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Bifurcation and Chaos in Delayed Cellular Neural Network Model

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Abstract

This paper deals with control of chaotic behavior of a delayed Cellular Neural Network (DCNN) model which is a one-dimensional regular array of four cells with continuous activation function. We investigate different dynamical behaviors including limit cycle, torus, and chaos for different range of weight parameters of the system. Regarding synaptic weight as parameter, Hopf bifurcations are obtained in the system without delay. In the delayed model condition for the Global asymptotic stability of the equilibrium point is presented. Numerical simulation and results are given to show the role of delay in chaos control of the CNNs.

1. Introduction

An artificial neural network (ANN) is a mathematical model that can mimic the biological process of human brain. An ANN can be presented by a directed graph composed of neurons as nodes and synapses or nerves as edges with an algorithm that show how impulses are conducted through the network. There are various types of network architecture. Cellular Neural Networks (CNNs) introduced by Chua and Yang [1] is one dimensional regular array of locally connected circuits (called cells), they are able to display various dynamics, such as limit cycle, torus, especially chaos. Neural oscillation can arise from interactions between different brain areas. Time delays can play an important role here because nerve impulse takes a time to travel the length of the axon to the target neuron which in turn takes the time to summate their inputs and produce response. Brain areas are assumed to be bidirectionally (BAM) coupled forming delayed feedback loops. An example of such a feedback loop is the connection between the thalamus and cortex [2] [3]. The malfunctioning of the neural system is often related to changes in the delay parameter causing unmanageable shifts in the phases of the neural signals. Some of the visible mechanical consequences could be explained: such as tremor in the fingers, difficulties in balancing; the increased danger of falling over for elderly people or even motion disorders in the case of bursts of epilepsy, Parkinson disease and so on [4].

Physiological experiments suggest that the main components of neural activity in olfactory systems are chaotic. In the olfactory system, the phase transition has the appearance of a change in the EEG from a chaotic, aperiodic fluctuation to a more regular nearly periodic oscillation. Chaotic activity enables the rapid state transitions

essential for information processing. Without this ability, the brain could not quickly concern itself with a new task. Thus, we can assume chaos for the rapid transitions between perceptual states. Without it, observation could be extremely slow. It is supposed that brains injured by Alzheimer's disease have electrophysiologically inactive neurons and/or synapses and show decreased chaotic behavior. Results of various research works in this area support the assumption that chaos plays an important role in brain function, for instance, learning and memory [5]-[13].

This paper is structured as follows: In Section 2 we analyze a new family of simplified 4-CNNs, as shown in **Figure 1**. In Section 3, we investigate different dynamical behaviors including chaos for different range of weight parameters of the system. Using Matlab software we can observe limit cycle, torus and chaos in a new family of 4-CNNs for different range of adjustable parameters of main diagonal of the weight matrix. Numerical results are discussed showing changes of dynamics of the system from unstable to stable (chaos control) one through bifurcation due to variation in the individual weight parameter. In Section 4, a delayed Cellular Neural Networks (DCNNs) is considered where the chaotic dynamics is controlled [see **Figure 2**]. A sufficient condition for the Global asymptotic stability of the equilibrium point in the delayed model is found. Finally some concluding remarks have been drawn on the implication of our results in the context of related work mentioned above.

2. Limit Cycle, Torus and Chaos in 4-CNNs

In this section, the dynamics of this system can be described by the following ordinary differential equations:

$$\dot{x}_i = -c_i x_i(t) + \sum_{j=1}^{4} W_{ij} f\left[x_j(t)\right], c_i > 0 \quad i = 1, 2, 3, 4 \tag{1}$$

where $W = (W_{ij})$ is a 4×4 weight matrix describing the strength of the connection from j th neuron on the i th neuron. $f[x_i(t)]$ is the output state of the i th neuron at time t.

For numerical simulation we consider $c = [1.1, \quad 1.5, \quad 4.5, \quad 15]^T$, $f(x_i) = \tanh(x_i)$ and the weight matrix W to be

$$W = \begin{pmatrix} 2.1 & 2.17 & 0 & 0 \\ -3.51 & 1 & 3.11 & 0 \\ 0 & -3 & p_3 & -1.1 \\ 0 & 0 & 20 & 30 \end{pmatrix} \tag{2}$$

where $p_3 = W_{33}$ is an adjustable parameter.

The phase portraits (**Figures 3-5**) show the evidence that there exist limit cycles, torus and chaotic attractors with different values of p_3. In **Figure 6**, chaotic nature is controlled by taking $p_3 = 6.4(> 6.1)$ which is also confirmed by bifurcation diagram (see **Figure 7**) with respect to parameter p_3.
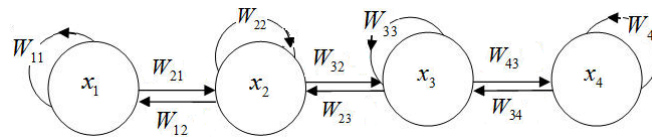

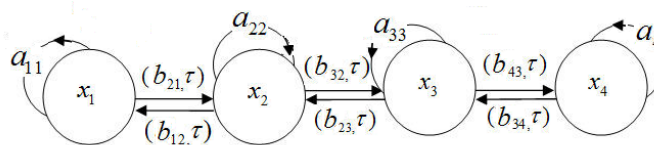**Figure 1.** Architecture of the network model 4-cell Cellular Neural Networks.

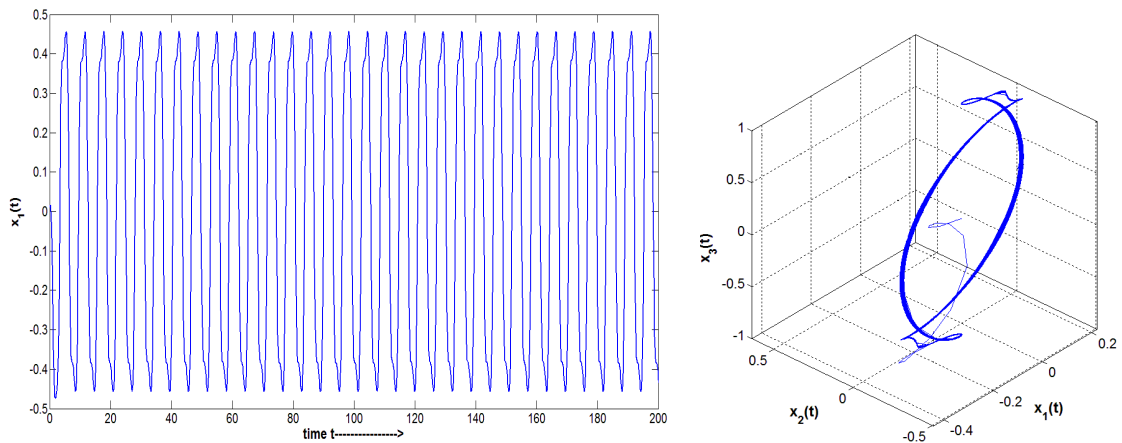**Figure 2.** Architecture of the network model 4-cell delayed Cellular Neural Networks.

Figure 3. Solution trajectory showing periodic behavior and corresponding phase portrait in (x_1, x_2, x_3) space with parameter $p_3 = 2$.
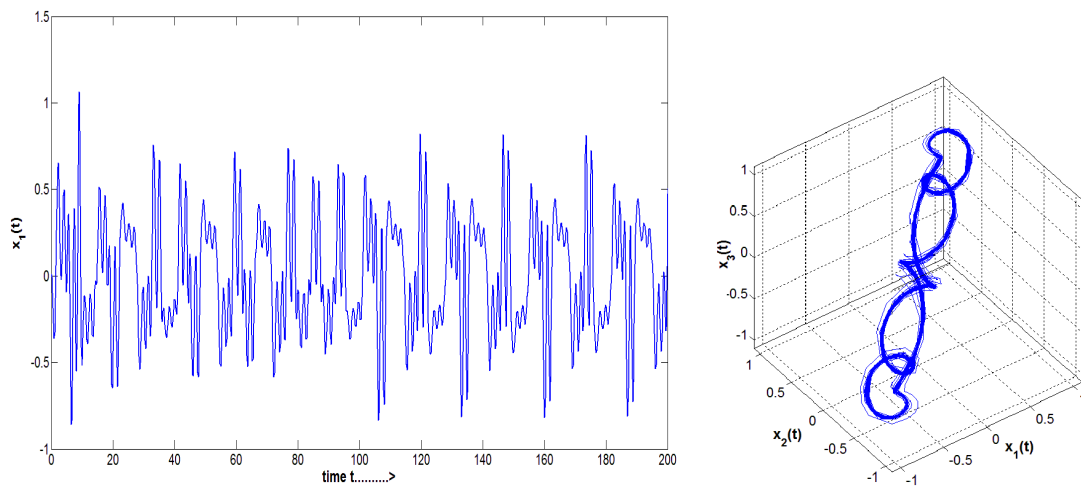

Figure 4. Solution trajectory showing chaotic behavior and corresponding phase portrait in (x_1, x_2, x_3) space with parameter $p_3 = 4.4$. $p_3 = 6.4$.
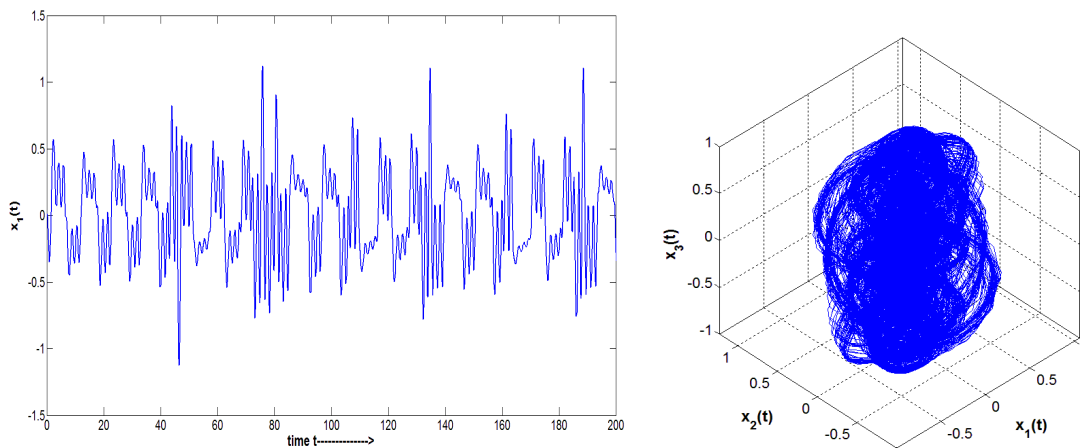

Figure 5. Solution trajectory showing chaotic behavior and corresponding phase portrait in (x_1, x_2, x_3) space with parameter $p_3 = 5.4$.
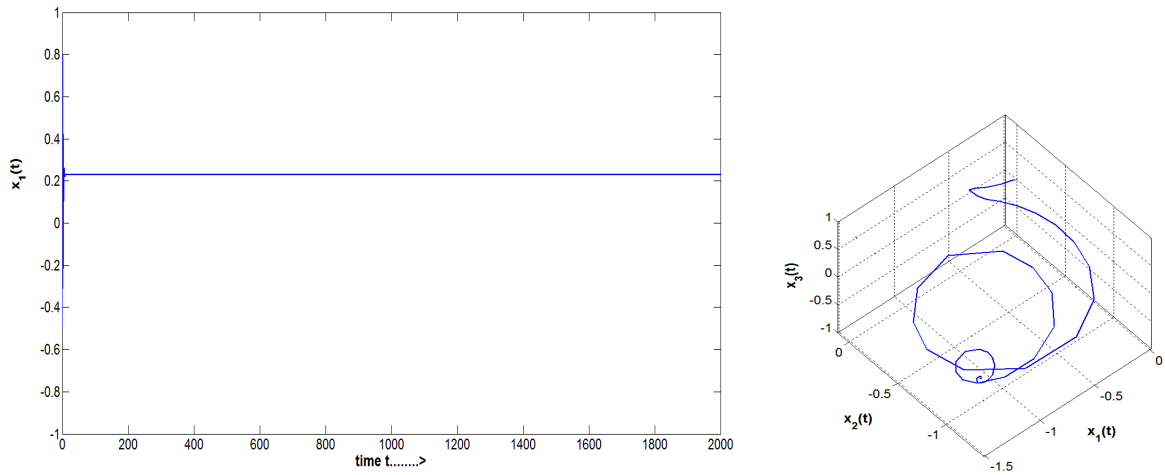
Figure 6. Solution trajectory showing stable behavior and corresponding phase portrait in (x_1, x_2, x_3) space with parameter $p_3 = 6.4$.
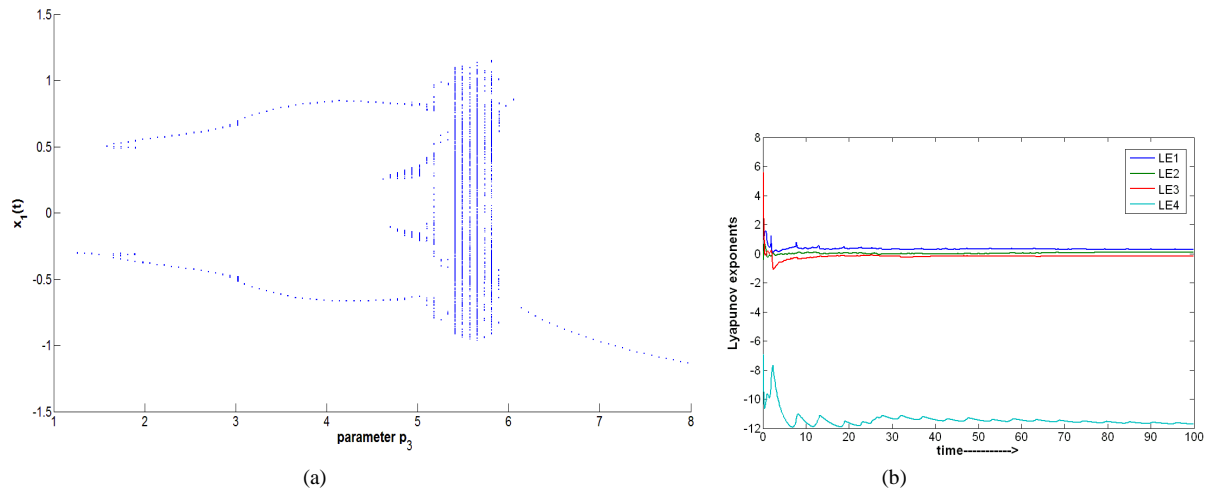

(a)   (b)

Figure 7. (a) Bifurcation diagram with respect to parameter p_3, (b) The four Lyapunov exponents when $p_3 = 5.4$.

Again, if we consider all other self-synaptic weights like $p_4 = W_{44}$ as adjustable parameter, then we get similar type of behavior which is shown in the following **Figure 8**.

3. Mathematical Model with Time Delay and Global Stability

We consider an artificial 4-neuron network model of Cellular Neural Networks time delayed connections between the neurons by the delay differential equations:

$$\dot{x}_i = -c_i x_i(t) + a_{ii} f\left[x_i(t)\right] + \sum_{\substack{j=1 \\ i \neq j}}^{4} b_{ij} f\left[x_j(t-\tau)\right], c_i > 0 \quad i = 1, 2, 3, 4 \tag{3}$$

with $x_i(t)$ is the activation state of i th neuron at time t, $f[x_i(t)]$ is the output state of the i th neuron at time t, a_{ii} is self-synaptic weight, b_{ij} is the strength of the j th neuron on the i th neuron at time $(t - \tau)$, τ is the signal transmission delay and c_i (decay rate) is the rate with which the i th neuron will reset its potential to the resting state in isolation when disconnected from the network. In the following, we assume that each of the relation between the output of the cell f and the state of the cell possess following properties:

(H1) f is bounded on R ;

(H2) There is a number $\mu > 0$ such that $\left| f(u) - f(v) \right| \le \mu \left| u - v \right|$ for any $u, v \in R$.

It is easy to find from (H2) that f is a continuous function on R. In particular, if output state of the cell is described by $f(x_i) = \tanh(x_i)$, then it is easy to see that the function f clearly satisfy the hypotheses (H1) and (H2).

Theorem 1 For the DCNN (3), suppose output of the cell $f[x_i(t)]$ satisfy the hypotheses (H1) and (H2) above and there exist constants α_{ij}^*, β_{ij}^*, $\eta_{ij}^*, \xi_{ij}^*, \alpha_{ij}, \beta_{ij}$, $\eta_{ij}, \xi_{ij} \in R$ such that

$$\left(\left| a_{ij} \right|^{2\eta_{ij}^*} \mu^{2\alpha_{ij}^*} + \left| a_{ii} \right|^{2\xi_{ij}^*} \mu^{2\beta_{ij}^*} + \sum_{\substack{j=1 \\ i \ne j}}^{4} \left| b_{ij} \right|^{2\eta_{ij}} \mu^{2\alpha_{ij}^*} + \left| b_{ji} \right|^{2\xi_j i} \mu^{2\beta_{ji}} \right) < 2c_i \qquad (4)$$

$i = 1, 2, 3, 4$, in which α_{ij}^*, β_{ij}^*, η_{ij}^*, ξ_{ij}^*, α_{ij}, β_{ij}, η_{ij}, ξ_{ij} $(i, j = 1, 2, 3, 4)$ are any real constants with $\alpha_{ij}^* + \beta_{ij}^* = 1$, $\eta_{ij}^* + \xi_{ij}^* = 1$, $\alpha_{ij} + \beta_{ij} = 1$, $\eta_{ij} + \xi_{ij} = 1$, μ is a constant number of the hypothesis (H2) above. Then the equilibrium x^* of the DCNN (3) is also globally asymptotically stable independent of delays.

4. Conclusion

This paper addresses chaos control by varying the adjustable parameter (self-connection weights) and introducing time delay [see **Figure 9**] in original model of 4-cell CNNs. These two approaches stabilize unstable
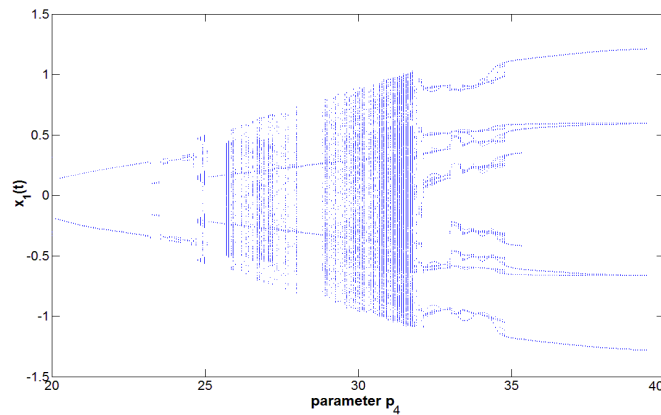

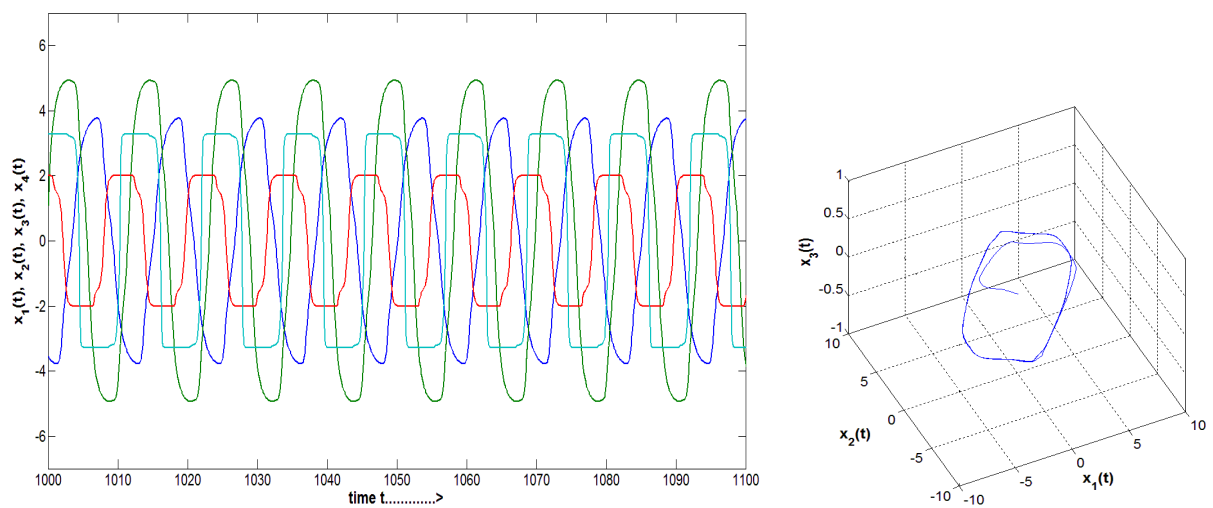**Figure 8.** Bifurcation diagram with respect to parameter p_4.

Figure 9. Control of chaos: Solution trajectory showing periodic behavior corresponding phase portrait in (x_1, x_2, x_3) space with parameter $p = 5.4$.
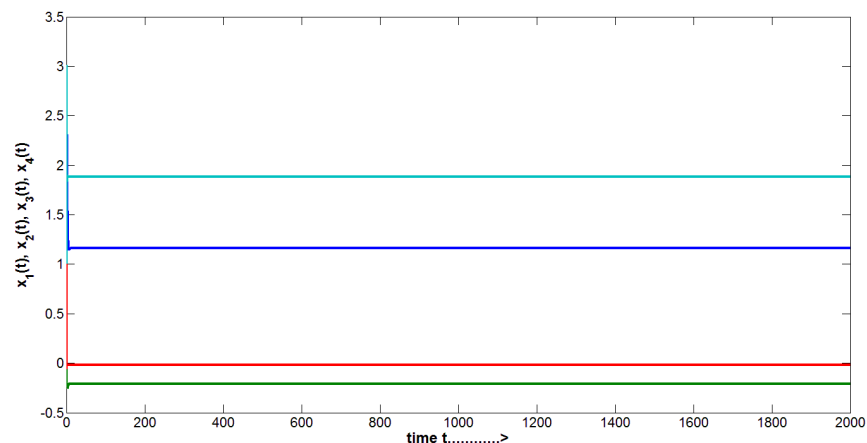
Figure 10. Solution trajectory showing stable behavior satisfying globally asymptotically stable conditions.

periodic orbits that a chaotic attractor encompasses. Furthermore, different from the work of Yang and Huang [14] where adjustable parameter lies off the main diagonal (self-connection weights), we consider it on the main diagonal which seems more meaningful for controlling chaos in CNNs. Besides global stability criteria ensures asymptotically stable behavior for DCNN model [see **Figure 10**]. The results obtained enforce more restrictions on synaptic weight and decay parameters.

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