Section 3 demonstrates the difficulties of convolution calculation and describes possible solution approaches. Finally, Section 4 draws a conclusion and indicates future steps.

2. Basic Modeling Elements of Network Calculus

The most important modeling elements of NC are the *arrival curve* (denoted by α in the figures) and the service curve (denoted by β in the figures) together with the *min-plus convolution*. The arrival and the service curve are the basis for the computation of maximum deterministic boundary values like backlog and delay bounds [1].

Definition 1 (Arrival curve) *Let* $\alpha(t)$ *be a non-negative, non-decreasing function. Flow* F *with input* $x(t)$ *at time t is constrained by or has arrival curve* $\alpha(t)$ *iff* $x(t)-x(s)\le\alpha(t-s)$ *for all* $t\ge s\ge 0$. *Flow* F *is also called* α *-smooth.*

Example 1 *A commonly used arrival curve is the token bucket constraint*:

$$\alpha_{r,b}(t)=\begin{cases} b+rt & : \quad t>0 \\ 0 & : \quad t\le 0 \end{cases} \tag{1}$$

Figure 1 shows an arrival curve which represents an upper limit for a given traffic flow $x(t)$ with an average rate r and an instantaneous burst b. Therefore:

$$x(t)-x(s)\le\alpha_{r,b}(t-s).$$

For $\Delta t := t-s$ and $\Delta t\to 0$:

$$\lim_{t\to s}\{x(t)-x(s)\}\le\lim_{\Delta t\to 0}\{r\cdot\Delta t+b\}=b \tag{2}$$

Next the convolution operation, which plays the most important role in Network Calculus will be defined.

Definition 2 (Min-plus convolution) *Let* $f(t)$ *and* $g(t)$ *be non-negative, non-decreasing functions that are* 0 *for* $t\le 0$. *A third function, called min-plus convolution is defined as*

$$(f\otimes g)(t)=\inf_{0\le s\le t}\{f(s)+g(t-s)\} \tag{3}$$

On this basis we can characterize the arrival curve $\alpha(t)$ with respect to a given $x(t)$ as: $x(t)\le(x\otimes\alpha)(t)$

In other words, arrival curves describe an upper bound to the input stream of a system. Considering the system's output, we might be interested in service guarantees: like a minimum of output $y(t)$, *i.e.* the amount of data that leaves the system. The following definition deals with this problem.

Definition 3 (Service curve) *Let a system* S *with input flow* $x(t)$ *and output flow* $y(t)$ *be given. The system provides a (minimum) service curve* $\beta(t)$ *to the flow, iff* $\beta(t)$ *is a non-negative, non-decreasing function with* $\beta(0)=0$ *and if* $y(t)$ *is lower bounded by the convolution of* $x(t)$ *and* $\beta(t)$, *i.e.*:

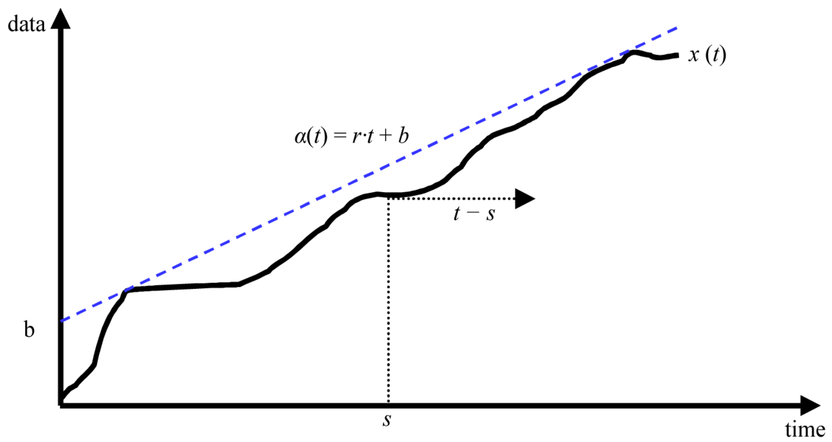

Figure 1. Token bucket arrival curve.

$$y(t) \geq (x \otimes \beta)(t) \tag{4}$$

Figure 2 shows $(x \otimes \beta)(t) = \inf_{0 \leq s \leq t} \{x(s) + \beta(t-s)\}$ as lower bound of output $y(t)$ and input $x(t)$. Such service curves are functions of the time and describe the service of network elements like routers or schedulers in an abstract manner [7].

Example 2 *A commonly used service curve of practical applications is the rate-latency function*:

$$\beta(t) = \beta_{R,T}(t) = R \cdot [t-T]^+ := R \cdot \max\{0, t-T\} \tag{5}$$

This function reflects a service element which offers a minimum service of rate R after a worst-case latency of T. Since we want to analyze the worst-case performance, it is possible to abstractly model the complex internal behavior of the node by just describing the worst-case service using this curve. This is very important in practical utilization.

In **Figure 4**, the (green) line depicts the rate-latency service curve with rate R and latency T.

Consider a system with input flow $x(t)$, arrival curve $\alpha(t)$, output flow $y(t)$ and service curve $\beta(t)$. According to [1] the following three bounds can be derrived:

- Backlog Bound v:

$$v(t) = x(t) - y(t) \leq \sup_{s \geq 0} \{\alpha(s) - \beta(s)\}$$

- Delay Bound d in case of FIFO service:

$$d \leq \sup_{s \geq 0} \{\inf\{\tau : \alpha(s) \leq \beta(s+\tau)\}\}$$

- Output Bound $\alpha^*(t)$:

$$\alpha^*(t) = \alpha \oslash \beta := \sup_{s \geq 0} \{\alpha(t+s) - \beta(s)\}$$

Remark:

The complete backlog $v(t) = x(t) - y(t)$ at time t within a system is also called *unfinished work* or *buffer* (t). The expression $\inf\{\tau : \alpha(s) \leq \beta(s+\tau)\}$ is called *virtual delay*: the minimal time distance which is necessary for input x for being served to output. Thus, the backlog and delay bound are the maximal vertical and horizontal deviation between arrival and service curve. **Figure 3** depicts d and v for a given arrival- and service curve.

Example 3 *Let a system with a token bucket-smooth input and rate-latency service be given, thus*:

$$x(t) - x(s) \leq \alpha_{r,b}(t-s)$$

and

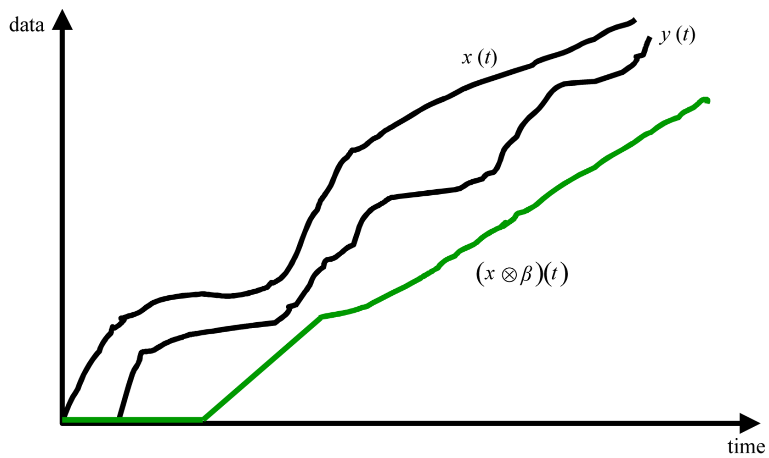

Figure 2. Convolution as a lower output bound.

$$y(t) \geq \inf_{s \leq t}\left\{x(s)+\beta_{R,T}(t-s)\right\}.$$

Based on the three bounds above we can calculate the delay bound as $d \leq T+\frac{b}{R}$, the output bound as $\alpha^*(t)=r(t+T)+b$, and the backlog bound as $v=b+rT$. **Figure 4** shows these results.

3. Difficulties of the Convolution Operation

Within the *Network Calculus* (NC) theory, the convolution operation of so-called min-plus algebra is considered the most essential operation. This is because—based on the arrival- and service curve—it computes the departure of a network element, therefore it is the main instrument of analytical performance evaluation. For example in [1], this operation is carried out "by hand" using pure analytical calculation.

Take Example 3 for which $\alpha_{r,b} \otimes \beta_{R,T}$ as the lower bound for output $y: y \geq \alpha_{r,b} \otimes \beta_{R,T}$ needs to be computed.

3.1. Analytical Computation of Convolution Operator ⊗

a) $0 \leq t \leq T$:

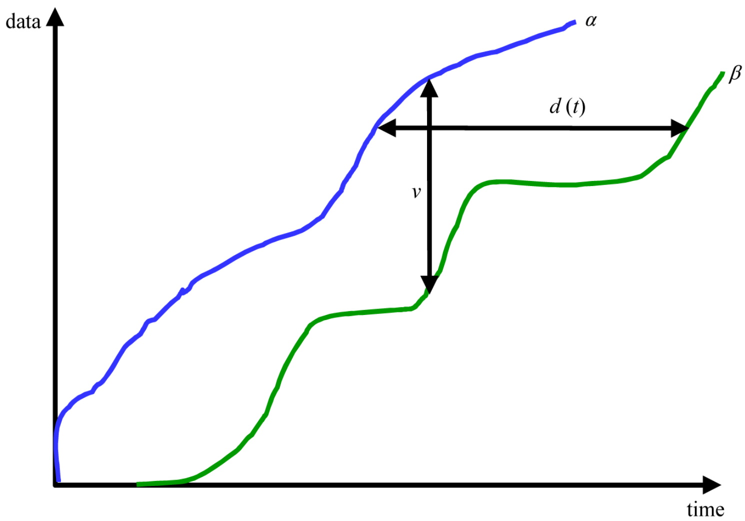

Figure 3. Backlog and delay bound.

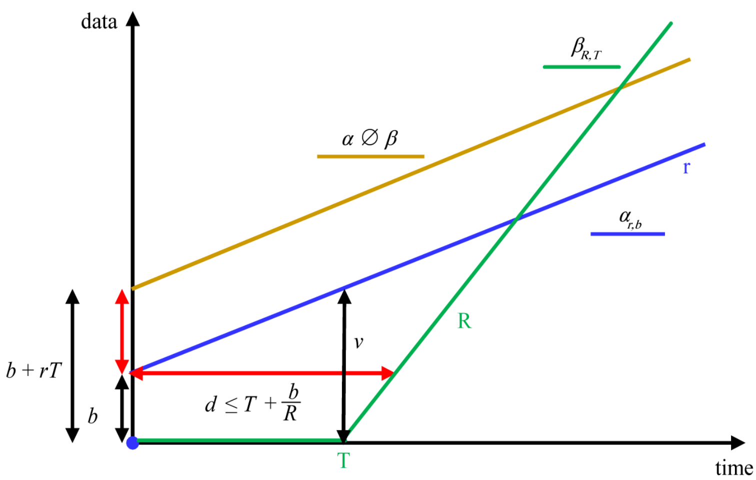

Figure 4. Example for the bounds.

$$\left(\alpha_{r,b}\otimes\beta_{R,T}\right)(t)=\inf_{s\le t}\left\{\alpha_{r,b}(t-s)+R[s-T]^{+}\right\}=\inf_{s\le t}\left\{\alpha_{r,b}(t-s)+0\right\}=\alpha_{r,b}(0)+0=0$$

b) $t>T$:

$$\begin{aligned}
&\left(\alpha_{r,b}\otimes\beta_{R,T}\right)(t)\\
&=\inf_{s\le t}\left\{\alpha_{r,b}(t-s)+R[s-T]^{+}\right\}\\
&=\inf_{s\le T}\left\{\alpha_{r,b}(t-s)+R[s-T]^{+}\right\}\wedge\inf_{T\le s\le t}\left\{\alpha_{r,b}(t-s)+R[s-T]^{+}\right\}\wedge\inf_{s=t}\left\{\alpha_{r,b}(t-s)+R[s-T]^{+}\right\}\\
&=\inf_{s\le T}\left\{b+r(t-s)+0\right\}\wedge\inf_{T<s<t}\left\{b+r(t-s)+R(s-T)\right\}\wedge\left\{0+R(t-T)\right\}\\
&=\left\{b+r(t-T)\right\}\wedge\left\{b+rt-RT+\inf_{T<s<t}\left\{(R-r)s\right\}\right\}\wedge\left\{R(t-T)\right\}\\
&=\left\{b+r(t-T)\right\}\wedge\left\{b+r(t-T)\right\}\wedge\left\{R(t-T)\right\}\\
&=\left\{b+r(t-T)^{+}\right\}\wedge\left\{R(t-T)^{+}\right\}.
\end{aligned}\tag{6}$$

Here $A\wedge B=\min\{A,B\}$ and convolution $\left(\alpha_{r,b}\otimes\beta_{R,T}\right)$ with result (6) is depicted in **Figure 5**.

For the following convolution of two rate-latency service curves (*concatenation*) a similarly complex calculation is required.

$$\beta=\beta_{R_1,T_1}\otimes\beta_{R_2,T_2}=\beta_{\min(R_1,R_2),T_1+T_2}\tag{7}$$

As we can see from above, this analytical approach is cumbersome and error-prone. But, similar to convolution of classical systems theory, the convolution $\otimes$ is of great importance for NC theory. This is why we are looking for other more elegant methods to computation.

Next, two alternative approaches called *Use of algebraic laws* and *Use of convex analysis* will be introduced.

3.2. Use of Algebraic Laws

In the following, the NC-relevant functions Φ are defined as non-negative, non-decreasing, and passing through the origin:

$$\Phi=\left\{f: f(t_1)\ge f(t_0)\ge 0\ \ \forall t_1\ge t_0,\ f(0)=0\right\}\tag{8}$$

Among others, the following algebraic properties for the set of functions Φ can be found in literature [1]

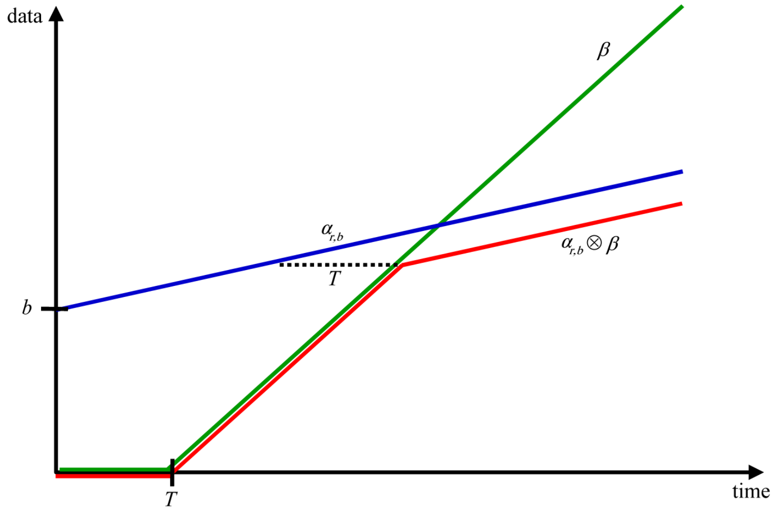

Figure 5. Convolution of $\alpha_{r,b}\otimes\beta_{R,T}$.

- Rule 1 (Commutativity of $\otimes$): $f \otimes g = g \otimes f$
- Rule 2 (Associativity of $\otimes$): $(f \otimes g) \otimes h = f \otimes (g \otimes h)$
- Rule 3 (Distributivity of $\otimes$ and $\wedge$): $h \otimes (f \wedge g) = (h \otimes f) \wedge (h \otimes g)$
- Rule 4 (Addition of a constant): $g \otimes (K + f) = K + (g \otimes f) \quad \forall K \geq 0$
- Rule 5 (Concave functions passing through the origin): $f \otimes g = \min\{f, g\}$
- Rule 6 (Shifting of f to right): $(f \otimes \delta_T)(t) = f(t-T)^+$ with $\delta_T(t) = 0$ for $t \leq T$ and ∞ otherwise.

Now, we only apply these algebraic rules to compute the convolution $\otimes$.

We consider Example 3 in an even more general form:

$\alpha_{r,b} \otimes (K + \beta_{R,T})$ with any constant K; let $\lambda_r = r \cdot t$

$$
\begin{aligned}
&\alpha_{r,b} \otimes (K + \beta_{R,T}) \\
&\overset{\text{(Rule1)}}{=} (K + \beta_{R,T}) \otimes \alpha_{r,b} \overset{\text{(Rule6)}}{=} (K + \delta_T \otimes \lambda_R) \otimes \alpha_{r,b} \overset{\text{(Rule4)}}{=} K + ((\delta_T \otimes \lambda_R) \otimes \alpha_{r,b}) \\
&\overset{\text{(Rule2)}}{=} K + (\delta_T \otimes (\lambda_R \otimes \alpha_{r,b})) \overset{\text{(Rule5)}}{=} K + (\delta_T \otimes (\lambda_R \wedge \alpha_{r,b})) \overset{\text{(Rule3)}}{=} K + (\delta_T \otimes \lambda_R) \wedge (\delta_T \otimes \alpha_{r,b}) \\
&\overset{\text{(Rule6)}}{=} K + [\beta_{R,T} \wedge (\delta_T \otimes (\lambda_r + b))] \overset{\text{(Rule4)}}{=} K + [\beta_{R,T} \wedge ((\delta_T \otimes \lambda_r) + b)] \overset{\text{(Rule6)}}{=} K + (\beta_{R,T} \wedge (\beta_{r,T} + b)) \\
&= K + \{b + r(t-T)^+\} \wedge \{R(t-T)^+\}.
\end{aligned}
\tag{9}
$$

Or take the concatenation example: $\beta = \beta_{R_1,T_1} \otimes \beta_{R_2,T_2}$:

$$
\begin{aligned}
\beta &= R_1(t-T_1)^+ \otimes R_2(t-T_2)^+ \overset{\text{(Rule6)}}{=} (R_1 t \otimes \delta_{T_1}(t)) \otimes (R_2 t \otimes \delta_{T_2}(t)) \overset{\text{(Rule2)}}{=} (R_1 t \otimes R_2 t) \otimes (\delta_{T_1}(t) \otimes \delta_{T_2}(t)) \\
&\overset{\text{(Rule5)}}{=} \min\{R_1, R_2\} t \otimes (\delta_{T_1}(t) \otimes \delta_{T_2}(t)) \overset{\text{(Rule6)}}{=} \min\{R_1, R_2\}(t - (T_1 + T_2)) = \beta_{\min(R_1,R_2),T_1+T_2}.
\end{aligned}
\tag{10}
$$

3.3. Use of Convex Analysis

The next approach is based on the theory of convex analysis [8]. Similar considerations related to NC theory can be found at [9]. However, we go another way considering "incomplete" convex functions. In Rockafellar's book [8] all necessary preconditions for the following definition and proofs of the next theorems can be found.

Definition 4 (Conjugate function) *Let* f *be any closed convex function on* R^n.

$f^*(s) := \sup_t \{\langle s,t \rangle - f(t) / t \in \mathrm{dom} f\}$ is called the conjugate of f or *Fenchel-transform*, with $s, t \in R^n$ $\langle \cdot,\cdot \rangle$ the inner product in R^n.

Theorem 1 (Conjugate f^{**}) *The conjugate* f^{**} *of* f^* *is* f:

$$f^{**}(t) = \sup_s \{\langle s,t \rangle - f^*(s) / s \in \mathrm{dom} f^*\}.$$

Theorem 2 (Convolution) *Let* $f_1, \cdots, f_n$ *be proper convex functions on* R^n. *Then*:

$$(f_1 \otimes \cdots \otimes f_n)^* = f_1^* + \cdots + f_n^*.$$

Remark:

In case of the convex NC functions of Φ: $R^n \equiv R^1$. From Theorem 1 and 2 we get

$$(f_1 \otimes \cdots \otimes f_n) = (f_1^* + \cdots + f_n^*)^*.$$

Example:

Again the aim is the concatenation $\beta = \beta_{R_1,T_1} \otimes \beta_{R_2,T_2}$ with

$$
\beta_{R_i,T_i}(t) = \begin{cases} R_i(t - T_i) & : \quad t \geq T_i \quad i = 1,2 \\ 0 & : \quad t < T_i \end{cases}
\tag{11}
$$

First of all, the conjugate of the rate-latency function $\beta_{R,T}$ *i.e.* $\beta^*_{R,T}$ needs to be determined. From definition 4 we know that

$$\beta^*(s) = \sup_t \{st - \beta(t) / t \in \text{dom}\beta\} = \sup_t \{st - R(t-T)\} \tag{12}$$

So, we get:
Case $s < 0$: $\beta^*(s) = \infty$ Case $s = 0$: $\beta^*(s) = 0$

and for Case $s > 0$: $\beta^*(s) = \begin{cases} Ts & : \quad 0 < s \le R \\ \infty & : \quad s > R \end{cases}$

$$\text{Altogether: } \beta^*(s) = \beta^*_{R,T}(s) = \begin{cases} \infty & : \quad s < 0 \\ Ts & : \quad 0 \le s \le R \\ \infty & : \quad s > R \end{cases} \tag{13}$$

Determination of $\beta^{**}(t)$ as conjugate of $\beta^*(s)$ with $s \in \text{dom}\beta^*$:

$$\beta^{**}(t) = \sup_s \{st - \beta^*(s)\} = \sup_s \left\{st - \begin{cases} \infty & : \quad s < 0 \\ Ts & : \quad 0 \le s \le R = \beta_{R,T}(t) \\ \infty & : \quad s > R \end{cases}\right. \tag{14}$$

This certainly confirms Theorem 1.
Applying Theorem 2:
$(\beta_1 \otimes \beta_2)^* = \beta_1^* + \beta_2^*$ and by applying Theorem 1: $(\beta_1 \otimes \beta_2)^{**} = (\beta_1 \otimes \beta_2)$; here $(\beta_1^* + \beta_2^*)^*$.
For the concatenation $\beta_{R_1,T_1} \otimes \beta_{R_2,T_2}$ we obtain
(let $\beta_1 = \beta_{R_1,T_1}$ and $\beta_2 = \beta_{R_2,T_2}$, $T_2 > T_1 > 0$, $R_2 > R_1 > 0$)

$$(\beta_1 \otimes \beta_2)^*(s) = \beta_1^*(s) + \beta_2^*(s) = \begin{cases} \infty & : \quad s < 0 \\ T_1 s & : \quad 0 \le s \le R_1 \\ \infty & : \quad s > R_1 \end{cases} = \begin{cases} \infty & : \quad s < 0 \\ T_2 s & : \quad 0 \le s \le R_2 \\ \infty & : \quad s > R_2 \end{cases} = \begin{cases} \infty & : \quad s < 0 \\ (T_1 + T_2)s & : \quad 0 \le s \le R_1 \\ \infty & : \quad s > R_1 \end{cases} \tag{15}$$

$$\Rightarrow (\beta_1^* + \beta_2^*)^*(t) = \sup_s \left\{st - \begin{cases} \infty & : \quad s < 0 \\ (T_1 + T_2)s & : \quad 0 \le s \le R_1 \\ \infty & : \quad s > R_1 \end{cases}\right. \tag{16}$$

But this is the same structure as in expression for $\beta^{**}(t)$ and therefore $(\beta_1^* + \beta_2^*)^*(t) = \beta_{R_1,T_1+T_2} = \beta_{\min(R_1,R_2),T_1+T_2}$ holds. **Figure 6** depicts this operation.

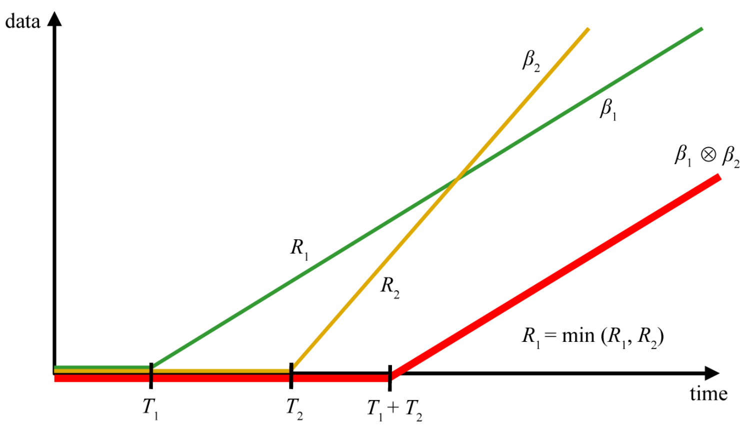

Figure 6. Concatenation of two rate-latency functions.

In conclusion, the use of convex analysis seems to be a further approach to determine the min-plus convolution operation—at least in the context of convex functions like the set of piecewise linear functions where $\beta_{R,T}$ belongs.

Question:

How to deal with non-convex functions $f \in \Phi$. Take Example 3 with input $x = \alpha_{r,b}$ and service curve $\beta_{R,T}$. The min-plus convolution realizes a guaranteed lower bound for the output: $y(t) \ge (\alpha_{r,b} \otimes \beta_{R,T})(t)$.

$$\text{But function } \alpha_{r,b}(t) = \begin{cases} rt+b & : \quad t>0 \\ 0 & : \quad t=0 \end{cases}$$

is not convex because the $\le$ relation in:

$$\alpha_{r,b}\left((1-\lambda)t_0 + \lambda t_1\right) \le (1-\lambda)\alpha_{r,b}(t_0) + \lambda \alpha_{r,b}(t_1) \quad \text{for} \quad t_0 = 0,\ t_1 \in R \quad \text{and} \quad 0 < \lambda < 1$$

is not fulfilled. Therefore Theorem 1 and 2 are not readily applicable.

However, in contrast to [9] other ways are possible. Here the following one is suggested: making functions convex by redefining their values at certain points, e.g. where unnatural discontinuities appear (e.g. $t = 0$). For example, we change $\alpha_{r,b}(t)$ to $\tilde{\alpha}_{r,b}(t)$ such that $\tilde{\alpha}_{r,b}(t)$ is convex and still reflects the arrival curve feature well:

$$\tilde{\alpha}_{r,b}(t) = rt + b \quad \text{for} \quad t \ge 0 \tag{17}$$

Now, $\tilde{\alpha}_{r,b}$ is convex and the principles of convex analysis are applicable. Although $\tilde{\alpha}_{r,b} \notin \Phi$ (since $\tilde{\alpha}_{r,b}(0) \ne 0$) it represents the important burst feature $\Delta t := (t-s) \to 0$ of the arrival curve $\alpha_{r,b}$ (of formula 2):

$$\lim_{t\to s}\{x(t) - x(s)\} \le \lim_{\Delta t\to 0}\{r\cdot\Delta t + b\} = \lim_{\Delta t\to 0}\alpha_{r,b}(\Delta t) = \lim_{\Delta t\to 0}\tilde{\alpha}_{r,b}(\Delta t) = b.$$

Since $\tilde{\alpha}_{r,b} \ne \alpha_{r,b}$ the convolution of

$$\tilde{\alpha}_{r,b} \otimes \beta_{R,T} = \left\{b + r(t-T)^+\right\} \quad \text{differs from} \quad \alpha_{r,b} \otimes \beta_{R,T} = \left\{b + r(t-T)^+\right\} \wedge \left\{R(t-T)^+\right\}.$$

The expression $\left\{b + r(t-T)^+\right\}$ is identical in both formulas. When considering rate-latency functions (c.f. Example 2) where an incoming input is served with minimal rate R after a maximal delay T, the output y is lower bounded not only by $\left\{b + r(t-T)^+\right\}$ but by $\min\left\{b + r(t-T)^+, R(t-T)^+\right\}$ Of course, in the long run for $t \to \infty$ the term $\left\{b + r(t-T)^+\right\}$ is the dominant one, as you can see in **Figure 5**.

4. Conclusions

The most important operation within the Network calculus is the min-plus convolution. But the calculation of this fundamental operation still is complex and error-prone. For this reason we introduced other computation approaches to perform the convolution: here called *Use of algebraic laws* and *Use of convex analysis*. In order to apply the convex analysis we transform non-convex into convex functions (e.g. keeping the *token-bucket* arrival curve property) which is different, for example, from the approach of [9].

This article reflects work in progress and the presented examples ought to demonstrate these techniques. In future work we will continue to investigate the procedures of subsection 3.2 and 3.3 to answer the following questions: Can we extend the principles to some classes of non-convex functions or even to mixed classes of convex and non-convex functions? Which kinds of applications are suited considering algebraic laws and convex analysis? Are there even cases where a combination of both methods is beneficial? To all of these, we want to find answers in future work.

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