

Gravitation in Flat Space-Time and General Relativity

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Abstract

A covariant theory of gravitation in flat space-time is stated and compared with general relativity. The results of the theory of gravitation in flat space-time and of general relativity agree for weak gravitational fields to low approximations. For strong fields the results of the two theories deviate from one another. Flat space-time theory of gravitation gives under some natural assumptions non-singular cosmological models with a flat space. The universe contracts to a positive minimum and then it expands for all times. Shortly, after the minimum is reached, the cosmological models of two theories approximately agree with one another if models in general relativity with zero curvature are considered. A flat space is proved by experiments.

Keywords

Gravitation, Flat Space-Time, Cosmology, Big Bounce, No Big Bang, Flat Space

1. Introduction

A previously studied covariant theory of gravitation in flat space-time is stated [1]. The energy-momentum of the gravitational field is a tensor. The source of the gravitational field is the total energy-momentum of all the fields inclusive that of gravitation. This is quite different from general relativity for which the energy-momentum of gravitation is not a tensor. Hence, the energy-momentum of the gravitational field cannot explicitly appear as source by virtue of the covariance of general relativity. Therefore, the Ricci tensor is used as differential operator yielding a non-Euclidean geometry. An extensive study exists of flat space-time theory of gravitation. It follows that the results of the two theories agree with one another for weak field approximations but there are differences if the gravitational fields are strong. Therefore, the theory of flat space-time theory is applied to homogeneous, isotropic cosmological models where only matter and radiation are considered. A cosmological constant could also be included [2]. The universe is non-singular under the assumption that the sum of the density parameters is a little bit greater than one. In the beginning of the universe there is no matter and no radiation. The universe contracts to a small minimum creating matter and radiation with very high temperature. All the densities of matter and radiation are always finite. After the minimum is reached the universe expands for all times. Shortly after the time when the minimum is reached the results of the two theories approximately agree if a vanishing curvature of general relativity is assumed. The space of flat space-time theory of gravitation is flat, *i.e.* there is no necessity of inflation in the beginning of the universe in contrast to general relativity where strong

curvature exists in the neighbourhood of the singularity which corresponds to the minimum of the universe by the use of gravitation in flat space-time.

2. Gravitation in Flat Space-Time

The covariant theory of gravitation in flat space-time [1] is shortly summarized. The metric of flat space-time is

$$(ds)^2 = -\eta_{ij} dx^i dx^j \quad (2.1)$$

where η_{ij} is a symmetric tensor. In the special case where (x^1, x^2, x^3) are the Cartesian coordinates, $x^4 = ct$ and

$$(\eta_{ij}) = (1, 1, 1, -1) \quad (2.2)$$

the space-time metric is the pseudo-Euclidean geometry. We put

$$\eta = \det(\eta_{ij}). \quad (2.3)$$

The gravitational field is described by a symmetric tensor (g_{ij}) . Let (g^{ij}) be defined by

$$g_{ik} g^{kj} = \delta_i^j \quad (2.4)$$

and put analogously to (2.3)

$$G = \det(g_{ij}) \quad (2.5)$$

Then, the proper-time τ is defined similarly to (2.1) by the quadratic form

$$(d\tau)^2 = -g_{ij} dx^i dx^j. \quad (2.6)$$

General relativity uses Equation (2.6) as metric.

The Lagrangian of the gravitational field is given by

$$L(G) = - \left(\frac{-G}{-\eta} \right)^{1/2} g_{ij} g_{kl} g^{mn} \left(g_{/m}^{ik} g_{/n}^{jl} - \frac{1}{2} g_{/m}^{ij} g_{/n}^{kl} \right) \quad (2.7)$$

where the bar / denotes the covariant derivative relative to the metric (2.1).

We mention that a Lagrangian of the form (2.7) for general relativity doesn't exist because the metric is given by Equation (2.6).

Put

$$\kappa = \frac{4\pi k}{c^4}. \quad (2.8)$$

Then, the energy-momentum of the gravitational field is

$$T(G)^i_j = \frac{1}{8\kappa} \left(\left(\frac{-G}{-\eta} \right)^{1/2} g_{kl} g_{mn} g^{ir} \left(g_{/j}^{km} g_{/r}^{ln} - \frac{1}{2} g_{/j}^{kl} g_{/r}^{mn} \right) + \frac{1}{2} \delta_j^i L(G) \right) \quad (2.9)$$

which is a tensor for this theory.

The energy-momentum of general relativity is not a tensor.

The energy-momentum tensor of matter is

$$T(M)^i_j = (\rho + p) g_{jk} u^k u^i + \delta_j^i p c^2. \quad (2.10)$$

Here, ρ , p and (u^i) denote the density, the pressure and the four-velocity $\left(\frac{dx^i}{d\tau} \right)$ of matter. It holds by virtue of Equation (2.6)

$$c^2 = -g_{ij} u^i u^j. \quad (2.11)$$

We define the covariant differential operator of order two in divergence form

$$D_j^i = \left\{ \left(\frac{-G}{-\eta} \right)^{1/2} g_{jk} g^{mn} g_{/n}^{ki} \right\}_{/m} . \quad (2.12)$$

Then, the Lagrangian gives the field equations

$$D_j^i - \frac{1}{2} \delta_j^i D_k^k = 4\kappa T_j^i, \quad (2.13)$$

with the relation

$$T_j^i = T(M)_j^i + T(G)_j^i \quad (2.14)$$

i.e. T_j^i is the total energy-momentum tensor inclusive that of the gravitational field.

The equations of motion of matter are

$$T(M)_{i/k}^k = \frac{1}{2} g_{kl/i} T(M)^{kl} \quad (2.15)$$

where

$$T(M)^{ij} = g^{jk} T(M)_k^i \quad (2.16)$$

is the symmetric energy-momentum tensor.

In addition, we have the conservation law of the total energy-momentum

$$T_{i/k}^k = 0. \quad (2.17)$$

The field Equations (2.13) with Equation (2.14) and the equations of motion (2.15) imply the conservation law of the total energy-momentum Equation (2.17). Conversely, the field Equations (2.13) with Equation (2.14) and the conservation law of the total energy-momentum Equation (2.17) yield the equations of motion Equation (2.15). All the stated equations are covariant.

General relativity is formally similar to the Equation (2.13) but it replaces

$$D_j^i = R_j^i, T_j^i = T(M)_j^i \quad (2.18)$$

where R_j^i is the Ricci tensor and T_j^i contains only the matter tensor. Hence, the equations of general relativity are also covariant relative to the metric given by Equation (2.6) but we get a non-Euclidean geometry. In addition, the condition of Einstein that any sort of energy is equal to matter is not fulfilled because gravitational energy is not contained as source.

It is worth mentioning that the theory of Maxwell is analogous to flat space-time theory of gravitation because the source of the electro-magnetic potentials (A_i) is the electrical four-current and the differential operator for the potentials is in divergence form of order two.

3. Homogeneous, Isotropic, Cosmological Model

Let us use the pseudo-Euclidean metric Equation (2.1) with Equation (2.2).

The matter tensor Equation (2.10) is given with

$$u^i = 0 (i = 1, 2, 3) \quad (3.1)$$

and

$$p = p_m + p_r, \rho = \rho_m + \rho_r \quad (3.2)$$

where the indices m and r denote matter and radiation.

The equations of state are

$$p_m = 0, p_r = \frac{1}{3} \rho_r. \quad (3.3)$$

The gravitational field has by virtue of Equation (3.1), the homogeneity and the isotropy the form

$$\begin{aligned}
g_{ij} &= a^2(t), (i = j = 1, 2, 3) \\
&= -1/h(t), (i = j = 4) \\
&= 0, (i \neq j)
\end{aligned} \tag{3.4}$$

The four-velocity is given by

$$(u^i) = (0, 0, 0, c\sqrt{h}). \tag{3.5}$$

The initial conditions at present time $t_0 = 0$ are

$$\begin{aligned}
a(0) &= h(0) = 1, a'(0) = H_0, h'(0) = h'_0, \\
\rho_m(0) &= \rho_{m0}, \rho_r(0) = \rho_{r0}.
\end{aligned} \tag{3.6}$$

Here, the prime denotes the t -derivative, H_0 is the Hubble constant and h'_0 doesn't appear by the use of general relativity because $h(t) = 1$. The condition $h(t) = 1$ is not possible by flat space-time theory of gravitation. Then, the field equations and the conservation of the total energy give after longer calculations

$$\left(\frac{a'}{a}\right)^2 = \frac{H_0^2}{(2\kappa c^4 \lambda t^2 + \varphi_0 t + 1)^2} (-\Omega_m K_0 + \Omega_r a^2 + \Omega_m a^3) \tag{3.7}$$

where λc^2 is the constant of the conservation of the total energy, Ω_m and Ω_r are the density parameters of matter and radiation, and

$$\varphi_0 = 3H_0 \left(1 + \frac{1}{6} \frac{h'_0}{H_0}\right), \Omega_m K_0 = \frac{1}{12} \left\{ \frac{8\kappa c^4 \lambda}{H_0^2} - \left(\frac{\varphi_0}{H_0}\right)^2 \right\}. \tag{3.8}$$

Furthermore, it holds

$$a^3 \sqrt{h} = 2\kappa c^4 \lambda t^2 + \varphi_0 t + 1. \tag{3.9}$$

It easily follows that non-singular solutions exist under the condition

$$\Omega_m K_0 > 0. \tag{3.10}$$

The inequality (3.10) implies

$$2\kappa c^4 \lambda t^2 + \varphi_0 t + 1 > 0 \text{ for all } t \in \mathbb{R}. \tag{3.11}$$

Relation (3.7) gives at present time $t_0 = 0$

$$\Omega_r + \Omega_m = 1 + \Omega_m K_0. \tag{3.12}$$

Let us furthermore assume

$$\Omega_m K_0 \ll 1. \tag{3.13}$$

Relation (3.7) implies the existence of a constant a_1 with $0 < a_1 \ll 1$ and

$$\Omega_r a_1^2 + \Omega_m a_1^3 = \Omega_m K_0. \tag{3.14}$$

Hence, there exists a time $t_1 < t_0 = 0$ such that

$$\frac{a'}{a} = \mp \frac{H_0}{2\kappa c^4 \lambda t^2 + \varphi_0 t + 1} (-\Omega_m K_0 + \Omega_r a^2 + \Omega_m a^3)^{1/2} \tag{3.15}$$

Here, the upper sign holds for $t \leq t_1$ and implies a contraction of the space till the time t_1 with $a(t_1) = a_1$ and the lower sign holds for $t \geq t_1$ yielding an expansion of space.

Let us introduce the time ζ by

$$d\zeta = \frac{1}{\sqrt{h(t)}} dt. \tag{3.16}$$

Then, Equation (3.15) for $t \geq t_1$ together with Equation (3.9) can be rewritten

$$\frac{1}{a} \frac{da}{d\zeta} = +H_0 \left(-\frac{\Omega_m K_0}{a^6} + \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} \right)^{1/2}. \quad (3.17)$$

Equation (3.17) is under the assumptions (3.13) and $a(t) > a_1$ the differential equation of general relativity for a universe with zero curvature.

Therefore, flat space-time theory of gravitation and general relativity give approximately the same result for the expanding, flat space. But in the beginning of the universe the results of both cosmological models are quite different, *i.e.* we have a bounce and not a big bang.

It is worth mentioning that a cosmological constant could also be included without changing the statements.

Furthermore, the cosmological models of gravitation in flat space-time also permit the interpretation of a non-expanding space. For this case the redshift is explained by the transformation of the different sorts of energy into one another whereas the conservation of the total energy is valid.

It is well-known that general relativity is only experimentally verified for weak fields.

More details of the theory of gravitation in flat space-time and the received results can be found in several articles of the author and in the book “A theory of gravitation in flat space-time” which appears soon in Science PG [3].

References

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